กรอบแนวคิดในการวิเคราะห์บทบาทของรัฐบาลในการส่งเสริมเทคโนโลยี ที่เป็นสาเหตุของความเหลื่อมล้ำในการกระจายรายได้

An Analytical Framework of Government Role in Technological Promotion as a Cause of Inequality

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หลังจากได้ทบทวนวรรณกรรมที่เกี่ยวข้อง งานศึกษาชิ้นนี้ได้ตั้งสมมติฐานเกี่ยวกับช่องว่างของค่าจ้างระหว่างแรงงานมีฝีมือและแรงงานไร้ฝีมือ ว่ามีสาเหตุจากความก้าวหน้าทางเทคโนโลยีที่เข้าข้างแรงงานมีฝีมือ (skilled-biased technological progress) อย่างไรก็ตาม งานวิจัยชิ้นนี้มีกรอบความคิดใหม่ที่แตกต่างจากงานวิจัยเดิมอื่นๆ โดยมุ่งเน้นในประเด็นบทบาทของรัฐบาลในการส่งเสริมความก้าวหน้าทางเทคโนโลยี การเปลี่ยนแปลงของสัดส่วนโครงสร้างแรงงาน (สัดส่วนของจำนวนแรงงานมีฝีมือต่อแรงงานไร้ฝีมือ) และการเปลี่ยนแปลงของราคาสัมพัทธ์ของสินค้าในตลาดโลก (สัดส่วนของราคาในตลาดโลกของสินค้าที่ใช้แรงงานมีฝีมือเข้มข้นต่อสินค้าที่ใช้แรงงานไร้ฝีมือเข้มข้น) ว่าปัจจัยเหล่านี้อาจเป็นสาเหตุของความเหลื่อมล้ำดังกล่าว

งานศึกษานี้มีเป้าหมายที่จะสร้างแบบจำลองด้านการค้าระหว่างประเทศ ที่มีผู้วางแผนส่วนกลางทำหน้าที่ในการส่งเสริมเทคโนโลยีด้วย แบบจำลองในงานศึกษานี้กำหนดให้มีช่วงเวลา 2 คาบ และนอกจากจะกำหนดพฤติกรรมของผู้บริโภคและผู้ผลิตแล้ว แบบจำลองนี้ยังกำหนดพฤติกรรมของผู้วางแผนส่วนกลางซึ่งมีเป้าหมายที่จะทำให้ผลรวมของรายได้ประชาชาติที่ใช้จ่ายได้จริงต่อหัว (per Capita disposable national income-pDNI) มีค่าสูงสุด โดยใช้เครื่องมือการคลังในการเก็บภาษีและการใช้จ่ายเพื่อเพิ่มประสิทธิภาพของแรงงานมีฝีมือและไร้ฝีมือในช่วงเวลาคาบที่ 1

เป้าหมายอีกประการหนึ่งของงานวิจัยนี้คือ เพื่อจะอธิบายผลกระทบของการเปลี่ยนแปลงของราคาสัมพัทธ์ของสินค้าในตลาดโลก และสัดส่วนโครงสร้างแรงงานต่อการกระจายรายได้ผ่านบทบาทของรัฐบาลในการส่งเสริมเทคโนโลยี โดยงานศึกษานี้ได้นิยามให้การกระจายรายได้วัดจากค่าจ้างสัมพัทธ์ (สัดส่วนของค่าจ้างแรงงานมีฝีมือต่อแรงงานไร้ฝีมือ) ที่อยู่ในแบบจำลอง

งานศึกษานี้อธิบายกลไกของการกระจายรายได้ผ่านทาง 3 สาเหตุ สาเหตุแรกคือรายได้ภาครัฐ เมื่อสมมติให้จำนวนแรงงานและราคาสินค้าในตลาดโลกเท่ากันตลอดช่วงเวลา 2 คาบ พบว่าภายใต้รัฐบาลที่มีเป้าหมายในการทำให้รายได้ประชาชาติมากที่สุดผ่านทางการส่งเสริมเทคโนโลยีนั้น รัฐบาลจะทำให้การกระจายรายได้เลวลงหากสัดส่วนโครงสร้างแรงงานมีค่ามากกว่าค่าวิกฤติ แต่ในทางตรงกันข้ามรัฐบาลจะทำให้การกระจายรายได้ดีขึ้นหากสัดส่วนโครงสร้างแรงงานมีค่าน้อยกว่าค่าวิกฤติ

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สาเหตุที่สองคือการเพิ่มขึ้นของสัดส่วนโครงสร้างแรงงานของช่วงเวลาคาบที่ 2 การเพิ่มขึ้นเล็กน้อยของสัดส่วนโครงสร้างแรงงานไม่ทำให้ค่าจ้างสัมพัทธ์เปลี่ยนแปลงในช่วงเวลาคาบที่ 1 แต่ทำให้ค่าจ้างที่แท้จริงของแรงงานมีฝีมือเพิ่มขึ้น แต่ค่าจ้างที่แท้จริงของแรงงานไร้ฝีมือลดลง จึงทำให้ค่าจ้างสัมพัทธ์เพิ่มขึ้น ซึ่งหมายถึงการกระจายรายได้เลวลงในช่วงเวลาคาบที่ 2 เมื่อเทียบกับกรณีที่สัดส่วนโครงสร้างแรงงานไม่เปลี่ยน

สาเหตุที่สามคือการลดลงของราคาสัมพัทธ์ของสินค้าในตลาดโลก ผลจากงานศึกษานี้สอดคล้องกับคำพยากรณ์ของทฤษฎีการค้าระหว่างประเทศของเฮคเชอร์-โอห์ลิน การเพิ่มขึ้นเล็กน้อยของราคาสัมพัทธ์ของสินค้าในตลาดโลกทำให้ค่าจ้างที่แท้จริงของแรงงานมีฝีมือลดลง แต่ค่าจ้างที่แท้จริงของแรงงานไร้ฝีมือเพิ่มขึ้น จึงทำให้ค่าจ้างสัมพัทธ์ลดลง ซึ่งหมายถึงการกระจายรายได้ดีขึ้นในช่วงเวลาที่ 2 คาบ ยิ่งไปกว่านั้นคือการเพิ่มขึ้นหรือลดลงของค่าจ้างสัมพัทธ์ในคาบที่ 2 จะรุนแรงกว่าคาบที่ 1

เมื่อรวมข้อสรุปจากข้างต้น การเพิ่มขึ้น (ลดลง) ของแรงงานมีฝีมือ (ไร้ฝีมือ) และการเพิ่มขึ้น (ลดลง) ของราคาสินค้าที่ใช้แรงงานมีฝีมือ (ไร้ฝีมือ) เข้มข้นในช่วงเวลาคาบที่ 2 จะเป็นตัวหน่วงให้ช้าลงหากการใช้จ่ายของรัฐบาลทำให้การกระจายรายได้ดีขึ้น แต่จะเป็นตัวเร่งหากการใช้จ่ายของรัฐบาลทำให้การกระจายรายได้เลวลง

ABSTRACT

After reviewing related literature, this study arises from the hypothesis that chronic wage gap between skilled and unskilled labors may come from skilled-biased technological progress. Unlike many studies which consider skilled-biased technological progress as exogeneous variable, this study points that government policy, a change in skilled to unskilled labors proportion and a change in relative world price may be principals of this phenomenon.

Firstly, this study aims at constructing the international traded-model which includes the role of government in technological promotion. The economy in this model is set up to last for two periods. Besides the production and consumer sides, this study sets up the per Capita disposable national income (pDNI) maximizing central planner who imposts taxation and expenses to promote efficiency of skilled and unskilled labors in the first period.

