How to Sell Retaliation in the WTO

Tanapong Potipiti *

Chulalongkorn University

email: Tanapong.P@chula.ac.th

March 2005

Abstract

We study revenue-maximizing mechanisms for selling retaliation in the WTO. The most interesting feature of this mechanism-design problem is the presence of positive externalities among buyers. We observe that the revenue-maximizing selling mechanisms have the following properties: i) The buyer with the lowest valuation for the good may get the good and the allocation is inefficient. ii) Numerical results suggest that the optimal mechanisms cannot be implemented by an ordinary auction. Moreover, we show that allowing retaliation to be tradable might weaken rather than strengthen the WTO enforcement system.

*I am grateful to Robert Staiger, Yeon-Cho Che, John Kennan, Santiago Olivos, Bo Chen and Kiriya Kulkolkarn.

1 Introduction

The main purpose of the World Trade Organization (WTO) is to promote international cooperation among governments and offer means to solve the term-of-trade problem.[1] For this cooperation to be sustained, the design of dispute settlement procedures is crucial. Although the WTO is a multilateral organization, its enforcement system is bilateral. Under the WTO rules, if country A violates an agreement that it has with country B, only country B can retaliate against country A by increasing tariffs on imports from country A.[2] A well known problem in the WTO dispute settlement procedures is that small countries find it difficult to retaliate against large countries. Even when authorized to do so, small countries such as the Netherlands and Ecuador, have not actually implemented retaliatory responses. It seems that under the current dispute settlement procedures, large countries have an advantage. Those concerned with this matter have supported moves to multilateralize retaliation (Lawrence, 2003, chapter 1). A natural way to multilateralize retaliation is to allow it to be tradable. Mexico has proposed in WTO (2002) to make retaliation tradable. In this proposal, if a country is authorized to retaliate by increasing tariffs against a country, it would be able to sell the right to retaliate to another country. In this paper, we study the outcomes of selling retaliation in optimal (revenue-maximizing) mechanisms.

Our work departs from the standard auction design (Myerson, 1981) because of the presence of externalities among buyers. In the standard auction design, buyers' payoffs - excluding transfers - when another buyer gets the good and when the seller keeps the good to himself are the same. However, in the presence of externalities, the two outcomes give different payoffs. Buyers get externalities when another buyer gets the good. Buyers get nothing when the seller keeps the good. In our WTO context, the good is retaliation and the externalities are term-of-trade externalities. When a buyer (country) retaliates by increasing his tariffs, the world price is affected and the other buyers (countries) get *positive* externalities. Existing studies on optimal auction designs in the presence of externalities focus on *negative* externality environments. Jehiel, Moldovanu and Stacchetti (1996) analyze optimal mechanisms when buyers have private information about their valuations for the good but the negative externalities that buyers suffer when another buyer gets the good are public information. Jehiel, Moldovanu and Stacchetti (1999) study optimal mechanisms in a similar environment in which buyers' valuations and externalities are their private information. Because of the negative externalities, if the seller decides to give the good to a buyer, other buyers are willing to pay to prevent that sale. It is sometimes optimal for the seller to collect those payments and keep the good. The authors also show that a modified second-price auction is optimal. In these papers, a strong assumption on the independence of buyers' valuations of the good and externalities they suffer is imposed. Brocas (2004) relaxes this assumption and studies optimal mechanisms in an environment in which the valuations and externalities of buyers

[1] This view was formalized in Bagwell and Staiger (1999).

[2] However, other international organizations, such as the UN, use multilateral enforcement. For example, if country A invades country B, countries C and D can retaliate against country A.

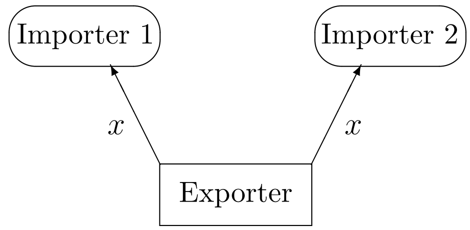

Figure 2.1: The Trading Environment.

are linearly dependent. He finds that a second-price auction with entry fees is optimal. To my knowledge, this paper is the first to study optimal mechanisms in the presence of positive externalities. Technically, our work is closest to Brocas (2004) in which buyers' valuations and the externalities are linearly dependent.

The other departure is that we study the effects of externalities between the seller and buyers. In our WTO context, when a buyer (country) gets a retaliation right and increases his tariffs, not only the other buyers but also the seller will be affected from changes in the world price.

Another important strand of related literature is the trade-agreement literature. Maggi (1999) studies a 3-country model and compares the efficiency of the grand multilateral agreement with the collection of 3 bilateral agreements. He finds that the multilateral agreement, through multilateral retaliation, is more effective in supporting the efficient outcomes. Bagwell, Mavroidis and Staiger (2003) study the outcomes of selling retaliation in the WTO in simple first-price auctions. Because of the positive externalities among buyers (importing countries), retaliation sometimes cannot be sold and allowing retaliation to be tradable might not serve its purpose. In this paper, we show that in optimal mechanisms, this problem disappears. However, retaliation trading might create another problem. It might make the WTO enforcement system less effective. Because of the positive externalities, it might be optimal for a seller who can retaliate effectively to sell retaliation to a buyer who is not very effective in retaliating. The seller therefore gets revenue from selling and enjoys a free ride in the form of a positive externality when the buyer retaliates.

The paper proceeds as follows. In section 2, we describe the basic economic model. Optimal mechanisms for selling retaliation are studied analytically and numerically in sections 3 and 4, respectively. In sections 3 and 4, we assume that the seller can not retaliate by himself. In section 5, we relaxes this assumption and study optimal mechanisms in the environment in which the seller can retaliate by himself. Technical proofs are provided in the appendix.

2 The Economic Model

Our model consists of three countries: Exporter, Importer 1 and Importer 2, and one good: good x. Exporter exports good x to Importer 1 and Importer 2. The trading environment is shown in figure 2.1.

The domestic demand (D_i) and domestic supply (Q_i) of Importer i are given by

$$D_i(P_i) = 1 - P_i$$

$$Q_i(P_i) = 1/8$$

for $i = 1, 2$. P_i is the domestic price of good x sold in Importer i. The domestic price is determined by

$$P_i = P_W + t_i \tag{2.1}$$

where t_i denotes the import tariff of Importer i and P_W denotes the world price. The import demand faced by Importer i is given by

$$M_i(P_i) = D_i(P_i) - Q_i(P_i) = 1 - P_i - 1/8.$$

Exporter has no demand for good x. It supplies 1/4 unit of good x to the two importing countries. The world market-clearing condition is

$$M_1(P_1) + M_2(P_2) = 1/4.$$

The world price and domestic prices in the equilibrium are

$$P_W(t_1, t_2) = \frac{3}{4} - \frac{t_1 + t_2}{2}$$

and

$$P_i = P_W + t_i = \frac{3}{4} + \frac{t_i - t_j}{2} \tag{2.2}$$

for $i, j \in \{1, 2\}$ and $i \neq j$.

2.1 Welfare Functions and Best-Response Tariffs

The government of Importer i is concerned about both national income and income distribution. Importer i's welfare is defined as

$$W_i \equiv CS(P_i) + TR_i(P_i, P_W) + \zeta_i \Pi_i(P_i)^3 \tag{2.3}$$

where CS_i and TR_i are Importer i's consumer surplus and tariff revenue, respectively. They are defined as:

$$CS_i(P_i) \equiv (1 - P_i)^2/2 \qquad \text{and} \qquad TR_i(P_i, P_W) \equiv (P_i - P_W)M_i(P_i),$$

where Π_i is Importer i's producer surplus and $\Pi_i(P_i) \equiv P_i/8$. ζ_i is the political-economy parameter of Importer i. When $\zeta_i = 1$, the government's welfare coincides with the national

[3] This welfare function was first introduced in Baldwin (1987). It can be considered as a reduced form of the Grossman and Helpman (1994) model.

welfare. We restrict our attention to the case in which each government weighs producer surplus more than consumer surplus:

Assumption 1 $\zeta_i \geq 1$.

