Reputation, Quality Choices and Optimal Product Discontinuation Policy

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- Imperfect information causes a wide range of market imperfections.
- Market failure when the product's quality is unobservable.
- Experience goods:able to evaluate the quality of the products after they have been bought and consumed
  - Durables: cars or used cars, electronic appliances.
  - Non-durables: wines.
  - Services:airlines or specialist professions.
- Problem occurs even when characteristics are observable but utility from the products requires a match between product features and consumer's taste.
- Solution?
  - Many mechanisms have been proposed
  - One possible mechanism: Reputation Mechanism

- Repeated interactions: frequently purchase the product and observe its realised quality, although still not able to observe the firm's efforts in quality
- The consumer's experiences provide imperfect information on a combination of
  - The firm's efforts,
  - A suiting of their tastes, and
  - Sheer luck.
- This collection of experiences forms an expectation of quality, and is the definition of reputation for this paper.

- Learning process gradually reveals information on the quality to the market but the firm has choice to stop the product before the quality is completely revealed.
    - Ex. the series of battery recalls from Lenovo in September 2006 (ThinkPad T43 caught fire at the LAX), Dell and Apple Computer in August 2006 (batteries were overheating or exploding, causing fires even when the machines were turned off)
- Products could also be discontinued by...
    - Advancement of a new technology e.g. computer processors.
    - Competition strategies e.g. Xbox 360 vs PS3.
    - Deterministic discontinuation policies e.g. Japanese car vs. German car manufacturers.
- We will be focusing on discontinuation policy from the realisation of quality.

- Quality choice and product discontinuation problems under a moral hazard when information about product quality is imperfect but symmetric.
- The firm's intended quality choice is difficult to control and is subject to a random shock.

- Principal-Agent framework similar to Career Concerns models (Holmström 1982)
- Incorporating decision of a firm to introduce/terminate its product
- A single product firm serving a large number of short-live identical consumers
- Firm chooses once and for all its desired level of product quality, q
- Actual product quality is $\tilde{q} = q + \gamma$ where $\gamma \sim N\left(0, \sigma_q^2\right)$ is an idiosyncratic quality disturbance term or a "match".

- Consumers are identical and live for only one period.
- Quality perceived by consumers is a noisy signal of the actual quality i.e.

$$u_t = \tilde{q} + \varepsilon_{u_t} \text{ where } \varepsilon_{u_t} \sim N\left(0, \sigma_u^2\right)$$

- ε_{u_t} can be interpreted as the consumers' random experience of using the product.

- Consumers make an imperfect report of their experience,

$$r_t = u_t + \varepsilon_{r_t}$$

where $\varepsilon_{r_t} \sim N\left(0, \sigma_r^2\right)$ is reporting error.

- The sequence of reports, $\{r_s\}_1^t$,are publicly observable.
- consumers are willing to pay up to their expected utility from the product. That is,

$$\begin{aligned} p_t &= E\left[u_t \left| r_1^{t-1}\right.\right] \\ &= E\left[q \left| r_1^{t-1}\right.\right] + E\left[\gamma \left| r_1^{t-1}\right.\right]. \end{aligned} \tag{1}$$

- If a product has life T, a risk neutral firm chooses quality level q^* satisfying

$$q^* = \arg\max_q \sum_{t=1}^{T} \delta^{t-1} \left[E\left[p_t \right] - C\left(q \right) \right] \tag{2}$$

- The market is not able to observe the firm's choice of quality, q, directly, it can be inferred by solving firm's problem (2)

At period $t+1$, observing r_t in equilibrium will be equivalent to observing the sequence

$$z_t \equiv r_t - q^* = q + \gamma + \varepsilon_{u_t} + \varepsilon_{r_t} - q^* \\ = q + \gamma + \varepsilon_t - q^* \tag{3}$$

At equilibrium $q = q^*$, then

$$z_t = \gamma + \varepsilon_t \tag{4}$$

where the composite noise, $\varepsilon_t \sim N(0, \sigma_\varepsilon^2 = \sigma_u^2 + \sigma_r^2)$, is the combination of two error terms, $\varepsilon_{u_t} + \varepsilon_{r_t}$.

Information structure is imperfect but symmetrics: γ is unknown to both firm and consumers......
........they share the same initial prior belief $m_1 = 0$ with precision h_1.