The other aim of this study is to explain the impact of the changes in the relative world price and skilled to unskilled labors proportion on the inequality through the channel of government's technological promotion. In this study, inequality refers to relative wage between skilled and unskilled labors.

This study explains three causes of inequality as follows. The first cause of inequality is government expenditure. Given the amount of labors and world prices being equal throughout two periods, under actions of national income maximizing government as technology promoter, government increases inequality if the labor proportion is more than critical value but decreases inequality if the labor proportion is less than critical value.

The second cause is the increase of skilled to unskilled labor proportion in the second period. While a small change in the labor proportion does not affect the relative wage in the first period, it increases the relative wage in the second period, comparing to the case of unchanged labor proportion.

The third cause is the decrease of the relative world price. The result from this study aligns with the prediction in the Stolper-Samuelson theorem; When the relative world price in the first and the second period increases (decreases) by the same small percentage, skilled labors real wages will decrease (increase) and unskilled labors real wages will increase (decrease). Moreover, the magnitude of increasing or decreasing of real wages in the second period is more extreme than the first period.

Integrating all of these conclusions, an increasing (a decrease) of skilled (unskilled) labors and an increase (a decrease) of price of goods which is skilled(unskilled)-labor intensive in the second period will retard government's inequality reduction if the labor proportion is more than critical value, but reinforce government's inequality creation if the labor proportion is more than critical value.

Keyword : Skilled-biased technological change, inequality, government, relative wage, relative world price, skilled labor, unskilled labor,

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Introduction

Inequality between the rich and the poor is the chronic economic and social problem in many economies. Thailand is another one of the most rapid growth country for the past several decades but inequality still exists. Increasing in degree of openness, Thailand's volumes of export and import have been increasing dramatically for a few decades. These facts do not align with the prediction in the Hecksher-Ohlin model which predicts that wages of unskilled labors will increase more than other factor's return when a country exports unskilled-labor intensive products after trade opening. Thus, there must be other important "black box" that influences inequality in Thailand.

The controversy about the causes of inequality has been debated for a long time. International trade liberalization and technological change have been generally accepted as the mutually causes of inequality (Romagosa, 2005). However, while some economists claim that skilled-biased technological change is the major cause of inequality, the others blame international trade liberalization. To combine both aspects, this study will focus the effect of technological change on inequality in the context of international trade.

Nowadays, technological progress grows rapidly and, moreover, asymmetrically: skilled-biased technological change. This study states the hypothesis about skilled biased technological change that this process cannot takes place by itself. On the contrary, it is a consequence from other changes. This study will focus on several suspicious principals.

Table 1

R&D expenditure, percentage of overall domestic's R&D expenditure in 2000 and 2001

Countries	Government sector	Industrial sector	Higher Education sector	others
China	25%	63%	12%	-
Malaysia	25%	58%	17%	-
Korea	15%	74%	11%	-
USA.	11%	73%	12%	4%
Thailand	46%	35%	18%	1%
Norway	23%	51%	26%	-

Source: Adapted from the National Research Council of Thailand[2]

The first suspicious principal is the government who plays a role as technology promoter. After reviewing the literature about technology progress, there are very few models in which include the roles of government. The small role of government in R&D sector in developed countries may be an explanation for why it has been neglected by western researchers. But in the context of some developing countries, especially Thailand, the government's role in R&D is more important than the private sector, according to table 1. Therefore, this research aims at constructing a model which includes the role of government as technology promoter.

The second suspicious principal is increasing of skilled labors. Acemoglu (2002) remarks that, over the past 60 years, the U.S. relative supply of skilled labors has increased rapidly. Contradicting to conventional ideas, this leaded to increasing in the college premium over this time period. Moreover, Acemoglu claims that skilled biased technology concerns with this phenomenon. For the case of Thailand, education system has been developed and causes a sharp increasing in skilled labors who graduate from colleges or universities.

[2] Downloaded from http://www.nrct.net/images/190404circlegraph-B.jpg

The last suspicious principal is world prices. Since Thai economy heavily depends on world economy, any fluctuations from the external can penetrate to local economy through swing of world prices and may also impact technology in production of Thailand.

This study aims at constructing the model which includes the role of government as technology promoter in the context of open trade economy and explaining the impact of changes in relative world price and labors force on the inequality through the channel of government's technological promotion.

Reviews of related literature

The Literature about inequality topics

There are many ways to measure income inequality. The classic one is Gini coefficient, measuring on a scale from zero (perfect equality) to one (perfect inequality). The other alternative is Quintiles share (20%) of national income. But these measures are limited for applying in theoretical model. In many theoretical papers, such as Acemoglu (2001), Zhu and Trefler (2005), use *relative wage*, the ratio of skilled wage and unskilled wage. The relative wage measure how difference of wage between skilled labors and unskilled labors. In this research, relative wage is applied for measurement of income inequality because it is suitable for the structure of model in this study.

Many researchers point that inequality has been increasing for decades, e.g., Acemoglu (2001,2002), Zhu (2005), Cline (1996). Juhn and Murphy(1995) study the contrasted change between wage inequality and the growth of demand and supply of skilled labor and find that the gap between skilled and unskilled labors is even worse though the supply of skilled labors increase. Cornia and Kiiski conclude that most of all countries faced the rising of inequality. Therefore, Rising of inequality in over times is widely common agreed by economists.

There are vast differences in explaining for the cause of inequality. Among many arguments for the causes of inequality, however, "trade-and-wage debate" is frequently cited by many economists. Difference between skilled and unskilled labors wages is caused by the rightward shift in relative demand for skilled labor. On one hand, some believe that skilled-biased technological change increases the demand and skilled labors wages. On the other hand, the others claim that international trade has an account.

Zhu and Trefler (2005) replicate continuum-of-goods Hecksher-Ohlin model (Dornbusch, Fischer and Samuelson 1980) and develop by allowing existence of technology gap between the North and the South. On their opinion, technological change, skilled-biased or not, is the major cause of inequality and international trade plays a role as channel of the effect from technological progress.

Acemoglu (2001) builds the model in which technical change is endogenous. In application of the model, Acomoglu points the fact that the amount of skilled labors and skilled to unskilled wage ratio have positively related. Acemoglu proposes the alternative explanation that increasing of skilled premium and skilled biased technical change are the mutually effect from the exogeneous causes, such as migration and international trade. In Acemoglu's opinion, skilled-biased technological change is just the one of consequences from international trade, which is one of causes of increasing inequality in his model.

Acemoglu's model can be applied for constructing model in this study. Given technological progress as endogeneous variable, the model has story behind the process of technology change. Technological progress does not take place by itself, but it is driven by other factors or actors in model. Unlike Acemoglu, this study aims at constructing the model which includes the role of government in technological promotion.

The Literature about the role of government in technology progress

Beije (1998) explained that government may play a role in stimulating R&D in two points: the use of patents and adopting policy. He sorts out technology-stimulating policy into science policy, which is concerned with higher education and public research, and technology policy, which focuses on subsidies or taxes to stimulate innovation in private firms directly.

The instrument of technology policies can be sorted out into direct and indirect instrument. Direct instruments try to stimulate private firms' R&D directly, for example, tax facilitations and subsidies. Indirect instruments try to facilitate creation, diffusion and application of technological knowledge.