Substituting P_i and P_W in terms of t_i and t_j, the welfare of Importer i in terms of the tariff rates and its political-economy parameter is

$$W_i = W_i(t_i, t_j, \zeta_i) = \frac{1}{32} - \frac{3t_i^2}{8} + \frac{t_j}{8} + \frac{t_j^2}{8} + \frac{t_i t_j}{4} + \zeta_i(\frac{3}{32} + \frac{t_i - t_j}{16}).$$

Its derivatives are

$$\frac{dW_i}{dt_i} = \frac{\zeta_i}{16} + \frac{t_j - 3t_i}{4} \quad \text{and} \quad \frac{dW_i}{dt_j} = \frac{1}{8}(1 - \frac{\zeta_i}{8} + 2t_i + 2t_j).$$

Solving $\frac{dW_i}{dt_i} = 0$, Importer i's best-response tariff is characterized by

$$t_i^{br}(\zeta_i, t_j) = \zeta_i/12 + t_j/3. \tag{2.4}$$

2.1.1 Retaliation

According to a prior agreement in the WTO, initially, all tariffs are zero. Exporter violates a prior agreement that it makes with a country. Consequently, a right to increase the tariff (retaliation right) on good x from zero to Δ[4] is granted to this country. However, this country does not import goods from Exporter and therefore cannot retaliate by itself. It can, however, sell the retaliation right to Importer 1 or Importer 2. The selling country is henceforth called the *seller*. We further assume that Δ is less than the best-response tariffs of the two importing countries:

Assumption 2 $\Delta \leq 1/12$.

This assumption is derived from equation (2.4) and $\zeta_i \geq 1$. Under this assumption, it is optimal for the importing country that gets the retaliation right to increase its tariff to Δ.

2.2 Outcomes and Welfare

In a selling mechanism, for Importer i, when it gets (*wins*[5]) the right and retaliates. Its *normalized* welfare is

$$\omega(\zeta_i; \Delta) \equiv W_i(\Delta, 0, \zeta_i) - W_i(0, 0, \zeta_i) = \frac{\Delta}{16}(\zeta_i - 6\Delta). \tag{2.5}$$

[4]In the WTO, the size of retaliation is determined in such a way that the retaliation will give equivalent compensation to countries who suffers from the violation.

[5]An importing country *wins* if it gets the retaliation right and then retaliates. An importing country *loses* if the other importing country wins.

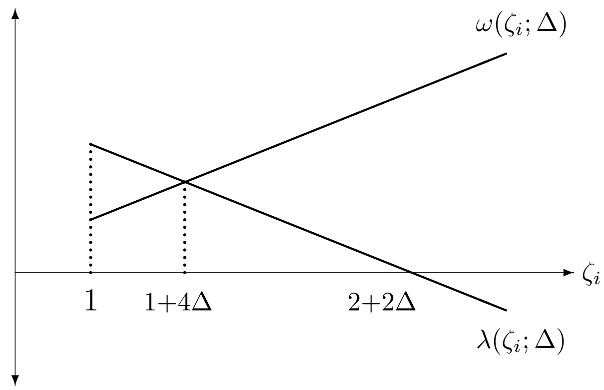

Figure 2.2: ω and λ.

$W_i(\Delta, 0, \zeta_i)$ is the welfare of Importer i when retaliating. $W_i(0, 0, \zeta_i)$ is the welfare when no retaliation occurs.

When Importer i *loses* and Importer j gets the right and retaliates, Importer i's normalized welfare (externality from losing) is

$$\lambda(\zeta_i; \Delta) \equiv W_i(0, \Delta; \zeta_i) - W_i(0, 0; \zeta_i) = \frac{\Delta}{16}(2 + 2\Delta - \zeta_i). \tag{2.6}$$

The functions ω and λ are shown in figure 2.2. As a result of normalization, the welfare of the importing countries when no retaliation occurs is 0. The normalized welfare will hereafter be referred as welfare. Properties of ω and λ are reported as follows:

Lemma 2.1 *(winning is better than nothing)* $\omega > 0$.

This result is due to assumption 2 that the retaliation tariff (Δ) is less than best-response tariffs.

Lemma 2.2 $\lambda(\zeta_i) \geq 0$ *(positive externality) if* $\zeta_i < 2 + 2\Delta$ *and* $\lambda(\zeta_i) < 0$ *(negative externality) otherwise.*

When Importer i loses and Importer j wins and retaliates by increasing its tariff, the world price decreases. Importer i's producers lose, but Importer i's consumers gain. When Importer i's government concerns more on its consumers ($\zeta_i < 2+2\Delta$), losing gives a *positive externality*. On the other hand, when its concerns on producers are high ($\zeta_i \geq 2+2\Delta$), losing gives a *negative externality*. We restrict our analysis to the positive externality case:

Assumption 3 *(Positive externality)* $\zeta_i \leq 2$.

This assumption can be interpreted to mean that the importing countries prefer their term-of-trade improvement. This assumption or its variants is generally assumed in the trade-agreement literature. This assumption and assumption 1 together are equivalent to $\zeta_i \in [1, 2]$.

Lemma 2.3 $\omega' = \frac{\Delta}{32}$, $\lambda' = -\frac{\Delta}{32}$ *and* $\omega' = -\lambda'$. $\lambda(\zeta_i) > \omega(\zeta_i)$ *if* $\zeta_i < 1+4\Delta$ *and* $\lambda(\zeta_i) \leq \omega(\zeta_i)$ *otherwise.*

The winning welfare is increasing in the political-economy parameter because when a country retaliates, its producer gains. The higher the weight that the government puts on the producer surplus, the higher the winning welfare. Conversely, the losing welfare is decreasing in the political-economy parameter. Therefore, winning is preferred to losing, when the political-economy parameter is sufficiently high. The functions ω and λ are linear. ω is increasing but λ is decreasing. Their slopes have the same magnitude but opposite signs.

Lemma 2.4 *It is efficient for the importing country with the higher political-economy parameter to retaliate or* $\omega(\zeta_i) + \lambda(\zeta_j) > \omega(\zeta_j) + \lambda(\zeta_i)$ *if* $\zeta_i > \zeta_j$.

This lemma is implied by $\omega' > 0$ and $\lambda' < 0$.

Lemma 2.5 $\frac{d\omega(\zeta_i;\Delta)}{d\Delta} = \frac{\zeta_i}{16} - \frac{3}{4}\Delta > 0$ *and* $\frac{d\lambda(\zeta_i;\Delta)}{d\Delta} = \frac{1}{8} + \frac{\Delta}{4} - \frac{\zeta_i}{16} > 0$.

The winning welfare and losing welfare (positive externality) are increasing in the retaliation tariff.

3 Optimal Mechanisms (Complete Information)

In this section, we study optimal (revenue-maximizing) mechanisms in an environment with complete information. In this environment, ζ_1 and ζ_2 are publicly known. For simplicity, we assume the two buyers[6] are symmetric: $\zeta_1 = \zeta_2 = \zeta$.

3.1 Simple Auctions are Not Optimal and are Inefficient

Before studying optimal mechanisms, we show that simple auctions without a reserve price are not optimal and are inefficient. In a complete information environment without externalities, it is well known that simple first-price auctions without a reserve price are optimal and efficient. However, in the presence of externalities, this is not the case. For simplicity, we consider only first-price auctions without a reserve price. Our analysis can be easily extended to first-price and second-price auctions with a reserve price.

Proof Consider the case that $\zeta = 1$. From figure 2.2, $\omega(1) < \lambda(1)$; buyers with $\zeta = 1$ prefer losing to winning. As a common practice in the auction literature, we restrict our analysis to symmetric equilibria. Suppose in a symmetric equilibrium, the good is always sold. A buyer would strictly gain by submitting a lower bid or not participating. Therefore, such equilibrium does not exist and in any symmetric equilibrium, the good is not always sold and the allocation is inefficient. ■

Lemma 3.1 *Simple first-price auctions are inefficient. The good may not be sold.*

[6] In this and subsequent sections, auction terminologies are used. The good is retaliation. The buyers are the importing countries.

Because of the public-good nature of the selling object (retaliation), buyers might prefer free riding on retaliation to retaliating by themselves. Sometimes none of the buyers are willing to buy the good. This result simply is a classic public-good problem. A similar result is found in Bagwell, Mavroidis and Staiger (2003) and Jehiel and Moldovanu (2000) in an environment with incomplete information.

Lemma 3.2 *In any symmetric equilibrium, if $\zeta > 1 + 4\Delta$, a negotiation (take-it-or-leave-it offer) with one buyer is better than simple first-price auctions.*

Proof From lemma 2.3, $\omega(\zeta_i) > \lambda(\zeta_i)$ for $\zeta_i > 1 + 4\Delta$. In the symmetric equilibrium, each buyer's bid value is his net gain from winning: $\omega(\zeta) - \lambda(\zeta) > 0$. The revenue is $\omega(\zeta) - \lambda(\zeta)$. However, if the seller offers the retaliation right with a price $= \omega(\zeta)$ to a buyer, and if that buyer rejects the offer, the seller keeps the good to himself and the game ends. In this mechanism, the revenue is $\omega(\zeta) > \omega(\zeta) - \lambda(\zeta)$. ■

In the auction with two buyers, positive externalities from losing reduce buyers' incentives to win and the expected revenue. In the take-it-or-leave-it offer, only one buyer participates, the externalities disappear and the revenue increases.[7] This result overturns a well known result in Bulow and Klemperer (1996). They show that a first-price auction without a reserve price is always better than negotiations with a buyer in a standard environment.