- Define the precision of ε_t as $h_\varepsilon = 1 \big/ \sigma_\varepsilon^2$ and precision of belief at time t as $h_t = 1 \big/ \sigma_t^2$ where σ_t^2 denotes variance of prior belief about γ at time t.
- The posterior distribution of γ will still be normal distribution with mean, variance and precision given by Bayes' rule as

$$m_{t+1} = \frac{h_t m_t + h_\varepsilon z_t}{h_t + h_\varepsilon} = \frac{h_1 m_1 + h_\varepsilon \sum_{s=1}^{t} z_s}{h_1 + t h_\varepsilon} \tag{5}$$

$$h_{t+1} = h_\varepsilon + h_t = t h_\varepsilon + h_1 \tag{6}$$

- Consumers' willingness to pay (1) can be rewritten as

$$p_t = E\left[u_t \middle| r_1^{t-1}\right] = q^* + E\left[\gamma \middle| r_1^{t-1}\right] = q^* + m_t \tag{7}$$

- Firm's problem (2) becomes

$$\max_q \sum_{t=1}^{T} \delta^{t-1} \left[q^* + \frac{h_1 m_1}{h_t} + \frac{(t-1)h_\varepsilon}{(t-1)h_\varepsilon + h_1} (m_1 + q - q^*) - C(q) \right] \tag{8}$$

- FOC ...

$$\frac{1-\delta}{1-\delta^T} \sum_{t=1}^{T} \delta^{t-1} \left[\frac{(t-1)h_\varepsilon}{(t-1)h_\varepsilon + h_1} \right] = C'(q^*) \tag{9}$$

- When the firm becomes more patient or δ increases, futures become more important; the incentive to bias consumers' learning increases, and higher quality is produced.
- As the precision of the first period prior belief, h_1, increases, implicit incentive to bias consumers'learning and hence equilibrium quality decreases.

- $h_\varepsilon = 1 \big/ \left(\sigma_u^2 + \sigma_r^2\right)$ can be viewed as the parameter determining the speed of consumers' inference about actual product quality or how noisy is the environment.

  $$m_{t+1} = \frac{h_t m_t + h_\varepsilon z_t}{h_t + h_\varepsilon}$$

  Low h_ε means a not very informative signal and consumers should put less weight on this when updating their belief.
- The quality offered by the firm increases with the length of time that the product is in the market

Decision to Discontinue

Deterministic Discontinuation Period

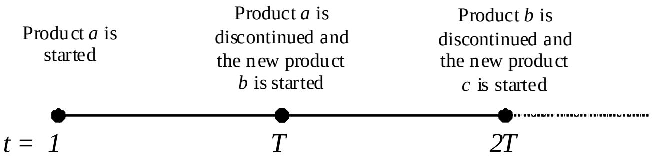


Figure: Deterministic Product Discontinuation Policy

Decision to Discontinue

Deterministic Discontinuation Period

Whenever a firm releases its new product, the quality choices must satisfy the FOC (9):

$$\frac{1-\delta}{1-\delta^{T}} \sum_{t=1}^{T} \delta^{t-1} \left[\frac{(t-1) h_{\varepsilon}}{(t-1) h_{\varepsilon} + h_{1}} \right] = C' \left(q^{*} \right)$$

Assumption

Mean and precision of initial prior belief about quality noise at the time of launching are the same for every product i.e.,

$$m_1 = m_{t_0}^b$$
$$h_1 = h_{t_0}^b$$

for all new product b launched at time t_0.

Remark

Under Assumption 1, optimal quality choices are identical for all products.

Let $y_t^a \equiv q^a + m_t^a - C\left(q_a^*\right)$ be the period t profit when product a has quality q^a. The total discounted return is

$$V = \sum_{i=1}^{T} \delta^{i-1} y_i^a + \delta^T \sum_{j=1}^{T} \delta^{j-1} y_j^b + \delta^{2T} \sum_{k=1}^{T} \delta^{k-1} y_k^c + \ldots$$

The expected return is

$$E\left[V\right] = E \sum_{i=1}^{T} \delta^{i-1} y_i^a + \delta^T E\left[\sum_{j=1}^{T} \delta^{j-1} y_j^b + \delta^T \sum_{k=1}^{T} \delta^{k-1} y_k^c + \ldots\right]$$

- Since quality choices are identical for all products by Remark 2,

$$E[V] = E\sum_{i=1}^{T}\delta^{i-1}y_i + \delta^T E[V]$$

Solving for $E[V]$...

$$E[V] = \frac{q^*(T) - C(q^*(T))}{(1-\delta)} \tag{10}$$

Choosing T that maximise $E[V]$....but T has no direct influence on the value function.... it has an indirect influence through optimal quality choice.

- Now the problem is to choose quality that maximises $E[V]$.