The model

The set up

The economy in this model is small and free trade economy. The economy lasts only two period, $t \in \{1, 2\}$. There are 2 goods, X and Y with the price $P_{X,t}$ and $P_{Y,t}$ in each period. Since the economy is small and free trade economy, $P_{X,t}$ and $P_{Y,t}$ are equal to the exogeneous world prices, $P_{X,t}^{W}$ and $P_{Y,t}^{W}$. There are 2 factors; skilled labors, H, and unskilled labors, L, whose wages rate at $w_{H,t}$ and $w_{L,t}$ respectively. The economy has the fix amount of skilled labors and unskilled labors in each period, H_t^S and L_t^S. We assume that all goods and factors markets are perfectly competitive.

Each good has production function as follow.

$$X_t^P = \left[\gamma_X \left(A_{L,t} L_{X,t} \right)^{\frac{\sigma-1}{\sigma}} + \left(1-\gamma_X\right)\left(A_{H,t} H_{X,t} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{1}$$

$$Y_t^P = \left[\gamma_Y \left(A_{L,t} L_{Y,t} \right)^{\frac{\sigma-1}{\sigma}} + \left(1-\gamma_Y\right)\left(A_{H,t} H_{Y,t} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \tag{2}$$

Where $A_{L,t} > 0$ and $A_{H,t} > 0$ are efficiencies of unskilled labors and skilled labors in period t respectively. γ_i is a distribution parameter which determines how important the two factors are. $L_{I,t}$ and $H_{I,t}$ are amount of unskilled and skilled labors used in I industries, $I_t \in \{X_t, Y_t\}$. This study assumes that $0 < \gamma_Y < \gamma_X < 1$ to set X as unskilled labors intensive relative to Y. $\sigma \in (0, \infty)$ is the elasticity of substitution between the two factors which are assumed to have the same σ. Note that we superscript P to specify equation (1) and (2) as the amount of X and Y produced in economy. They are not necessary equals to the amount of X and Y consumed in the economy, X_t^C and Y_t^C, which will be shown in consumer side topic.

In the first period, the economy's central planner collects ad-varolem tax at rate τ equally on every labor's income and distributes total amount of revenue are into G_L and G_H for promoting efficiency of unskilled labors and skilled labors of next period (i.e. , increase $A_{L,2}$ and $A_{H,2}$). In the second period, Central planner will do nothing. Growth of A_L and A_H can be explained by the equations as follow,

$$A_{L,2} = A_{L,1}\left(\alpha_L G_L\right)^{\delta} \tag{3}$$

$$A_{H,2} = A_{H,1}\left(\alpha_H G_H\right)^{\delta} \; ; \quad 0 < \delta < 1 \tag{4}$$

Where α_L and α_H are coefficients of G_j where $j \in \{L, H\}$. Note that G_L and G_H are not specified period because government can only make a decision in the first period. The objective of central planner is to maximize country's per capita Disposable National Income (pDNI).

The production side

There are 2 groups of equilibrium conditions. The first set is Zero-profit conditions. Since goods markets are competitive, the prices are equal to their marginal cost,

$$P_{X,t}^W = MC_{X,t}\left(w_{L,t}, w_{H,t}\right) \tag{5}$$

$$P_{Y,t}^W = MC_{Y,t}\left(w_{L,t}, w_{H,t}\right) \tag{6}$$

Note that our marginal cost functions are independent of X_t^P and Y_t^P because their production functions are constant return to scale.

The others conditions are full employment conditions. Each factor can be employed in 2 sectors and assumed to be full employment. According to Shephard's lemma, these condition are written as follows

$$L_t^S = X_t^P \frac{\partial MC_{X,t}\left(w_{L,t}, w_{H,t}\right)}{\partial w_{L,t}} + Y_t^P \frac{\partial MC_{Y,t}\left(w_{L,t}, w_{H,t}\right)}{\partial w_{L,t}} \tag{7}$$

$$H_t^S = X_t^P \frac{\partial MC_{X,t}\left(w_{L,t}, w_{H,t}\right)}{\partial w_{H,t}} + Y_t^P \frac{\partial MC_{Y,t}\left(w_{L,t}, w_{H,t}\right)}{\partial w_{H,t}} \tag{8}$$

Since the equations (5), (6), (7) and (8) are 4 equilibrium conditions and there are 4 endogeneous variables, $w_{L,t}, w_{H,t}, X_t^P, Y_t^P$, thus we can solve for an unique equilibrium $\overline{w}_{L,t}, \overline{w}_{H,t}, \overline{X}_t^P, \overline{Y}_t^P$.

To find marginal cost function, the cost functions of X and Y must be derived previously from minimizing cost. We set the cost minimization problem of X as follows.

$$\underset{L_{X,t}, H_{X,t}}{Min} \quad w_{L,t} L_{X,t} + w_{H,t} H_{X,t}$$

$$S.T. \qquad X_t^P\left(L_{X,t}, H_{X,t}\right) = \left[\gamma_X \left(A_{L,t} L_{X,t}\right)^{\frac{\sigma-1}{\sigma}} + \left(1-\gamma_X\right)\left(A_{H,t} H_{X,t}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

After solving above problem, we yield the cost function of goods X as follows.

$$C_{X,t}\left(w_{L,t}, w_{H,t}, X_t^P\right) = X_t^P \left[\gamma_X^\sigma A_{L,t}^{\sigma-1} w_{L,t}^{1-6} + \left(1-\gamma_X\right)^\sigma A_{H,t}^{\sigma-1} w_{H,t}^{1-6}\right]^{\frac{1}{1-\sigma}} \tag{9}$$

Do the same pattern with goods Y and then yield

$$C_{Y,t}\left(w_{L,t}, w_{H,t}, Y_t^P\right) = Y_t^P \left[\gamma_Y^\sigma A_{L,t}^{\sigma-1} w_{L,t}^{1-6} + \left(1-\gamma_Y\right)^\sigma A_{H,t}^{\sigma-1} w_{H,t}^{1-6}\right]^{\frac{1}{1-\sigma}} \tag{10}$$

Since $MC_{I,t} = \dfrac{\partial C_{I,t}\left(w_{L,t}, w_{H,t}, I_t^P\right)}{\partial I_t^P}$, $I_t^P \in \left\{X_t^P, Y_t^P\right\}$, and equations (5) and (6), the zero profit conditions of X and Y can be rewritten as

$$P_{X,t}^W = MC_{X,t}\left(w_{L,t}, w_{H,t}\right) = \left[\gamma_X^\sigma A_{L,t}^{\sigma-1} w_{L,t}^{1-6} + \left(1-\gamma_X\right)^\sigma A_{H,t}^{\sigma-1} w_{H,t}^{1-6}\right]^{\frac{1}{1-\sigma}} \tag{11}$$

$$P_{Y,t}^W = MC_{Y,t}\left(w_{L,t}, w_{H,t}\right) = \left[\gamma_Y^\sigma A_{L,t}^{\sigma-1} w_{L,t}^{1-6} + \left(1-\gamma_Y\right)^\sigma A_{H,t}^{\sigma-1} w_{H,t}^{1-6}\right]^{\frac{1}{1-\sigma}} \tag{12}$$

There are 2 conditions (equations), equations (11) and (12), with 2 unknowns wages, $w_{L,t}, w_{H,t}$, so we can solve this system for equilibrium wages of skilled and unskilled labors and yield

$$\overline{w}_{H,t} = A_{H,t} \left[\frac{\gamma_X^\sigma \left(P_{Y,t}^W\right)^{1-\sigma} - \gamma_Y^\sigma \left(P_{X,t}^W\right)^{1-\sigma}}{\gamma_X^\sigma \left(1-\gamma_Y\right)^\sigma - \left(1-\gamma_X\right)^\sigma \gamma_Y^\sigma}\right]^{\frac{1}{1-\sigma}} \tag{13}$$

$$\overline{w}_{L,t} = A_{L,t} \left[\frac{\left(1-\gamma_Y\right)^\sigma \left(P_{X,t}^W\right)^{1-\sigma} - \left(1-\gamma_X\right)^\sigma \left(P_{Y,t}^W\right)^{1-\sigma}}{\gamma_X^\sigma \left(1-\gamma_Y\right)^\sigma - \left(1-\gamma_X\right)^\sigma \gamma_Y^\sigma}\right]^{\frac{1}{1-\sigma}} \tag{14}$$