Lemma 3.3 *In simple first-price auctions, the revenue is sometimes decreasing in the size of the retaliation tariff (Δ).*

Proof Consider the case that $\zeta \in (1 + 4\Delta, 1 + 8\Delta)$. From the proof of lemma 3.2, the revenue from the first-price auction is $\omega(\zeta) - \lambda(\zeta)$. Diffentiating and subtracting (2.5) from (2.6), we have $\frac{\partial \omega(\zeta;\Delta)}{\partial \Delta} - \frac{\partial \lambda(\zeta;\Delta)}{\partial \Delta} = \frac{1}{8}(\zeta - 1 - 8\Delta)$. The derivative is negative and the revenue is decreasing in Δ if $\zeta < 1 + 8\Delta$. ■

The revenue does not only depend on the winning welfare: $\omega(\zeta;\Delta)$, but it also depends on the difference between the winning and losing welfare: $\omega(\zeta;\Delta) - \lambda(\zeta;\Delta)$. Although the winning welfare and the losing welfare are increasing in the retaliation tariff, their difference is not. Therefore, higher retaliation tariffs do not always imply higher revenue.

PROPOSITION 1 *In a complete information environment, simple first-price auctions are not optimal and inefficient. The revenue is sometimes decreasing in Δ.*

3.2 Optimal Mechanisms (Complete Information)

To find an optimal mechanism, the seller maximizes the expected revenue subject to individual rationality constraints.[8] A mechanism is *optimal* if the game it defines has a Bayes-Nash

[7] A similar result is found in Agastya and Daripa (2002). They show that a negotiation is better than standard selling procedures in the presence of high positive externalities. Standard selling procedures require that the seller cannot extract payments from buyers with zero winning probability. Standard selling procedures include standard first and second-price auctions.

[8] Under a complete information environment, incentive-compatible constraints are ignored.

equilibrium, that gives the seller the highest revenue. In order to maximize the revenue, the seller picks the most severe threat in which the buyers get the lowest welfare when not participating. Under assumptions 1 and 3 , $\zeta_i \in [1, 2]$ and the lowest welfare of buyers is 0. The buyers get the lowest welfare when the seller keeps the good to himself.[9] The individual rationality constraint for the buyer i is

$$u_i(\zeta_i) \equiv \omega(\zeta_i)Q_i + \lambda(\zeta_i)Q_j - M_i \geq 0 \tag{IR}$$

where u_i is the buyer i's payoff when participating, Q_i and M_i are his winning probability and payment, respectively. Using this individual rationality constraint, the mechanism-design problem is

$$\begin{aligned} \mathcal{P}_C : \max_{Q_i, M_i} \;& M_1 + M_2 \\ \text{s.t.} \quad & Q_i\omega(\zeta_i) + Q_j\lambda(\zeta_i) - M_i \geq 0 \quad \forall i \qquad \text{(IR)} \\ & Q_1 + Q_2 \quad \leq 1 \\ & Q_i \geq 0 \quad \forall i. \end{aligned}$$

PROPOSITION 2 *In the optimal mechanisms, the revenue is $\omega(\zeta) + \lambda(\zeta)$ and the revenue is increasing in Δ.*

Proof This result is obtained by solving the linear program $\mathcal{P}_C$. Under the symmetry: $\zeta_1 = \zeta_2 = \zeta$, one solution of this optimization problem is $Q_1 = 1$, $Q_2 = 0$, $M_1 = \omega(\zeta)$ and $M_2 = \lambda(\zeta)$. The revenue from this optimal mechanism is $\omega(\zeta) + \lambda(\zeta)$. From lemma (2.5), $\frac{d\omega}{d\Delta} + \frac{d\lambda}{d\Delta} = \frac{1}{8} - \frac{\Delta}{2} > 0$. ■

PROPOSITION 3 *The auction with the following rules is optimal and efficient.*
Case 1: $\omega(\zeta) \geq \lambda(\zeta)$ ($\zeta \geq 1 + 4\delta$)

- *The seller requires an entry fee of $\lambda(\zeta)$ from each participant.*
- *Buyers simultaneously choose to participate or not participate.*
- *The game proceeds only if both buyers participate. Otherwise, the game ends and no retaliation occurs.*
- *Each participant submits a sealed bid in a first-price auction.*
- *The bidder with the **highest** bid gets the good and retaliates.*

Case 2: $\omega(\zeta) < \lambda(\zeta)$ ($\zeta < 1 + 4\delta$)

- *The mechanism is the same as the above except that the entry fee is $\omega(\zeta)$ and the bidder with the **lowest** bid gets the good and retaliates.*

[9] If $\zeta_i > 2 + 2\Delta$, the lowest welfare of buyers is negative. In this case, if a buyer does not participate, it is optimal for the seller to give the good to the other buyer.

Proof We prove only the first case. The proof of the second case is analogous to that of the first. To show that this mechanism is optimal, it is sufficient to show that in an equilibrium, both buyers' bid values are $\omega(\zeta) - \lambda(\zeta)$ and the revenue is $\omega(\zeta) + \lambda(\zeta)$.

We are to show that in an equilibrium both buyers participate and their bid values are $\omega(\zeta) - \lambda(\zeta)$. Submitting a bid higher than $\omega(\zeta) - \lambda(\zeta)$ gives a negative payoff and is not profitable for a buyer. Not participating or submitting a lower bid gives nothing and is not a profitable deviation. Therefore, there is an equilibrium in which both buyers' bid values are $\omega(\zeta) - \lambda(\zeta)$. In this equilibrium, the winner pays $\omega(\zeta) = \lambda(\zeta) + [\omega(\zeta) - \lambda(\zeta)] =$ entry fee + bid value, and the loser pays $\lambda(\zeta)$. ■

Although, this proposition seems complicated, it is intuitive. The entry fee is used to extract positive externalities. Entry fees are generally used to extract surplus from losing bidders in the existing literature on optimal auctions in the presence of externalities. When buyers' political-economy parameters are high: $\omega(\zeta) \geq \lambda(\zeta)$ and buyers prefer retaliating by themselves to free riding, the seller sells the retaliation. On the other hand, when the political-economy parameters are low: $\lambda(\zeta) < \omega(\zeta)$, the seller sells positive externalities from free riding the retaliation.

4 Optimal Mechanisms (Incomplete Information)

4.1 Analytical Results

We now consider an incomplete information environment. The political-economy parameter of each buyer is private information (type). The types of buyers are identically and independently distributed according to a density function f. The support of f is $[\zeta_l, \zeta_u] \subseteq [1, 2]$. By the revelation principle (Myerson, 1981), there is no loss of generality in considering only direct revelation mechanisms. Let $\Phi \equiv \{(x_1, x_2) | x_1 \geq 0, x_2 \geq 0, \text{and } x_1 + x_2 \leq 1\}$. A direct mechanism $(\mathbf{Q}, \mathbf{M})$ consists of a pair of functions $\mathbf{Q} : \mathrm{Z}^2 \to \Phi$ and $\mathbf{M} : \mathrm{Z}^2 \to \mathbf{R}^2$, where $Q_i(\zeta_i, \zeta_j)$ is the probability that the buyer i gets the good and $M_i(\zeta_i, \zeta_j)$ is his payment, given that the two buyers report (ζ_i, ζ_j). Define q_i, $\bar{q}_i$ and m_i as follows:

$$\begin{aligned} q_i(\zeta_i) &\equiv \int_{\zeta_l}^{\zeta_u} Q_i(\zeta_i, \zeta_j) f(\zeta_j) d\zeta_j \\ \bar{q}_i(\zeta_i) &\equiv \int_{\zeta_l}^{\zeta_u} Q_j(\zeta_j, \zeta_i) f(\zeta_j) d\zeta_j \\ m_i(\zeta_i) &\equiv \int_{\zeta_l}^{\zeta_u} M_i(\zeta_i, \zeta_j) f(\zeta_j) d\zeta_j. \end{aligned}$$

The terms $q_i(\zeta_i)$ and $\bar{q}_i(\zeta_i)$ are winning and losing probability assignments to a buyer with type ζ_i, respectively. The term $m_i(\zeta_i)$ is the expected payment paid by a buyer with type ζ_i.