Theorem

The optimal deterministic discontinuation period is to set $T \to \infty$.

- A Product with a longer selling period is of better quality
- Better to produce one product with a relatively long life than to produce many shorter life products...

- In fact, the firm never stop its first product
- This simple model cannot explain why different firms choose different selling periods for their products.

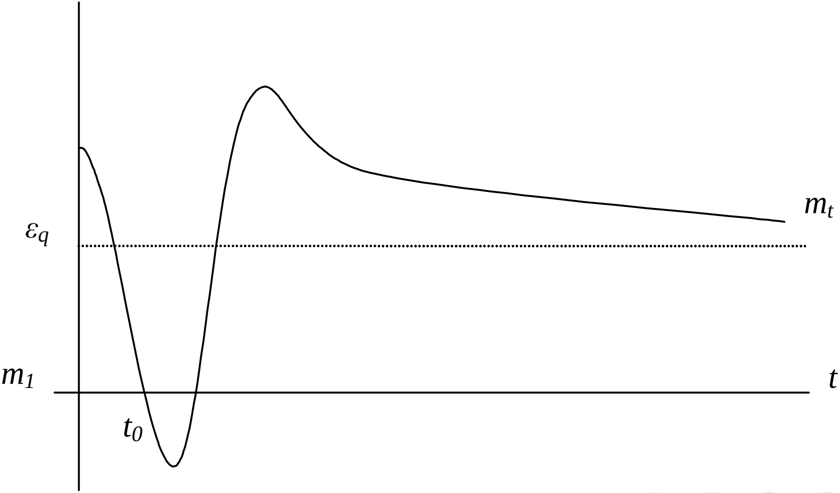
- If a firm can terminate its old product and relaunch the new *one once and only once*....
- ....when should it stop the old product?

Proposition

There exists a unique m^* such that a new product is launched at period t_0 if and only if $m_{t_0} < m^*$ and if all new products have the same initial prior belief as stated in Assumption 1, $m^* = m_1$.

Decision to Discontinue

When to Start a New Product: A Simple Illustration for Stochastic Discontinuation Period



- Stochastic discontinuation period: product's lifespan is not fixed in advance...
- ...product discontinuation policy based on realised quality or product reputation.
- Two decision variables: Discontinuation policy and quality choice.
- Firm can stop and restart products as many time as it wants.

Product Discontinuation Policies

Structure of Product Discontinuation Problem

- In each period, after observing prior m_t, firm chooses whether to stop or continue the product.
- The strategy of the firm is given by

$$\psi : \mathbb{R} \longrightarrow \{0, 1\} \, .$$

- The optimal product stopping policy consists of the sequence of these functions

$$\bar{} = (\psi_1, \psi_2, \ldots) \, .$$

that maximises the total expected discounted return

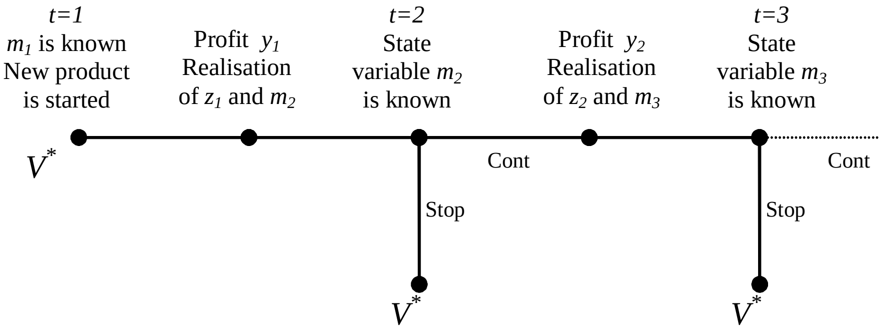


Figure: Structure of the stopping problem

- Let

$$y_t^a \equiv q_a^* + m_t^a - C\left(q_a^*\right)$$

- The total discounted return is

$$V = \sum_{i=1}^{T_a} \delta^{i-1} y_i^a + \delta^{T_a} \sum_{j=1}^{T_b} \delta^{j-1} y_j^b + \delta^{T_a+T_b} \sum_{k=1}^{T_c} \delta^{k-1} y_k^c + \ldots$$