Equations (13) and (14) show the skilled and unskilled labors wages in the case of no taxation, i.e. $\tau = 0$. For the case of $\tau > 0$, wages in the first period must be sorted out into 2 types: *the market wages* which are the rate for producer paying for labors, and *the disposable wages* which are the rate that labors actually receive after tax. Since elasticity of supply for labors are zero (due to H_t^S and L_t^S are fix amount), labors will bare the full burden of tax. Equations (13) and (14) can be rewritten to identify wages as follows,

$$\overline{w}_{H,1}^{M} = A_{H,1}\left[\frac{\gamma_X^{\sigma}\left(P_{Y,1}^{W}\right)^{1-\sigma} - \gamma_Y^{\sigma}\left(P_{X,1}^{W}\right)^{1-\sigma}}{\gamma_X^{\sigma}\left(1-\gamma_Y\right)^{\sigma} - \left(1-\gamma_X\right)^{\sigma}\gamma_Y^{\sigma}}\right]^{\frac{1}{1-\sigma}} \tag{15}$$

$$\overline{w}_{L,1}^{M} = A_{L,1}\left[\frac{\left(1-\gamma_Y\right)^{\sigma}\left(P_{X,1}^{W}\right)^{1-\sigma} - \left(1-\gamma_X\right)^{\sigma}\left(P_{Y,1}^{W}\right)^{1-\sigma}}{\gamma_X^{\sigma}\left(1-\gamma_Y\right)^{\sigma} - \left(1-\gamma_X\right)^{\sigma}\gamma_Y^{\sigma}}\right]^{\frac{1}{1-\sigma}} \tag{16}$$

$$w_{H,1}^{D} = \left(1-\tau\right)\overline{w}_{H,1}^{M} \tag{17}$$

$$w_{L,1}^{D} = \left(1-\tau\right)\overline{w}_{L,1}^{M} \tag{18}$$

$$w_{H,2} = A_{H,2}\left[\frac{\gamma_X^{\sigma}\left(P_{Y,2}^{W}\right)^{1-\sigma} - \gamma_Y^{\sigma}\left(P_{X,2}^{W}\right)^{1-\sigma}}{\gamma_X^{\sigma}\left(1-\gamma_Y\right)^{\sigma} - \left(1-\gamma_X\right)^{\sigma}\gamma_Y^{\sigma}}\right]^{\frac{1}{1-\sigma}} \tag{19}$$

$$w_{L,2} = A_{L,2}\left[\frac{\left(1-\gamma_Y\right)^{\sigma}\left(P_{X,2}^{W}\right)^{1-\sigma} - \left(1-\gamma_X\right)^{\sigma}\left(P_{Y,2}^{W}\right)^{1-\sigma}}{\gamma_X^{\sigma}\left(1-\gamma_Y\right)^{\sigma} - \left(1-\gamma_X\right)^{\sigma}\gamma_Y^{\sigma}}\right]^{\frac{1}{1-\sigma}} \tag{20}$$

Where $\overline{w}_{H,1}^{M}$ and $\overline{w}_{L,1}^{M}$ are the skilled labor market wage and the unskilled labor market wage. $w_{H,1}^{D}$ and $w_{L,1}^{D}$ are the skilled labor disposable wage and the unskilled labor disposable wage.

Equations (15) and (16) show that market wages before and after tax are the same because employers (producers) do not bare the tax burden at all. Equations (17) and (18) show that labors, both skilled and unskilled, bare the full burden of tax. Equations (19) and (20) have the same forms as equation (13) and (14) since no tax are not collected in the second period. Note that wages in equations from (17) to (20) are not equilibrium wages until solving for equilibrium central planner's tax rate and expenditure.

For positiveness of wages, relative world price must lie in these range,

$$\left(\frac{1-\gamma_X}{1-\gamma_Y}\right)^{\frac{\sigma}{1-\sigma}} < \frac{P_{X,t}^{W}}{P_{Y,t}^{W}} < \left(\frac{\gamma_X}{\gamma_Y}\right)^{\frac{\sigma}{1-\sigma}} \quad if\ 0<\sigma<1 \tag{21}$$

$$\left(\frac{\gamma_Y}{\gamma_X}\right)^{\frac{\sigma}{\sigma-1}} < \frac{P_{X,t}^{W}}{P_{Y,t}^{W}} < \left(\frac{1-\gamma_Y}{1-\gamma_X}\right)^{\frac{\sigma}{\sigma-1}} \quad if\ \sigma>1 \tag{22}$$

Now turn to solve for full employment conditions. After differentiating equation (11) and (12) by $w_{L,t}, w_{H,t}$ and substituting in equation (7) and (8), the rewritten full employment conditions are

$$L_t^S = X_t^P\left(p_{X,t}^{W}\right)^{\sigma}\gamma_X^{\sigma}A_{L,t}^{\sigma-1}\overline{w}_{L,t}^{-\sigma} + Y_t^P\left(p_{Y,t}^{W}\right)^{\sigma}\gamma_Y^{\sigma}A_{L,t}^{\sigma-1}\overline{w}_{L,t}^{-\sigma} \tag{23}$$

$$H_t^S = X_t^P\left(p_{X,t}^{W}\right)^{\sigma}\left(1-\gamma_X\right)^{\sigma}A_{H,t}^{\sigma-1}\overline{w}_{H,t}^{-\sigma} + Y_t^P\left(p_{Y,t}^{W}\right)^{\sigma}\left(1-\gamma_Y\right)^{\sigma}A_{H,t}^{\sigma-1}\overline{w}_{H,t}^{-\sigma} \tag{24}$$

Note that we use $\overline{w}_{H,t}$ and $\overline{w}_{L,t}$ from equations (13) and (14). Since equation (15), (16), (19) and (20) show that rate of wages which producers pay for labors are the same rate whether in the case of taxation or not, equations (13) and (14) are applicable in both cases either. After solving, the equilibrium outputs are

$$\overline{X}_t^P = \left(p_{X,t}^W\right)^{-\sigma} \frac{\left(1-\gamma_Y\right)^{\sigma} A_{L,t}^{1-\sigma}\overline{w}_{L,t}^{6} L_t^S - \gamma_Y^{\sigma} A_{H,t}^{1-\sigma}\overline{w}_{H,t}^{6} H_t^S}{\gamma_X^{\sigma}\left(1-\gamma_Y\right)^{\sigma} - \left(1-\gamma_X\right)^{\sigma}\gamma_Y^{\sigma}} \tag{25}$$

$$\overline{Y}_t^P = \left(p_{Y,t}^W\right)^{-\sigma} \frac{\gamma_X^{\sigma} A_{H,t}^{1-\sigma}\overline{w}_{H,t}^{6} H_t^S - \left(1-\gamma_X\right)^{\sigma} A_{L,t}^{1-\sigma}\overline{w}_{L,t}^{6} L_t^S}{\gamma_X^{\sigma}\left(1-\gamma_Y\right)^{\sigma} - \left(1-\gamma_X\right)^{\sigma}\gamma_Y^{\sigma}} \tag{26}$$