The expected payoff of a buyer with type ζ_i when reporting $\hat{\zeta}_i$ is

$$u_i(\hat{\zeta}_i, \zeta_i) \equiv q_i(\hat{\zeta}_i)\omega(\zeta_i) + \bar{q}_i(\hat{\zeta}_i)\lambda(\zeta_i) - m_i(\zeta_i). \tag{4.1}$$

A direct mechanism $(\mathbf{Q}, \mathbf{M})$ is *incentive compatible* if

$$u_i(\zeta_i) \equiv u_i(\zeta_i, \zeta_i) \geq u_i(\hat{\zeta}_i, \zeta_i) \quad \forall i, \zeta_i, \hat{\zeta}_i. \tag{IC}$$

As shown above, under assumption 3, the *individual rationality* constraint is

$$u_i(\zeta_i) \geq 0 \quad \forall i, \zeta_i. \tag{IR}$$

To maximize the expected revenue, the seller solves the following program.

$$\mathcal{P}_I : \max_{Q_i, M_i} \; E[m_1] + E[m_2]$$

$$\text{s.t.} \quad u_i(\zeta_i) \geq u_i(\hat{\zeta}_i, \zeta_i) \quad \forall i,\, \zeta_i,\, \hat{\zeta}_i \tag{IC}$$

$$u_i(\zeta_i) \geq 0 \quad \forall i,\, \zeta_i,\, \hat{\zeta}_i \tag{IR}$$

$$Q_i(\zeta_i, \zeta_j) + Q_j(\zeta_j, \zeta_i) \leq 1 \quad \forall i, \zeta_i,\, \zeta_j$$

$$Q_i(\zeta_i, \zeta_j) \geq 0 \quad \forall i,\, \zeta_i,\, \zeta_j.$$

This program can be simplified as the following.

Lemma 4.1 *Program $\mathcal{P}_I$ is equivalent to*

$$\max_{Q_i, u_i(\zeta_l)} \sum_i \int_{\zeta_l}^{\zeta_u} \int_{\zeta_l}^{\zeta_u} (v\omega(\zeta_i) + v\lambda(\zeta_j)) Q_i(\zeta_i, \zeta_i) f(\zeta_i) f(\zeta_j) d\zeta_i d\zeta_j - \sum_i u_i(\zeta_l)$$

$$s.t. \quad u_i(\zeta_i) = u_i(\zeta_l) + \int_{\zeta_l}^{\zeta_i} (\omega' q_i(t) + \lambda' \bar{q}_i(t))\, dt, \quad \forall i,\, \zeta_i \tag{IC1}$$

$$q_i - \bar{q}_i \textit{ is non-decreasing in } \zeta_i \tag{IC2}$$

$$u_i(\zeta_i) \geq 0 \quad \forall i,\, \zeta_i \tag{IR}$$

$$Q_i(\zeta_i, \zeta_j) + Q_j(\zeta_j, \zeta_i) \leq 1 \quad \forall i, j, \zeta_i,\, \zeta_j$$

$$Q_i(\zeta_i, \zeta_j) \geq 0 \quad \forall i,\, \zeta_i,\, \zeta_j$$

where $v\omega(x) \equiv \omega(x) - \omega' h^i(x)$, $v\lambda(x) \equiv \lambda(x) - \lambda' h^i(x)$ and $h^i(x) = (1 - F(x))/f(x)$.[10]

With this lemma, an optimal mechanism can be identified by Q_i and $u_i(\zeta_l)$. Because the closed-from solutions of optimal mechanisms for the general case are difficult to obtain analytically, to proceed, we focus on the case in which the political parameters are uniformly distributed. The analytical results are reported as follows.

[10] In the auction literature, $v\omega(\zeta_i)$ and $v\lambda(\zeta_i)$ are called virtual values of winning and losing respectively.

PROPOSITION 4 *If f is a continuous uniform density function over interval $[\zeta_l, \zeta_u]$ and $1 \leq \zeta_l < \zeta_u \leq 2$, any optimal mechanism $(\mathbf{Q}, \mathbf{M})$ has the following properties:*

(i) (No auction failure) $\forall \zeta_1, \zeta_2, Q_1(\zeta_1, \zeta_2) + Q_2(\zeta_2, \zeta_1) = 1$.

(ii) (Lower types sometimes win.) $\exists \zeta_i < \zeta_j,\ Q_i(\zeta_i, \zeta_j) > 0$.

4.1.1 Intuition

Property (i) shows that there is no auction failure or exclusion of low types. With a uniform f, positive externalities are sufficiently high. It is always optimal for the seller to sell the good to a buyer and extract positive externalities.[11] While in standard first-price auctions, positive externalities create auction failure (lemma 3.1), in the optimal mechanism, positive externalities prevent auction failure.

Properly (ii) seems counter-intuitive and worth discussing in detail. In the optimal mechanisms, the buyer with a lower valuation sometimes gets the good. As a result of positive externalities, there is a trade-off between the proportion of surplus that the seller can extract from the buyers and the economic surplus (efficiency). For example, when $Q_1(\zeta_1, \zeta_2) = Q_2(\zeta_2, \zeta_1) = 0.5$ for all ζ_1, ζ_2. The economic surplus is

$$\frac{\omega(\zeta_1) + \lambda(\zeta_2)}{2} + \frac{\omega(\zeta_2) + \lambda(\zeta_1)}{2} = \frac{\Delta}{8}(1 - 2\Delta)$$

for all ζ_1, ζ_2. This allocation rule is purely random and inefficient, and the economic surplus is not maximized. Under this random allocation rule, the expected surplus (excluding transfers) of each buyer is independent of his true type and the type he reports. Each buyer's expected surplus (excluding transfers) is $\frac{\Delta}{16}(1 - 2\Delta)$. The seller can simply extract *all* of the economic surplus by collecting payment $= \frac{\Delta}{16}(1 - 2\Delta)$ from each buyer.[12]

On the other hand, under the efficient allocation rule in which the higher type always wins, the economic surplus is maximized. However, the seller cannot extract all of the surplus. Under the efficient rule, the expected surplus (excluding transfers) of each buyer depends on his true type and the type he reports. To ensure that buyers tell the truth, the seller has to give information rents to some buyers. With such trade-off, the efficient allocation rule does not always maximize the revenue. The next section reports numerical results. The results confirm that the seller exploits this trade-off to maximize the expected revenue.

[11] The same result is found in optimal mechanisms in the standard environment in which the valuation of the lowest type is sufficiently high.

[12] It is important to note that such allocation rule in which the seller can extract all the economic surplus is possible because $\omega' > 0$, $\lambda' < 0$ and $\lambda' + \omega' = 0$.

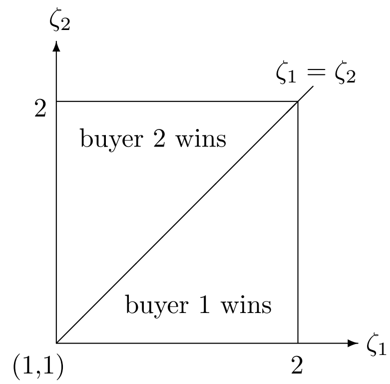

Figure 4.1: The optimal allocation rule in the standard environment.

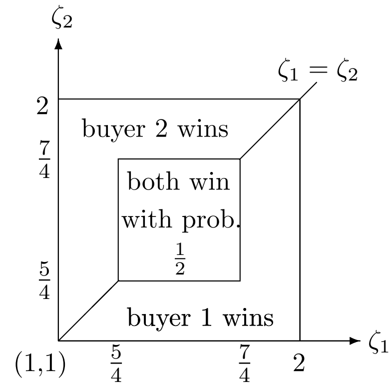

Figure 4.2: The allocation rule that solves $\mathcal{P}_D$.