- The expected return is

$$E\left[V\right] = E \sum_{i=1}^{T_a} \delta^{i-1} y_i^a + E\delta^{T_a} E\left[\sum_{j=1}^{T_b} \delta^{j-1} y_j^b + \delta^{T_b} \sum_{k=1}^{T_c} \delta^{k-1} y_k^c + \ldots\right] \tag{11}$$

$$= E \sum_{i=1}^{T_a} \delta^{i-1} y_i^a + E\delta^{T_a} E\left[V\right]$$

- Firm chooses product discontinuation policy that maximises

$$E\left[V\right] = \frac{E\sum_{t=1}^{T} \delta^{t-1} y_t}{\left(1-\delta\right) E\sum_{t=1}^{T} \delta^{t-1}}$$

- Let $\Im$ denote the class of product discontinuation rules,

$$\Im = \left\{ T : T \geq 1, 0 < \sum_{i=1}^{T} \delta^{i-1} < \infty \right\} \tag{12}$$

- The objective is to find a discontinuation policy $T \in \Im$ to achieve the supremum in

$$V^* = \sup_{T \in \Im} \frac{E\sum_{t=1}^{T} \delta^{t-1} y_t}{\left(1-\delta\right) E\sum_{t=1}^{T} \delta^{t-1}} \tag{13}$$

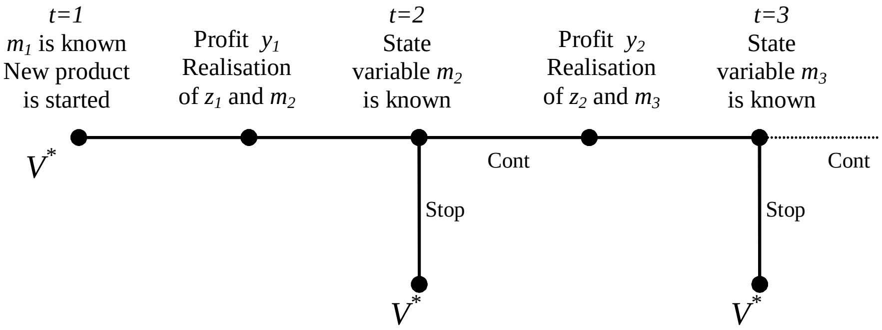


Figure: Structure of the stopping problem

- $(1-\delta)\, E\left[V\right]$ can be viewed as firm's expected *rate of return* or *per period return* from doing business

$$E\left[V\right] = \frac{E\sum_{t=1}^{T}\delta^{t-1}y_t}{(1-\delta)\, E\sum_{t=1}^{T}\delta^{t-1}}$$

- In maximising expected profit, firm, in fact, chooses optimal product discontinuation policy T^* that maximises expected rate of return.

Lemma

If for some λ, $\sup_{T\in\Im} E\left(\sum_{t=1}^{T}\delta^{t-1}\left[y_t-\lambda\left(1-\delta\right)\right]\right)=0$*, then*

$$V^* = \sup_{T\in\Im}\frac{E\sum_{t=1}^{T}\delta^{t-1}y_t}{\left(1-\delta\right)E\sum_{t=1}^{T}\delta^{t-1}} = \lambda$$

Proposition

If $\sup_{T\in\Im} E\left(\sum_{t=1}^{T}\delta^{t-1}\left[y_t-\lambda\left(1-\delta\right)\right]\right)=0$ *is attained at* T^**, then* T^* *is optimal for maximising*

$$\sup_{T\in\Im}\frac{E\sum_{t=1}^{T}\delta^{t-1}y_i}{\left(1-\delta\right)E\sum_{t=1}^{T}\delta^{t-1}}.$$

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Existence of Optimal Product Discontinuation Policies

Proposition

There exists a product discontinuation policy T^* such that

$$E\left[\sum_{t=1}^{T^*} \delta^{t-1}\left[y_t - \lambda\left(1-\delta\right)\right]\right] = \overline{V}$$

where $\overline{V} = \sup_{T \in \Im} E\left[\sum_{t=1}^{T} \delta^{t-1}\left[y_t - \lambda\left(1-\delta\right)\right]\right]$.

Optimal Product Discontinuation Policy

Proposition

Optimal product discontinuation policy T^* *for the transformed problem ,* $\sup_{T \in \Im} E\left(\sum_{i=1}^{T} \delta^{i-1}\left[y_i - \lambda\left(1-\delta\right)\right]\right)$, *is in the form*

$$T^* = \min\left\{t > 1 : m_t \leq m^*\left(\lambda, q^*\right)\right\}$$

where $m^*\left(\lambda, q^*\right) = \lambda\left(1-\delta\right) - \left[q^* - C\left(q^*\right)\right]$.

Optimal Product Discontinuation Policy

Corollary

Under optimal product discontinuation policy and for any period t, a firm continues a product if profit from continuation in that period is at least as large as the optimal per period profit, or otherwise discontinues.