These imply that the production of $\overline{X}_t^P$ and $\overline{Y}_t^P$ in equation (25) and (26) are not affected by tax due to insensitivity of the market wages to tax. To force quantity of both outputs to be greater than zero, following conditions must be set up.

$$\left(1-\gamma_Y\right)^{\sigma} A_{L,t}^{1-\sigma}\overline{w}_{L,t}^{6} L_t^S - \gamma_Y^{\sigma} A_{H,t}^{1-\sigma}\overline{w}_{H,t}^{6} H_t^S > 0 \tag{27}$$

$$\gamma_X^{\sigma} A_{H,t}^{1-\sigma}\overline{w}_{H,t}^{6} H_t^S - \left(1-\gamma_X\right)^{\sigma} A_{L,t}^{1-\sigma}\overline{w}_{L,t}^{6} L_t^S > 0 \tag{28}$$

For positiveness of outputs, relative world price must lie in these range, at any relative world price that satisfies given range of in equations (21) and (22), equation (29) must be satisfied to guarantee that both goods X and Y are produced, i.e. $\overline{X}_t^P > 0$ and $\overline{Y}_t^P > 0$.

$$\frac{\left(1-\gamma_X\right)^{\sigma}}{\gamma_X^{\sigma}}\Gamma_t\left(\frac{P_{X,t}^W}{P_{Y,t}^W}\right) < \frac{A_{H,t}H_t^S}{A_{L,t}L_t^S} < \frac{\left(1-\gamma_Y\right)^{\sigma}}{\gamma_Y^{\sigma}}\Gamma_t\left(\frac{P_{X,t}^W}{P_{Y,t}^W}\right) \tag{29}$$

where $\Gamma_t\left(\frac{P_{X,t}^W}{P_{Y,t}^W}\right) = \left[\frac{\left(1-\gamma_Y\right)^{\sigma}\left(\frac{P_{X,t}^W}{P_{Y,t}^W}\right)^{1-\sigma} - \left(1-\gamma_X\right)^{\sigma}}{\gamma_X^{\sigma} - \gamma_Y^{\sigma}\left(\frac{P_{X,t}^W}{P_{Y,t}^W}\right)^{1-\sigma}}\right]^{\frac{\sigma}{1-\sigma}}$

The consumer side

Assuming that entire population in the economy have the same preference, the social welfare function is

$$U_t = X_t^C Y_t^C \tag{30}$$

Where X_t^C and Y_t^C are the amounts of X and Y which are consumed in economy.

The consumer's maximization problem is

$$\underset{X_t^C, Y_t^C}{Max} \quad \sum_{t=1}^{2} U_t = \sum_{t=1}^{2} \beta_C^{t-1} \; X_t^C Y_t^C$$

$$S.T. \qquad P_{X,1}^W X_1^C + P_{Y,1}^W Y_1^C = w_{L,1}^D L_1^S + w_{H,1}^D H_1^S$$

$$P_{X,2}^W X_2^C + P_{Y,2}^W Y_2^C = w_{L,2} L_2^S + w_{H,2} H_2^S$$

After solving above problem, the equilibrium consumption of goods X and Y in both periods are as follow

$$X_2^C = \frac{w_{L,2}L_2^S + w_{H,2}H_2^S}{2P_{X,2}^W} \tag{31}$$

$$Y_2^C = \frac{w_{L,2}L_2^S + w_{H,2}H_2^S}{2P_{Y,2}^W} \tag{32}$$

$$X_1^C = \frac{w_{L,1}^D L_1^S + w_{H,1}^D H_1^S}{2P_{X,1}^W} = (1-\tau)\frac{w_{L,1}^M L_1^S + w_{H,1}^M H_1^S}{2P_{X,1}^W} \tag{33}$$

$$Y_1^C = \frac{w_{L,1}^D L_1^S + w_{H,1}^D H_1^S}{2P_{Y,1}^W} = (1-\tau)\frac{w_{L,1}^M L_1^S + w_{H,1}^M H_1^S}{2P_{Y,1}^W} \tag{34}$$

Again, note that X_t^C and Y_t^C are not equilibrium consumption until solving for equilibrium central planner's tax rate and expenditure.

GDP , DNI and pDNI of Economy

There are 3 approaches for calculating Gross Domestic Product (GDP), output approach, expenditure approach and income approach. The definitions of three approaches are shown in equations as follows.

$$GDP_t^O \equiv p_X^W \bar{X}_t^P + p_Y^W \bar{Y}_t^P \tag{35}$$

$$GDP_2^E \equiv p_{X,2}^W X_2^C + p_2^W Y_2^C \tag{36}$$

$$GDP_1^E \equiv p_{X,1}^W X_1^C + p_{Y,1}^W Y_1^C + G_L + G_H \tag{37}$$

$$GDP_2^I \equiv w_{L,2}L_2^S + w_{H,2}H_2^S \tag{38}$$

$$GDP_1^I \equiv w_{L,1}^M L_1^S + w_{H,1}^M H_1^S \tag{39}$$

Since there is central planner's expenditure in the first period, the definition of GDP in expenditure approach in the first period must includes central planner's expenditure, $G_L + G_H$. Since there are two types of wages in the first period and the GDP in income approach must be calculated by market rate of wages, the definition of GDP in income approach in the first period applies market wages for calculating. There exists indifference among these three approaches though the proof is omitted due to limitation of pages.

Moreover, indifference among three approaches also implies that central planner's taxation (or expenditure) dose not affect GDP in the first period because the value of GDP_1^O is not affected by tax. On the contrary, central planner's action affect economy's GDP at the second period due to increasing of $A_{H,2}$ and $A_{L,2}$.

Disposable National Income (DNI) is the summation of every labor's incomes which are able to spend for purchasing, while per capita Disposable National Income (pDNI) is average DNI per one labor in economy. DNI and pDNI are defined as follows,

$$DNI_t \equiv GDP_t - total\ tax \tag{40}$$

$$pDNI_t \equiv \frac{DNI_t}{L_t^S + H_t^S} \tag{41}$$

Since there is no taxation in the second period, $DNI_2 = GDP_2$ and $pDNI_2 = \frac{GDP_2}{L_2^S + H_2^S}$. But in the first period, taxation affects DNI_1 and $pDNI_1$ as follows

$$DNI_1 = (1-\tau)\, w_{L,1}^M L_1^S + w_{H,1}^M H_1^S = w_{L,1}^D L_1^S + w_{H,1}^D H_1^S \tag{42}$$

$$pDNI_1 = (1-\tau)\frac{w_{L,1}^M L_1^S + w_{H,1}^M H_1^S}{L_1^S + H_1^S} = \frac{w_{L,1}^D L_1^S + w_{H,1}^D H_1^S}{L_1^S + H_1^S} \tag{43}$$

Central planner's problem

Since central planner's objective is to maximize overall per capita disposable income, the central planner's problem is set up as follows

$$\underset{\tau, G_L, G_H}{Max} \quad \sum_{t=1}^{2} \beta_G^{t-1} pDNI \ \ ; \ 0 < \beta_G < 1$$

$$S.T. \qquad \tau\left(w_{L,1}^M L_1^S + w_{H,1}^M H_1^S\right) = G_L + G_H \quad , \quad 0 < \tau < 1$$

where β_G is discount factor of central planner.