4.2 Numerical Results

We now study an optimal mechanism numerically by using the $\Delta = 1/12$ and the following discrete f:

$$
\begin{aligned}
f(\zeta_i) &= 1/100 \quad \text{for } \zeta_i = 1.01, 1.02, ..., 2 \\
f(\zeta_i) &= 0 \qquad\quad \text{otherwise.}
\end{aligned}
$$

This distribution can be considered as a fine discrete approximation of the continuous uniform distribution from 1 to 2. With a discrete f, the mechanism-design problem is

$$
\mathcal{P}_D : \max_{Q_i, M_i} \sum_{\zeta_1, \zeta_2} (M_1(\zeta_1, \zeta_2) + M_2(\zeta_1, \zeta_2)) f(\zeta_1) f(\zeta_2)
$$

$$
u_i(\zeta_i, \zeta_i) \geq u_i(\hat{\zeta}_i, \zeta_i) \quad \forall i, \zeta_i, \hat{\zeta}_i \tag{IC}
$$

$$
u_i(\zeta_i, \zeta_i) \geq 0, \quad \forall i, \hat{\zeta}_i, \zeta_i \tag{IR}
$$

$$
Q_1(\zeta_1, \zeta_2) + Q_2(\zeta_1, \zeta_2) \leq 1, \quad \forall \zeta_1, \zeta_2
$$

$$
Q_i(\zeta_i, \zeta_j) \geq 0, \quad \forall i, \zeta_i, \zeta_j
$$

$$
u_i(\hat{\zeta}_i, \zeta_i) = \sum_{\zeta_j} f(\zeta_j)(Q_i(\hat{\zeta}_i, \zeta_j)\omega(\zeta_i) + Q_j(\zeta_j, \hat{\zeta}_i)\lambda(\zeta_i) - M_i(\hat{\zeta}_i, \zeta_j)) \quad \forall i, \zeta_i.
$$

Program $\mathcal{P}_D$ is a simple linear program. It was numerically solved by the NEOS optimization server (http://www-neos.mcs.anl.gov/). The results are shown in figures 4.2 - 4.5. Figure 4.1 shows the optimal allocation rule in which bidders' valuations are uniformly distributed in [1, 2] in the standard environment. Figure 4.2 shows the optimal allocation rule that numerically solves $\mathcal{P}_D$. In the environment with positive externalities, it is sometimes optimal to allocate the good to the buyer with a lower valuation. The numerical results show that with probability 1/8, the good is allocated to the buyer with a lower valuation. The equilibrium payoff (u_i)

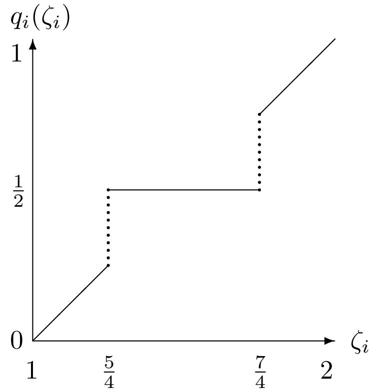

Figure 4.3: Winning prob. (q_i).

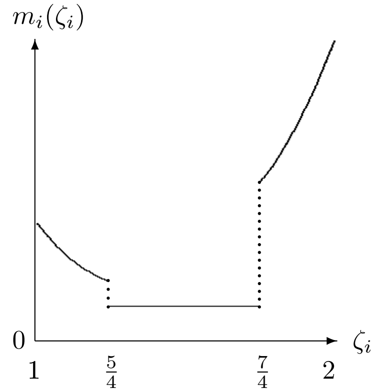

Figure 4.4: Payment (m_i).

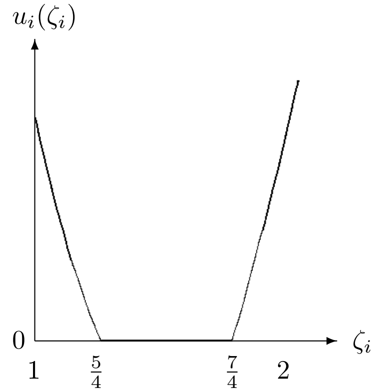

Figure 4.5: Payoff (u_i).

of each type is shown in figure 4.5. The equilibrium payoff is not monotonic in buyers' types. The lowest and highest types get the highest payoffs. The winning probability (q_i) is shown in figures 4.3. The function q_i is (weakly) increasing. However, the expected payment (m_i) in figure 4.4 is not monotonic in ζ_i.

Notice that for the intermediate types $\zeta_i \in [\frac{5}{4}, \frac{7}{4}]$, q_i, m_i and u_i are flat. Their winning probabilities are 0.5. Their equilibrium payoffs are 0 and their expected payments are the same.

Another interesting property is that the expected payment is not positively related in the winning probability. In ordinary observed auctions, the winning probability is positively related to the expected payment. These results suggest that the optimal mechanism cannot be implemented by typical auctions. To get some intuition behind this result, we consider the optimal auction in the complete information. From proposition 3, in the complete information, the optimal auction rules depend on the types of the buyers. When the types of the buyers are high ($\zeta_i > 1 + 4\Delta$), the buyer with the highest bid gets the retaliation right and retaliates. On the other hand, when the types of buyers are low ($\zeta_i \geq 1 + 4\Delta$), the buyer with the lowest bid gets the retaliation right and the buyer with the highest bid free rides the retaliation. To have an optimal auction, the complete information on the types of the buyers is necessary. However, in the incomplete information environment, this information is not available and it is not possible to have an ordinary optimal auction.

5 The Seller as an Importing Country

In the previous sections, we assume that the seller does not import good x. Therefore, the seller cannot retaliate by himself. Under this assumption, allowing the seller to sell his retaliation right obviously improves the effectiveness of retaliation and punishment in the WTO. In this section, we relax this assumption and the seller is now an importing country and is able to retaliate by himself. We show that in an incomplete information environment, allowing retaliation to be tradable can decrease rather than increase the effectiveness of retaliation. Allowing retaliation to be tradable may weaken punishment in the WTO. To show this result, we assume the following:

Assumption 5.1 *The seller is an importing country.*

Importer 1 and Importer 2 in section 2 are relabeled as the seller (S) and the buyer (B), respectively. In this section, there are one seller and one buyer. Both import good x and are able to retaliate.

Assumption 5.2 *The retaliation right is such that any country who gets it can increase its tariff as much as it wants.*[13]

Under this assumption, when an importing country gets the retaliation right and retaliate, its tariff is increased to its best-response level defined in equation (2.4). Countries with different types have different best-response tariffs.

5.1 Welfare

When the buyer or seller retaliates, he raises his tariff to the best-response level. His normalized welfare is

$$\omega^*(\zeta_i) \equiv W(t^{br}(\zeta_i), 0, \zeta_i) - \eta(\zeta_i) = \frac{\zeta_i^2}{384}$$

for $i \in \{S, B\}$, where S denotes the seller and B denotes the buyer, and t^{br} is the best-response tariff function defined in equation (2.4). The welfare of the seller or buyer when the other retaliates is

$$\lambda^*(\zeta_i, \zeta_j) \equiv W_i(0, t^{br}(\zeta_j), \zeta_i) - \eta(\zeta_i) = \frac{\zeta_j}{96}(1 + \frac{\zeta_j}{12} - \frac{\zeta_i}{2})$$

for $i, j \in \{S,\ B\}$ and $i \neq j$. The functions ω^* and λ^* are similar to ω and λ except that the retaliator chooses its best-response tariff rather than Δ. The functions ω^* and λ^* have the following properties:

Lemma 5.1 $\frac{d\omega^*(\zeta_i)}{d\zeta_i} > 0, \frac{d\lambda^*(\zeta_i,\zeta_j)}{d\zeta_i} < 0$ *and* $\frac{d\lambda^*(\zeta_i,\zeta_j)}{d\zeta_j} > 0.$

The first two derivatives are similar to those of ω and λ. The last derivative shows that the positive externality from the retaliation is increasing in the type of the retaliator. A country with a high political-economy parameter has a high best-response tariff and creates high positive externalities when retaliating.

Lemma 5.2 $\omega^*(\zeta_S) + \lambda^*(\zeta_B, \zeta_S) \geq \omega^*(\zeta_B) + \lambda^*(\zeta_S, \zeta_B)$ *if* $\zeta_S \geq \zeta_B$.

It is efficient for the buyer or seller with the higher political-economy parameter to retaliate. This lemma is similar to lemma 2.4.

[13] This simple assumption is sufficient for our results but not necessary. The necessary assumption for our results is that the size of the retaliation allowed (Δ) is higher than the best-responses of some types. Therefore, a retaliator does not always increase his tariff to Δ.

5.2 Optimal Mechanisms (Complete Information)

Now, we are ready to study optimal mechanisms under a complete information environment. In section 3.2, in the optimal mechanisms, the seller uses a threat such that if a buyer does not participate, the retaliation right is not allocated and no retaliation occurs. However, in the environment in which the seller has an ability to retaliate, this threat is not credible. In this section, we assume the following:

Assumption 5.3 *If the seller cannot sell the retaliation, he retaliates by himself.*

With one seller and buyer, optimal mechanisms are take-or-leave-it offers. Specifically, an optimal mechanism has the form:

1. The seller offers a price p.
2. The buyer accepts or rejects the offer.
3. If the buyer accepts the offer, the buyer pays the price and then retaliates.
4. If the buyer rejects, the seller retaliates by himself.