Optimal Product Discontinuation Policy

There have been shown that

- It has been shown that optimal discontinuation policy exists.
- The firm discontinues its product if product reputation or prior belief m_t falls below certain thresholds....
- ....or period profit falls below optimal per period profit.
- But how does the optimal discontinuation policy change as model's parameters vary?
- Similar to the case when discontinuation period is fixed, we need to solve for discontinuation policy m^* and perform comparative static analysis

Optimal Product Discontinuation Policy

Consider the special case where the firm implements discontinuation policy T^* and it tells the firm to continue the product forever. Or, $m_t \geq m^*(\lambda, q^*)$ for every $t \in \mathbb{N}$.

$$
\begin{aligned}
& \sup_{T \in \Im} E\left[\sum_{t=1}^{T} \delta^{t-1}\left[y_t - \lambda(1-\delta)\right]\right] \\
= \quad & \sum_{t=1}^{\infty} \kappa_t(\lambda)\, \delta^{t-1} E\left[y_t - \lambda(1-\delta) \middle| m_t \geq m^*(\lambda, q^*)\right] \\
= \quad & \sum_{t=1}^{\infty} \kappa_t(\lambda)\, \delta^{t-1}\left[E\left(m_t \middle| m_t \geq m^*(\lambda, q^*)\right) - m^*(\lambda, q^*)\right]
\end{aligned}
$$

where $\kappa_t(q, \lambda) = P\left[\bigcap_{j \leq t}\left(m_j \geq m^*(\lambda, q^*)\right)\right]$ is the probability that the product survives in period t

Optimal Product Discontinuation Policy

- From Lemma 5, the firm's expected profit attains it supremum at $V^* = \lambda^*$ where λ^* satisfies

$$\sum_{t=1}^{\infty} \kappa_t (\lambda^*) \delta^{t-1} \left[E \left(m_t \middle| m_t \geq m^* (\lambda^*, q^*) \right) - m^* (\lambda^*, q^*) \right] = 0 \quad (14)$$

- In determining m^*, firm views future m_t as a random variable.
- Since every random variable are normally distributed and m_t is a linear combination of these variable....m_t are normally distributed.

$$m_t = \frac{h_1 m_1 + h_\varepsilon \sum_{s=1}^{t-1} z_s}{h_1 + (t-1) h_\varepsilon}$$

- Equation 14 is central for comparative static analysis: how does m^* change as h_1, h_ε and δ vary...

Comparative Static Analysis

- Define

$$F\left(m^*,\delta,h_{1,}h_\varepsilon\right)=\sum_{t=1}^{\infty}\kappa_t\delta^{t-1}\left[E\left(m_t\middle|\,m_t\geq m^*\right)-m^*\right]$$

Definition

Given any product discontinuation policy $T=\min\left\{t>1:m_t\leq m^T\right\}$, a product discontinuation policy $T'=\min\left\{t>1:m_t\leq m^{T'}\right\}$ is said to be **a more stringent (lenient) quality control** if T' calls to discontinue the product sooner (later) almost surely or $m^{T'}>m^T$ $(m^{T'}<m^T)$.

Comparative Static Analysis

Lemma

The probability that the product will be continued in period t, κ_t, **decreases** *as*

1. m^* *increases or* $\frac{\partial \kappa_t}{\partial m^*} < 0$ *for all* $m^* \in \mathbb{R}$,
2. h_1 *increases or* $\frac{\partial \kappa_t}{\partial h_1} < 0$ *for all* $h_1 \in \mathbb{R}^+$ *and*
3. h_ε *decreases or* $\frac{\partial \kappa_t}{\partial h_\varepsilon} > 0$ *for all* $h_\varepsilon \in \mathbb{R}^+$.

Comparative Static Analysis

Proposition

The function $f(m^*) = E(m_t | m_t \geq m^*) - m^*$ *is decreasing.*

Lemma

$E(m_t | m_t \geq m^*) - m^*$ **decreases** *if*

1. h_1 *increases and*
2. h_ε *increases for* $t > 3 + \frac{h_1}{h_\varepsilon}$ *or* h_ε *decreases for* $t < 3 + \frac{h_1}{h_\varepsilon}$

Proposition

As the discount factor (δ) increases, the optimal product discontinuation policy becomes more stringent.

Proposition

As the precision of initial prior belief (h_1) increases, the optimal product discontinuation policy becomes more lenient.

- The effect of h_ε on optimal product discontinuation policy m^* is ambiguous.

Discussions, Questions and Comments

- That's about it!!
- Any comment?
- Thank you for attending!