After solving above problem, the equilibrium central planner's expenditure and tax rate are as follows

$$\bar{G}_L = \left[\beta_G \alpha_L^\delta\, \delta A_{L,1} \frac{L_2^S\left(L_1^S + H_1^S\right)}{L_2^S + H_2^S}\left[\frac{(1-\gamma_Y)^\sigma \left(P_{X,2}^W\right)^{1-\sigma} - (1-\gamma_X)^\sigma \left(P_{Y,2}^W\right)^{1-\sigma}}{\gamma_X^\sigma (1-\gamma_Y)^\sigma - (1-\gamma_X)^\sigma \gamma_Y^\sigma}\right]^{\frac{1}{1-\sigma}}\right]^{\frac{1}{1-\delta}} \tag{44}$$

$$\bar{G}_H = \left[\beta_G \alpha_H^\delta \delta A_{H,1} \frac{H_2^S\left(L_1^S + H_1^S\right)}{L_2^S + H_2^S}\left[\frac{\gamma_X^\sigma \left(P_{Y,2}^W\right)^{1-\sigma} - \gamma_Y^\sigma \left(P_{X,2}^W\right)^{1-\sigma}}{\gamma_X^\sigma (1-\gamma_Y)^\sigma - (1-\gamma_X)^\sigma \gamma_Y^\sigma}\right]^{\frac{1}{1-\sigma}}\right]^{\frac{1}{1-\delta}} \tag{45}$$

$$\bar{\tau}=\left(\beta_G\delta\frac{L_1^S+H_1^S}{L_2^S+H_2^S}\right)^{\frac{1}{1-\delta}}\left(A_{L,1}L_1^S\left[\frac{(1-\gamma_Y)^{\sigma}\left(P_{X,1}^W\right)^{1-\sigma}-(1-\gamma_X)^{\sigma}\left(P_{Y,1}^W\right)^{1-\sigma}}{\gamma_X^{\sigma}(1-\gamma_Y)^{\sigma}-(1-\gamma_X)^{\sigma}\gamma_Y^{\sigma}}\right]^{\frac{1}{1-\sigma}}+A_{H,1}H_1^S\left[\frac{\gamma_X^{\sigma}\left(P_{Y,1}^W\right)^{1-\sigma}-\gamma_Y^{\sigma}\left(P_{X,1}^W\right)^{1-\sigma}}{\gamma_X^{\sigma}(1-\gamma_Y)^{\sigma}-(1-\gamma_X)^{\sigma}\gamma_Y^{\sigma}}\right]^{\frac{1}{1-\sigma}}\right)^{-1}$$
$$\times\left[\alpha_L^{\delta}A_{L,1}L_2^S\left[\frac{(1-\gamma_Y)^{\sigma}\left(P_{X,2}^W\right)^{1-\sigma}-(1-\gamma_X)^{\sigma}\left(P_{Y,2}^W\right)^{1-\sigma}}{\gamma_X^{\sigma}(1-\gamma_Y)^{\sigma}-(1-\gamma_X)^{\sigma}\gamma_Y^{\sigma}}\right]^{\frac{1}{1-\sigma}}+\alpha_H^{\delta}A_{H,1}H_2^S\left[\frac{\gamma_X^{\sigma}\left(P_{Y,2}^W\right)^{1-\sigma}-\gamma_Y^{\sigma}\left(P_{X,2}^W\right)^{1-\sigma}}{\gamma_X^{\sigma}(1-\gamma_Y)^{\sigma}-(1-\gamma_X)^{\sigma}\gamma_Y^{\sigma}}\right]^{\frac{1}{1-\sigma}}\right]^{\frac{1}{1-\delta}} \tag{46}$$

Every parameters and exogeneous variables in equation (46) are positive, therefore $\bar{\tau}>0$ .To force $\bar{\tau}<1$, we must assume that

$$\left(\beta_G\delta\frac{L_1^S+H_1^S}{L_2^S+H_2^S}\right)^{\frac{1}{1-\delta}}\left(A_{L,1}L_1^S\left[\frac{(1-\gamma_Y)^{\sigma}\left(P_{X,1}^W\right)^{1-\sigma}-(1-\gamma_X)^{\sigma}\left(P_{Y,1}^W\right)^{1-\sigma}}{\gamma_X^{\sigma}(1-\gamma_Y)^{\sigma}-(1-\gamma_X)^{\sigma}\gamma_Y^{\sigma}}\right]^{\frac{1}{1-\sigma}}+A_{H,1}H_1^S\left[\frac{\gamma_X^{\sigma}\left(P_{Y,1}^W\right)^{1-\sigma}-\gamma_Y^{\sigma}\left(P_{X,1}^W\right)^{1-\sigma}}{\gamma_X^{\sigma}(1-\gamma_Y)^{\sigma}-(1-\gamma_X)^{\sigma}\gamma_Y^{\sigma}}\right]^{\frac{1}{1-\sigma}}\right)^{-1}$$
$$\times\left[\alpha_L^{\delta}A_{L,1}L_2^S\left[\frac{(1-\gamma_Y)^{\sigma}\left(P_{X,2}^W\right)^{1-\sigma}-(1-\gamma_X)^{\sigma}\left(P_{Y,2}^W\right)^{1-\sigma}}{\gamma_X^{\sigma}(1-\gamma_Y)^{\sigma}-(1-\gamma_X)^{\sigma}\gamma_Y^{\sigma}}\right]^{\frac{1}{1-\sigma}}+\alpha_H^{\delta}A_{H,1}H_2^S\left[\frac{\gamma_X^{\sigma}\left(P_{Y,2}^W\right)^{1-\sigma}-\gamma_Y^{\sigma}\left(P_{X,2}^W\right)^{1-\sigma}}{\gamma_X^{\sigma}(1-\gamma_Y)^{\sigma}-(1-\gamma_X)^{\sigma}\gamma_Y^{\sigma}}\right]^{\frac{1}{1-\sigma}}\right]^{\frac{1}{1-\delta}}<1 \tag{47}$$

Equation (47) must be satisfied to guarantee validity of result from central planner’ problem

Defining terms

In this study, Inequality refers to the skilled to unskilled wage proportion (relative wage, hereafter), which is mathematically expressed as $\frac{\bar{w}_{H,t}}{\bar{w}_{L,t}}$. In the case that central planner collects tax, the relative wage in the first period refers to “relative disposable wage”. Therefore, dividing equation (17) by (18), the relative wage in the first period is

$$\frac{\bar{w}_{H,1}^D}{\bar{w}_{L,1}^D}=\frac{A_{H,1}}{A_{L,1}}\left[\frac{\gamma_X^{\sigma}-\gamma_Y^{\sigma}\left(\frac{P_{X,1}^W}{P_{Y,1}^W}\right)^{1-\sigma}}{(1-\gamma_Y)^{\sigma}\left(\frac{P_{X,1}^W}{P_{Y,1}^W}\right)^{1-\sigma}-(1-\gamma_X)^{\sigma}}\right]^{\frac{1}{1-\sigma}} \tag{48}$$

Note that the relative disposable wage is the same as the relative market wage because tax rate is ad valorem. In the second period, dividing equation (19) by (20), the relative wage is

$$\frac{\bar{w}_{H,2}}{\bar{w}_{L,2}}=\frac{\bar{A}_{H,2}}{\bar{A}_{L,2}}\left[\frac{\gamma_X^{\sigma}-\gamma_Y^{\sigma}\left(\frac{P_{X,2}^{W}}{P_{Y,2}^{W}}\right)^{1-\sigma}}{\left(1-\gamma_Y\right)^{\sigma}\left(\frac{P_{X,2}^{W}}{P_{Y,2}^{W}}\right)^{1-\sigma}-\left(1-\gamma_X\right)^{\sigma}}\right]^{\frac{1}{1-\sigma}} \tag{49}$$

Unlike the first period, the term $\frac{\bar{A}_{H,2}}{\bar{A}_{L,2}}$ is endogeneous variable, so it may be changed by some changes of exogeneous variables.