PROPOSITION 5 *The subgame perfect equilibrium has the following properties:*

1. *The price the seller charges when selling is* $p^* = \omega^*(\zeta_B) - \lambda^*(\zeta_B, \zeta_S)$ *and* $\frac{dp^*}{d\zeta_S} < 0$.
2. *The seller retaliates by himself if* $\zeta_S > \zeta_B$ *and the outcome is efficient.*

Proof The optimal price that the seller charges from the buyer is

$$p^* = \omega^*(\zeta_B) - \lambda^*(\zeta_B, \zeta_S) \tag{5.1}$$

The RHS is the net benefit the buyer gets when retaliating. It can be negative or positive. A negative price implies that the seller pays the buyer to retaliate in order to free ride the positive externality from the retaliation. From lemma 5.1 and equation (5.1), $\frac{dp^*}{d\zeta_S} = \frac{-d\lambda^*(\zeta_B,\zeta_S)}{d\zeta_S} > 0$. If the seller sells the good, his payoff is

$$\lambda^*(\zeta_S, \zeta_B) + p^* = \lambda^*(\zeta_S, \zeta_B) + \omega^*(\zeta_B) - \lambda^*(\zeta_B, \zeta_S).$$

The payoff when the seller retaliates by himself is $\omega^*(\zeta_S)$. It is optimal for the seller to sell if

$$\lambda^*(\zeta_S, \zeta_B) + \omega^*(\zeta_B) - \lambda^*(\zeta_B, \zeta_S) > \omega^*(\zeta_S).$$

This statement is true if $\zeta_B > \zeta_S$. From lemma 5.2, the allocation outcome is efficient. ■

The price that the seller charges is the buyer's net benefit when retaliating. Because the positive externality that the buyer gets when the seller retaliates is increasing in the seller's type, the price that the seller can charge is decreasing in his type. The efficient outcome can be considered as a special case of the Coase theorem.

5.3 Incomplete Information

We now analyze an incomplete information environment in which the seller's type (ζ_S) is private information but the buyer's type is public information. The game is as follows:

1. Nature picks a value of $\zeta_S \in \{1, \zeta_S^h\}$ such that $prob(\zeta_S = 1) = prob(\zeta_S = \zeta_S^h) = 0.5$.
2. The seller offers a price p.
3. The buyer with type ζ_B accepts or rejects the offer.
4. If the buyer rejects, the seller retaliates by himself.

The seller with $\zeta_S = 1$ and $\zeta_S = \zeta_S^h$ is henceforth called the high type and low type, respectively.

Lemma 5.3 *If $\zeta_B = 1.5$ and $\zeta_S^h \in (1.5, 1.7)$, a pooling sequential equilibrium is the following. The low type and high type offer $p^* = \omega^*(\zeta_B) - 0.5(\lambda^*(\zeta_B, 1) + \lambda^*(\zeta_B, \zeta_S^h)) > 0$. The buyer's strategy is to accept the offer if $p \leq p^*$. Define $b(p)$ as the buyer's belief that the seller is the low type ($\zeta_S = 1$) when observing p. A belief to support this sequential equilibrium is*

$$
\begin{aligned}
b(p) &= 0.5 \qquad \textit{if } p = p^* \\
b(p) &= 0 \qquad\ \ \textit{otherwise.}
\end{aligned}
$$

Proof Obviously, the belief is Bayesian. As usual in signaling games, Bayesian beliefs are all consistent. The buyer's strategy is clearly a best-response. Its expected benefit from the retaliation is equal to p^*. For the seller, offering a lower price is not a profitable for both types. Offering a higher price is not profitable because the buyer will reject the offer. It is remained to show that for both types, retaliating by themselves is not a profitable deviation. Simple algebra shows that $\omega^*(1) < \lambda^*(1, \zeta_S)$ for $\zeta_S = (1.5, 1.7)$, and $\omega^*(\zeta_B) - .5(\lambda^*(\zeta_B, 1) + \lambda^*(\zeta_B, \zeta_S^h)) + \lambda^*(\zeta_S^h, \zeta_B) > \omega^*(\zeta_S)$ for $\zeta_B = 1.5$ and $\zeta_S^h \in (1.5, 1.7)$. Therefore, not selling and retaliating are not a profitable deviation for both types. ■

Lemma 5.4 *The equilibrium proposed in lemma 5.3 satisfies the Cho-Krep intuitive criterion.*

Proof From proposition 5, $\frac{dp^*}{d\zeta_S} < 0$. The low type can charge a higher price. It is therefore not profitable for the high type to distinguish himself from the low type. Suppose the low type separates himself. To have a profitable deviation, the lower type must offer a higher price. However, it would be also profitable for the high type to offer a higher price. Hence, there is no signal that the low type may profitably use to separate himself. ■

PROPOSITION 6 *In an equilibrium, a buyer with $\zeta_B < \zeta_S$ gets the retaliation right. In this equilibrium, allowing the retaliation to be tradable weakens the punishment.*

Proof If the seller cannot trade the retaliation, he always retaliate by himself, and the size of the retaliatory tariff is $t^{br}(\zeta_S)$. When the seller can trade, lemma 5.3 shows that he may sell it to a buyer with $\zeta_B \leq \zeta_S$. From equation (2.4), $t^{br}(\zeta_B) < t^{br}(\zeta_S)$ for $\zeta_B \leq \zeta_S$. The retaliation is less severe when it is tradable. ■

Allowing retaliation to be tradable may create inefficiency. The seller who should retaliate by himself may exploit the trading opportunity by selling it to the buyer, extracting the buyer's surplus and free riding the positive externality. Clearly, this inefficiency results from the externalities between the buyer and seller.

6 Conclusion

We have studied optimal mechanisms for selling retaliation in the WTO. Because of externalities among buyers, losing buyers can enjoy positive externalities when the other buyer retaliates. In this environment, simple auctions are not optimal. We have shown that the optimal mechanisms exhibit interesting properties: i) the buyer with the lowest valuation sometimes wins and the allocation is inefficient. ii) Numerical results suggest that optimal mechanisms cannot be implemented by an ordinary auction. We have also studied selling mechanisms in the presence of externalities between the seller and buyers. With such externalities, we show that allowing retaliation to be tradable might weaken rather than strengthen the WTO enforcement system.

7 References

Agastya, M. and Daripa, A.(2002): "To Auction or to Negotiate," mimeo, University of Sydney.

Bagwell, K., Mavroidis P. C. and Staiger, R. W. (2003): "The Case of Auctioning Countermeasures in the WTO," mimeo, University of Wisconsin-Madison.

Bagwell, K. and Staiger, R. W. (1999): "An Economic Theory of GATT," *American Economic Review*, 89, 215-48.

Baldwin, R. (1987): "Politically Realistic Objective Functions and Trade Policy," *Economic Letter*, 24, 287-90.

Brocas, I. (2004): "Auctions with Type-dependent and Negative Externalities: the Optimal Mechanism," mimeo, University of Southern California.

Bulow, J. and Klemperer, P. (1996): "Auctions versus Negotiations," *American Economic Review*, 86, 180-94.

Groosman, G. M. and Helpman, E. (1994): "Protection for Sale," *American Economic Review*, 84, 833-50.

Jehiel, P. and Moldovanu, B. (2000): "Auctions with Downstream Interaction among Buyers," *Rand Journal of Economics*, 31, 768-791.

Jehiel, P., Moldovanu, B., Stacchetti, E. (1996): "How (Not) to Sell Nuclear Weapons," *American Economic Review* 86, 814-29.

Jehiel, P., Moldovanu, B., Stacchetti, E. (1999): "Multidimensional mechanism design for auctions with externalities," *Journal of Economic Theory* 85, 258-93.

Krishma, V. (2002): *Auction Theory*, Academic Press.

Maggi, G. (1999): "The Role of Multilateral Institutions in International Trade Cooperation," *American Economic Review*, 89, 190-214.

Myerson, R.B. (1981): "Optimal Auction Design," *Mathematics of Operation Research*, 6, 58-73.

Lawrence, R. Z. (2003): *Crime & Punishments? Retaliation Under the WTO*, Institute for International Economics.

WTO (2002): "Dispute Settlement Body - Special Session - Negotiations on Improvements and Clarifications of the Dispute Settlement Understanding - Proposal by Mexico," November 4, catalogue record TN/DS/W/23.

8 Appendix

Proof of Lemma 4.1

Claim 1: (IC) implies (IC1).
Proof The expected payoff of type ζ_i is

$$u_i(\zeta_i) \equiv q_i(\zeta_i)\omega(\zeta_i) + \bar{q}_i(\zeta_i)\lambda(\zeta_i) - m_i(\zeta_i).$$

Hence,

$$m_i(\zeta_i) = q_i(\zeta_i)\omega(\zeta_i) + \bar{q}_i(\zeta_i)\lambda(\zeta_i) - u_i(\zeta_i). \tag{8.1}$$

From (IC),

$$u_i(\zeta_i) = \max_{\hat{\zeta}_i}(q_i(\hat{\zeta}_i)\omega(\zeta_i) + \bar{q}_i(\hat{\zeta}_i)\lambda(\zeta_i) - m_i(\hat{\zeta}_i)).$$

$u_i(\zeta_i)$ is differentiable almost everywhere because it is the maximum of a family of affine functions. By the envelope theorem,

$$u_i'(\zeta_i) = q_i(\zeta_i)\omega' + \bar{q}_i(\zeta_i)\lambda'.$$

Integrating both sides yields

$$u_i(\zeta_i) = u_i(\zeta_l) + \int_{\zeta_l}^{\zeta_i} (\omega' q_i(t) + \lambda' \bar{q}_i(t))dt. \quad \square$$

Claim 2: (IC) implies (IC2).

Proof Using $\omega' = -\lambda'$, (IC) can be written as

$$u_i(\zeta_i) \geq u_i(\hat{\zeta}_i) + \omega'(q_i(\hat{\zeta}_i) - \bar{q}_i(\hat{\zeta}_i))(\zeta_i - \hat{\zeta}_i).$$

Interchanging $\hat{\zeta}_i$ and ζ_i, we obtain

$$u_i(\hat{\zeta}_i) \geq u_i(\zeta_i) + \omega'(q_i(\zeta_i) - \bar{q}_i(\zeta_i))(\hat{\zeta}_i - \zeta_i).$$

Adding the two inequalities above and rearranging give

$$q_i(\zeta_i) - \bar{q}_i(\zeta_i) \geq q_i(\hat{\zeta}_i) - \bar{q}_i(\hat{\zeta}_i), \quad \forall \zeta_i \geq \hat{\zeta}_i. \quad \square$$

Claim 3: (IC1) and (IC2) imply (IC).

Proof From (IC1) and $\omega' = -\lambda'$,

$$u_i(\zeta_i) = u_i(\hat{\zeta}_i) + \omega' \int_{\hat{\zeta}_i}^{\zeta_i} (q_i(t) - \bar{q}_i(t))\, dt \geq u_i(\hat{\zeta}_i) \quad \forall \zeta_i \geq \hat{\zeta}_i.$$

The last inequality uses (IC2). $\square$

Claim 4: The expected revenue is

$$R = \sum_i \int_{\zeta_l}^{\zeta_u} \int_{\zeta_l}^{\zeta_u} (v\omega(\zeta_i) + v\lambda(\zeta_j))Q_i(\zeta_i, \zeta_j) f(\zeta_i) f(\zeta_j) d\zeta_i d\zeta_j - \sum_i u_i(\zeta_l).$$

Proof Integrating equation (8.1) over $[\zeta_l, \zeta_u]$ gives

$$E[m_i] = -u_i(\zeta_l) + \int_{\zeta_l}^{\zeta_u} (q_i(\zeta_i)\omega(\zeta_i) + \bar{q}_i(\zeta_i)\lambda(\zeta_i))f(\zeta_i)d\zeta_i - \omega' \int_{\zeta_l}^{\zeta_u}\int_{\zeta_l}^{\zeta_i} (q_i(t) - \bar{q}_i(t))f(\zeta_i)dtd\zeta_i.$$

Interchanging the order of integrations in the last term results in

$$\int_{\zeta_l}^{\zeta_u}\int_{\zeta_l}^{\zeta_u} (q_i(t) - \bar{q}_i(t))f(\zeta_i)dtd\zeta_i = \int_{\zeta_l}^{\zeta_u} (1 - F(t))(q_i(t) - \bar{q}_i(t))dt.$$

Using this equation and $w' = -\lambda'$, we have

$$R = \sum_i E[m_i(\zeta_i)] = \sum_i \int_{\zeta_l}^{\zeta_u} (v\omega(\zeta_i)q_i(\zeta_i) + v\lambda(\zeta_i)\bar{q}_i(\zeta_i))f(\zeta_i)d\zeta_i - \sum_i u_i(\zeta_l). \tag{8.2}$$

The second summation can be written as

$$\sum_i \int_{\zeta_l}^{\zeta_u} (v\omega(t)q_i(t) + v\lambda(t)\bar{q}_{-i}(t))f(t)dt.$$

By substituting $q_i(t)$ and $\bar{q}_i(t)$, we have

$$\int_{\zeta_l}^{\zeta_u} (v\omega(t)q_i(t) + v\lambda(t)\bar{q}_{-i}(t))f(t)dt = \int_{\zeta_l}^{\zeta_u}\int_{\zeta_l}^{\zeta_u} (vw(t)Q_i(t,x) + v\lambda(t)Q_{-i}(x,t))f(x)f(t)dxdt.$$

Summing this equation across i and using $Q_{-1} = Q_2$ and $Q_{-2} = Q_1$, and substituting the result into equation (8.2), we obtain

$$R = -\sum_i u_i(\zeta_l) + \sum_i \int_{\zeta_l}^{\zeta_u}\int_{\zeta_l}^{\zeta_u} (v\omega(\zeta_i) + v\lambda(\zeta_j))Q_i(\zeta_i,\zeta_j)f(\zeta_i)f(\zeta_j)d\zeta_i d\zeta_j. \quad \square$$

Claims 1 to 5 complete the proof of lemma 4.1. ■

Proof of Proposition 4

Proof of (i) From lemma 4.1, any truth-telling and feasible mechanism can be identified by $(Q_i, u_i(\zeta_l))$. Suppose there exists a feasible mechanism $(Q_i, u_i(\zeta_l))$ in which $Q_1(\zeta_1,\zeta_2) + Q_2(\zeta_2,\zeta_1) < 1$ for some ζ_1, ζ_2. From claim 5 in the proof of lemma 4.1, the expected payment of the buyer i is

$$E[m_i] = \int_{\zeta_l}^{\zeta_u}\int_{\zeta_l}^{\zeta_u} (v\omega(\zeta_i) + v\lambda(\zeta_j))Q_i(\zeta_i,\zeta_j)f(\zeta_i)f(\zeta_j)d\zeta_i d\zeta_j.$$

Consider mechanism $(Q^*, u_i^*(\zeta_l))$ such that $u_i^*(\zeta_l) = u_i(\zeta_l)$ and

$$Q_i^*(\zeta_i, \zeta_j) = Q_i(\zeta_i, \zeta_j) + \frac{1 - \sum\limits_i Q_i(\zeta_i, \zeta_j)}{2}.$$

It follows that $q_i^* - \bar{q}_i^* = q_i - \bar{q}_i$ and $(Q^*, u_i^*(\zeta_l))$ satisfies (IC1), (IC2) and (IR). Therefore, $(Q^*, u_i^*(\zeta_l))$ is implementable. The expected payment of the buyer i in this mechanism is

$$E[m_i^*] = \int\limits_{\zeta_l}^{\zeta_u} \int\limits_{\zeta_l}^{\zeta_u} (v\omega(\zeta_i) + v\lambda(\zeta_j)) Q_i^*(\zeta_i, \zeta_j) f(\zeta_i) f(\zeta_j) d\zeta_i d\zeta_j.$$

The difference between $E[m_i^*]$ and $E[m_i]$ is

$$E[m_i^*] - E[m_i] = \int\limits_{\zeta_l}^{\zeta_u} \int\limits_{\zeta_l}^{\zeta_u} (v\omega(\zeta_i) + v\lambda(\zeta_j))(Q_i^*(\zeta_i, \zeta_j) - Q_i(\zeta_i, \zeta_j)) f(\zeta_i) f(\zeta_j) d\zeta_i d\zeta_j.$$

Using $F(\zeta_i) = \frac{\zeta_i - \zeta_l}{\zeta_u - \zeta_l}$ and $f(\zeta_i) = \frac{1}{\zeta_u - \zeta_l}$ and substituting ω and λ explicitly, one can verify that

$$v\omega' > 0,\ v\lambda' < 0 \text{ and } v\omega(\zeta_i) + v\lambda(\zeta_j) \geq v\omega(\zeta_l) + v\lambda(\zeta_u) = \tfrac{\Delta}{8}(1 - \tfrac{\Delta}{2}) > 0.$$

By construction,

$$\int\limits_{\zeta_l}^{\zeta_u} \int\limits_{\zeta_l}^{\zeta_u} (Q_i^*(\zeta_i, \zeta_j) - Q_i(\zeta_i, \zeta_j)) f(\zeta_i) f(\zeta_j) d\zeta_i d\zeta_j = \int\limits_{\zeta_l}^{\zeta_u} \int\limits_{\zeta_l}^{\zeta_u} \left(\frac{1 - \sum\limits_i Q_i^*(\zeta_i, \zeta_j)}{2}\right) f(\zeta_i) f(\zeta_j) d\zeta_i d\zeta_j > 0.$$

Therefore, $E[m_i^*] - E[m_i] > 0$. This result contradicts the supposition that $(Q_i, u_i(a))$ is optimal and completes the proof of (i). $\blacksquare$

Proof of (ii) The sketch of proof is as follows. First, we find the maximum revenue that mechanisms in which a lower type never wins can give. Then, we show that there exists a mechanism that gives a higher revenue.