There is perfect equality when $\frac{\bar{w}_{H,t}}{\bar{w}_{L,t}}=1$. this study assumes that initial relative wage is more than one, or skilled labors wages are greater than unskilled labors wages. Therefore, an increase of inequality refers to increasing of relative wage while a decrease of inequality refers to decreasing of relative wage which is not so great that make $\frac{\bar{w}_{H,t}}{\bar{w}_{L,t}}<1$.

Dynamic inequality in this study refers to comparison between the relative wage in the first and the second period.

To analyst central planner's behavior, dividing equation (45) by (44) yields

$$\frac{\bar{G}_H}{\bar{G}_L}=\left[\frac{\alpha_H^{\delta}}{\alpha_L^{\delta}}\frac{A_{H,1}}{A_{L,1}}\frac{H_2^S}{L_2^S}\left[\frac{\gamma_X^{\sigma}-\gamma_Y^{\sigma}\left(\frac{P_{X,2}^{W}}{P_{Y,2}^{W}}\right)^{1-\sigma}}{\left(1-\gamma_Y\right)^{\sigma}\left(\frac{P_{X,2}^{W}}{P_{Y,2}^{W}}\right)^{1-\sigma}-\left(1-\gamma_X\right)^{\sigma}}\right]^{\frac{1}{1-\sigma}}\right]^{\frac{1}{1-\delta}} \tag{50}$$

The value of $\frac{\bar{G}_H}{\bar{G}_L}$ in equation (50), which is called the "expenditure ratio", increase if central planner expenses more on promoting efficiency of skilled labors, relatively to unskilled labors'. The explanation is opposite if the expenditure ratio decreases.

The "efficiency ratio" is defined as $\frac{A_{H,t}}{A_{L,t}}$ and can be applied for comparing efficiency of skilled labors and unskilled labors. In the first period, $\frac{A_{H,1}}{A_{L,1}}$ is exogeneous. But efficiency ratio in the second period $\frac{A_{H,2}}{A_{L,2}}$ is determined by central planner's expenditure in the first period. According to equations (3) and (4), the efficiency ratio in the second period is

$$\frac{\bar{A}_{H,2}}{\bar{A}_{L,2}}=\frac{A_{H,1}}{A_{L,1}}\left(\frac{\alpha_H\bar{G}_H}{\alpha_L\bar{G}_L}\right)^{\delta} \tag{51}$$

A change in the efficiency ratio expresses a direction of technology progress; which one between skilled labors or unskilled labors that their efficiency increases more rapid? Therefore, we define that technology progress is skilled-biased when efficiency ratio in the second period increases from the first period and it is opposite in the definition of unskilled-biased.

The growth patterns of labor force are stylized as follows.

$$L_2^S = n_L L_1^S \tag{52}$$

$$H_2^S = n_H H_1^S \tag{53}$$

$0 < n_j < 1$ when the amount of j labors decrease and $1 < n_j < \infty$ when the amount of j labors increase ; $j \in \{L, H\}$. After dividing equation (53) by (52), yields

$$\frac{H_2^S}{L_2^S} = \frac{n_H}{n_L}\frac{H_1^S}{L_1^S} \equiv n\frac{H_1^S}{L_1^S} \tag{54}$$

According to above equation, n is called labor growth ratio. $0 < n < 1$ when skilled to unskilled labor proportion in the second period is less than that in the first period because the growth of skilled labors is less than unskilled labors, i.e. $n_H < n_L$. $n > 1$ when skilled to unskilled labor proportion in the second period is more than that in the first period because the growth of skilled labors is more than unskilled labors, i.e. $n_H > n_L$

We define skilled to unskilled labor proportion (the labor proportion, hereafter) as $\Phi_t = H_t^S / L_t^S$, therefore

$$\Phi_2 = n\Phi_1 \tag{55}$$

Equation (55) implies that increasing (decreasing) of skilled to unskilled labor proportion (labor proportion, hereafter) in the second period, Φ_2, may come from increasing (decreasing) of labor proportion endowment, Φ_1, and/or increasing (decreasing) of labor growth ratio, n.

Hereafter, the term “endowment” refers to exogeneous variables, except for relative world price, in the first period. For examples, “technology endowment” is the general term for any functions of $A_{H,1}$ and/or $A_{L,1}$, or “labor proportion endowment” refer to Φ_1.

The next section will analyze the dynamic inequality when central planner is technology promoter and other things being equal throughout two periods. Then analyze the dynamic of relative wage in the case of the labor proportion or relative world price is changeable.

Central planner’s expenditure and dynamic inequality

This section will show that, though all exogeneous variables do not change, central planner still plays an important role in changing the relative wage by himself. Given the same relative world price $\frac{P_{X,t}^W}{P_{Y,t}^W}$, the relative wages in both periods are the same if efficiency ratio in the second period is equal to efficiency ratio endowment. Therefore, we set the condition that makes indifferent relative wages between both periods as follows.

$$\frac{A_{H,1}}{A_{L,1}} = \frac{\bar{A}_{H,2}}{\bar{A}_{L,2}}. \tag{56}$$

After substituting equation (51) in equation (56) and rearranging,

$$\Phi_2^* = \left[\frac{\alpha_H}{\alpha_L} \frac{A_{H,1}}{A_{L,1}} \left[\frac{\gamma_X^\sigma - \gamma_Y^\sigma \left(\frac{P_{X,2}^W}{P_{Y,2}^W} \right)^{1-\sigma 1-\sigma}}{\left(1-\gamma_Y\right)^\sigma \left(\frac{P_{X,2}^W}{P_{Y,2}^W} \right)^{1-\sigma 1-\sigma} - \left(1-\gamma_X\right)^\sigma} \right]^{\frac{1}{1-\sigma}} \right]^{-1} \tag{57}$$

The right-hand side of equation (57) is defined as $\Psi\left(\frac{P_{X,2}^W}{P_{Y,2}^W}\right)$. Given every exogeneous variables unchanged throughout two periods, i.e. $P = \frac{P_{X,1}^W}{P_{Y,1}^W} = \frac{P_{X,2}^W}{P_{Y,2}^W}$ and $\Phi = \Phi_1 = \Phi_2$, equation (57) can be rewritten as follows.

$$\Phi^* = \Psi\left(P\right) \tag{58}$$

Φ^* is called the critical value of the labor proportion. According to equation (56), the critical value of the labor proportion is the labor proportion in the second period that makes the efficiency ratio in the second period equal to its endowment, i.e. $\frac{A_{H,1}}{A_{L,1}} = \frac{\bar{A}_{H,2}}{\bar{A}_{L,2}}$ if $\Phi_2 = \Phi^*$.

Given every exogeneous variables unchanged throughout two periods, therefore, we can conclude that there are three cases of the dynamic inequality as follows.

(1) Inequality does not change throughout two periods if the labor proportion in the second period is equal to its critical value, i.e. $\frac{\bar{w}_{H,2}}{\bar{w}_{L,2}} = \frac{\bar{w}_{H,1}^D}{\bar{w}_{L,1}^D}$ if $\Phi_2 = \Phi^*$.

(2) Inequality in the second period increases from the first period if the labor proportion in the second period more than its critical value, i.e. $\frac{\bar{w}_{H,2}}{\bar{w}_{L,2}} > \frac{\bar{w}_{H,1}^D}{\bar{w}_{L,1}^D}$ if $\Phi_2 > \Phi^*$.