Claim 1: In any optimal mechanism,

$$u_i(\zeta_l) = \bar{u}_i(\zeta_l) = \omega' \max_x \int_{\zeta_l}^{x} (\bar{q}_i(\zeta_i) - q_i(\zeta_i))\, d\zeta_i = \omega' \max_x \int_{\zeta_l}^{x} (1 - 2q_i(\zeta_i))\, d\zeta_i.$$

Proof From (i), $Q_1(\zeta_1, \zeta_2) + Q_2(\zeta_2, \zeta_1) = 1,\ \forall \zeta_1, \zeta_2$. Therefore, $\bar{q}_i(\zeta_i) = 1 - q_i(\zeta_i)$ and $\bar{q}_i(\zeta_i) - q_i(\zeta_i) = 1 - 2q_i(\zeta_i)$. Suppose $u_i(\zeta_l) < \bar{u}_i(\zeta_l)$. (IR) is violated. Suppose $u_i(\zeta_l) > \bar{u}_i(\zeta_l)$. The mechanism is not optimal because the expected revenue can be increased by using $u_i(\zeta_l) = \bar{u}_i(\zeta_l)$. $\square$

Suppose mechanism $(Q, u_i(\zeta_l))$ is optimal and $Q_i(\zeta_i, \zeta_j) = 0,\ \forall \zeta_i < \zeta_j$. Property (i) and the supposition imply that $Q_i(\zeta_i, \zeta_j) = 1,\ \forall \zeta_i > \zeta_j$ and $q_i(\zeta_i) = F(\zeta_i)$. Define $v(\zeta_i, \zeta_j) \equiv v\omega(\zeta_i, \zeta_j) + v\lambda(\zeta_i, \zeta_j)$ and $f(\zeta_i, \zeta_j) \equiv f(\zeta_i) f(\zeta_j)$. From claim 1 and equation (8.2), the revenue from this mechanism is

$$R = -\omega' \sum_i \int\limits_{\zeta_l}^{c} (1 - 2q_i(\zeta_i)) d\zeta_i + \sum_i \int\limits_{\zeta_l}^{\zeta_u} \int\limits_{\zeta_l}^{\zeta_u} v(\zeta_i, \zeta_j) Q_i(\zeta_i, \zeta_j) f(\zeta_i, \zeta_j) d\zeta_i d\zeta_j$$

where $c = \arg\max_x \int_{\zeta_l}^{x} (1 - 2q_i(\zeta_i))d\zeta_i$.. In the uniform case, $c = \frac{\zeta_l + \zeta_u}{2}$. Substituting $q_i(t)$ explicitly, we obtain

$$\begin{aligned}
R &= -\omega' \sum_i \int_{\zeta_l}^{c} \int_{\zeta_l}^{\zeta_u} (1 - 2Q_i(\zeta_i, \zeta_j)) f(\zeta_j) d\zeta_j d\zeta_i + \sum_i \int_{\zeta_l}^{\zeta_u} \int_{\zeta_l}^{\zeta_u} v(\zeta_i, \zeta_j) Q_i(\zeta_i, \zeta_j) f(\zeta_i, \zeta_j) d\zeta_i d\zeta_j \\
&= -\omega' \sum_i \int_{\zeta_l}^{c} f(\zeta_j) d\zeta_j d\zeta_i + \sum_i \int_{\zeta_l}^{c} \int_{\zeta_l}^{\zeta_u} (v(\zeta_i, \zeta_j) + \frac{2\omega'}{f(\zeta_i)}) Q_i(\zeta_i, \zeta_j) f(\zeta_i, \zeta_j) d\zeta_i d\zeta_j \\
&\quad + \sum_i \int_{c}^{\zeta_u} \int_{\zeta_l}^{\zeta_u} v(\zeta_i, \zeta_j) Q_i(\zeta_i, \zeta_j) f(\zeta_i, \zeta_j) d\zeta_i d\zeta_j.
\end{aligned}$$

Note that

$$(v(\zeta_i, \zeta_j) + \frac{2\omega'}{f(\zeta_i)}) f(\zeta_i, \zeta_j) > v(\zeta_j, \zeta_i) f(\zeta_i, \zeta_j) \tag{8.3}$$

for $\zeta_u \geq \zeta_j > c > \zeta_i \geq \zeta_l$. The inequality uses

$$v(\zeta_j, \zeta_i) - v(\zeta_i, \zeta_j) = \frac{\Delta}{8}(\zeta_j - \zeta_i) < \frac{2}{f(\zeta_i)} = 2(\zeta_u - \zeta_l).$$

Ignoring the constant $-\omega' \sum_i \int_{\zeta_l}^{c} f(\zeta_j) d\zeta_j d\zeta_i$ in R, the LHS of (8.3) is the revenue that the seller can extract when setting $Q_i(\zeta_i, \zeta_j) = 1$. The RHS is the revenue that the seller gets when setting $Q_j(\zeta_j, \zeta_i) = 1$. Note that $Q_i(\zeta_i, \zeta_j) + Q_j(\zeta_j, \zeta_i) \leq 1$.

In the mechanism in which the lower type never wins, $Q_i(\zeta_i, \zeta_j) = 0$ for $\zeta_i < \zeta_j$. This mechanism cannot be optimal because the seller can get higher revenue by increasing $Q_i(\zeta_i, \zeta_j)$ for some $\zeta_j > c > \zeta_i$ in a continuous and smooth way such that c is unchanged, and (IC) and (IR) are not violated. A specific mechanism that gives higher revenue is the following $(Q^*, u_i^*(\zeta_l))$, where

$$\begin{aligned}
Q_i^*(\zeta_i, \zeta_j) &= \frac{1}{4} \quad \text{for } (\zeta_i, \zeta_j) \text{ such that } \quad \zeta_i \in (a - \varepsilon,\, a + \varepsilon), \zeta_j \in (b - \varepsilon,\, b + \varepsilon) \\
&\qquad \text{and} \quad |\zeta_i - a| > |\zeta_j - b| \\
Q_i^*(\zeta_i, \zeta_j) &= 1 \quad \text{for } (\zeta_i, \zeta_j) \text{ such that } \quad \zeta_i \notin (a - \varepsilon,\, a + \varepsilon), \zeta_j \notin (b - \varepsilon,\, b + \varepsilon) \quad \text{and} \quad \zeta_i > \zeta_j \\
u_i^*(\zeta_l) &= \omega' \int_{\zeta_l}^{c^*} (q_i^*(t) - \bar{q}_i^*(t)) dt \\
c^* &= \arg\max_x \int_{\zeta_l}^{x} (q_i^*(\zeta_i) - \bar{q}_i^*(\zeta_i)) d\zeta_i = \frac{\zeta_l + \zeta_u}{2}.
\end{aligned}$$

where a and b are such that $\zeta_l < a < c < b < \zeta_u$ and ε is a small positive number, and

$$\begin{aligned}
q_i^*(\zeta_i) &= \frac{\zeta_i - \zeta_l}{\zeta_u - \zeta_l} + \frac{\varepsilon}{2(\zeta_u - \zeta_l)}(1 - \frac{\varepsilon - |\zeta_i - a|}{\varepsilon}) \text{ for } \zeta_i \in (a - \varepsilon,\, a + \varepsilon) \\
q_i^*(\zeta_i) &= \frac{\zeta_i - \zeta_l}{\zeta_u - \zeta_l} + \frac{\varepsilon}{2(\zeta_u - \zeta_l)}(\frac{\varepsilon - |\zeta_i - b|}{\varepsilon} - 1) \text{ for } \zeta_i \in (b - \varepsilon,\, b + \varepsilon) \\
q_i^*(\zeta_i) &= \frac{\zeta_i - \zeta_l}{\zeta_u - \zeta_l}, \text{ otherwise} \\
\bar{q}_i^*(\zeta_i) &= 1 - q_i^*(\zeta_i).
\end{aligned}$$

It follows that $c = c^*$, $q_i^* - \bar{q}_i^*$ is non-decreasing and $(Q_i^*, u_i^*(\zeta_l))$ satisfies (IC) and (IR). ■