(3) Inequality in the second period decreases from the first period if the labor proportion in the second period less than its critical value, i.e. $\frac{\bar{w}_{H,2}}{\bar{w}_{L,2}} < \frac{\bar{w}_{H,1}^D}{\bar{w}_{L,1}^D}$ if $\Phi_2 < \Phi^*$.

The second case of above conclusion, for example, can be explained as follows. Since $GDP_2^I \equiv w_{L,2}L_2^S + w_{H,2}H_2^S \equiv DNI_2$ Central planner may increase wage of skilled and unskilled labors in the second period through the expenditures which promote their efficiency. By this reason, according to above equation, the number of skilled and unskilled labors plays the important roles as the multiplier of the increasing wages. To maximize total pDNI of economy, central planner tends to expend more on promoting efficiency of the larger group rather than the smaller group of labors in the second period because the expenditure will more effectively increase pDNI in the second period. That is, when the labor proportion in the second period is high, the expenditure ratio tends to be high.

Since central planner's expenditure is the determinant of labor's efficiency in the second period, increasing in the expenditure ratio leads to increasing in the efficiency ratio, in the other word, skilled biased technology progress. Given other things being equal, increasing in the efficiency ratio increases the relative wage in the second period is from the first period, that is increasing of inequality. The third case of above conclusion can be explained by the same intuition.

Change in the labor proportion and dynamic inequality

In the previous section, we have discussed dynamic inequality when there are no changes in any exogeneous variables. This section will show how a change in the labor proportion affects inequality. Remind that we have defined labor proportion as $\Phi_t = H_t^S / L_t^S$. This section will analyze impact of a change in the labor proportion on dynamic inequality.

Note that any changes in the labor proportion mentioned in this section are the small changes. In more specifying, the labor proportion after changing still lies in the range which satisfies the condition in equation (29),

The labor proportion elasticity of the relative wage

Since elasticity is necessary for analyzing both direction and size of impact, we firstly investigate by consider the value of it. The elasticity is defined as follow.

$$\omega_{\Phi,t} \equiv \frac{\partial\left(\overline{w}_{H,t} / \overline{w}_{L,t}\right)}{\partial\Phi_t} \times \frac{\Phi_t}{\overline{w}_{H,t} / \overline{w}_{L,t}} \tag{59}$$

where $\omega_{\Phi,t}$ is the labor proportion elasticity of the relative wage in the t^{th} period, $t \in \{1,2\}$.

Considering equation (48), the labor proportion elasticity of the relative wage in the first period is

$$\omega_{\Phi,1} = 0 \tag{60}$$

Surprisingly, equation (60) implies that relative disposable wage is not affected by a small change in labor proportion. Therefore, in the first period, the number of labors has nothing to do with inequality. This result aligns with the prediction in Factor Price Insensitivity Lemma (Feenstra, 2003).

Before finding impact of the change in labor proportion on relative wage in the second period, consider equations (49), (50) and (51), the derivative of the relative wage with respective the labor proportion in the second period is

$$\frac{d\left(\bar{w}_{H,2}/\bar{w}_{L,2}\right)}{d\Phi_2}=\frac{\partial\left(\bar{w}_{H,2}/\bar{w}_{L,2}\right)}{\partial\left(\bar{A}_{H,2}/\bar{A}_{L,2}\right)}\frac{\partial\left(\bar{A}_{H,2}/\bar{A}_{L,2}\right)}{\partial\left(\bar{G}_H/\bar{G}_L\right)}\frac{\partial\left(\bar{G}_H/\bar{G}_L\right)}{\partial\Phi_2} \tag{61}$$

After calculating above equation, the labor proportion elasticity of the relative wage in the second period is

$$\omega_{\Phi,2}=\frac{\delta}{1-\delta} \tag{62}$$

Since $0<\delta<1$, then $\omega_{\Phi,2}>0$. Equation (62) means that, in the second period, a small increase (decrease) of labor proportion leads to an increase (decrease) of relative wage. Unlike the first period, the labor proportion elasticity of relative wage in the second period implies that a change in the labor proportion affects the relative wage in the second period.

Above implication can be explained as follows. Comparing to the case of unchanged labor proportion, if the labor proportion in the second period increases, central planner will expect such change and sacrifice more budget for promoting efficiency of skilled labors. Therefore technology is biased to skilled labors in the second period, i.e. the efficiency ratio increases. Increasing of the efficiency ratio leads to increasing of the relative wage in the second, comparing to the case of unchanged labor proportion.

Change in labor proportion and real wages

Since change in inequality can not be applied for welfare analysis, we must investigate that how skilled and unskilled labor real wages change when the labor proportion change. Given relative world price being equal, increasing (decreasing) of wages can be referred to increasing (decreasing) of real wages. Therefore, we can know how a change in the labor proportion affects real wages by considering the elasticity as follows.

$$\varepsilon_{\Phi,t}^{I}\equiv\frac{\partial\bar{w}_{I,t}}{\partial\Phi_t}\times\frac{\Phi_t}{\bar{w}_{I,t}} \tag{63}$$

where $\varepsilon_{\Phi,t}^{I}$ is the labor proportion elasticity of i^{th} labor wage in the t^{th} period, $I\in\{H,L\}$ and $t\in\{1,2\}$.

According to equations (17) and (18), the labor proportion elasticity of skilled and unskilled labor wage in the first period are

$$\varepsilon_{\Phi,1}^{H}=\varepsilon_{\Phi,1}^{L}=0 \tag{64}$$

Equation (64) insists that, in the first period, a change in the labor proportion not only disaffects relative wage between skilled and unskilled labors, but also skilled and unskilled wages.

For analyzing the impact in the second period, Consider equations (3), (4), (19), (20), (45) and (44) and then

$$\frac{\partial \overline{w}_{H,2}}{\partial \Phi_2} = \frac{\partial \overline{w}_{H,2}}{\partial \overline{A}_{H,2}} \times \frac{\partial \overline{A}_{H,2}}{\partial \overline{G}_H} \times \frac{\partial \overline{G}_H}{\partial H_2^S} \times \frac{\partial H_2^S}{\partial \Phi_2} = \frac{\partial \overline{w}_{H,2}^D}{\partial \overline{A}_{H,2}} \times \frac{\partial \overline{A}_{H,2}}{\partial \overline{G}_H} \times \frac{\partial \overline{G}_H}{\partial H_2^S} \times L_2^S \tag{65}$$

$$\frac{\partial \overline{w}_{L,2}}{\partial \Phi_2} = \frac{\partial \overline{w}_{L,2}}{\partial \overline{A}_{L,2}} \times \frac{\partial \overline{A}_{L,2}}{\partial \overline{G}_L} \times \frac{\partial \overline{G}_L}{\partial H_2^S} \times \frac{\partial H_2^S}{\partial \Phi_2} = \frac{\partial \overline{w}_{L,2}}{\partial \overline{A}_{L,2}} \times \frac{\partial \overline{A}_{L,2}}{\partial \overline{G}_L} \times \frac{\partial \overline{G}_L}{\partial H_2^S} \times L_2^S \tag{66}$$

After solving above equations, the labor proportion elasticity of skilled and unskilled labor wage in the second period are

$$\varepsilon_{\Phi,2}^H = \delta \frac{L_2^S}{L_2^S + H_2^S} \tag{67}$$

$$\varepsilon_{\Phi,2}^L = -\delta \frac{H_2^S}{L_2^S + H_2^S} \tag{68}$$

According to equations (67) and (68), $\varepsilon_{\Phi,2}^H > 0$ and $\varepsilon_{\Phi,2}^L < 0$. Equations (67) and (68) mean that, if labor proportion in the second period increases (decreases), skilled labor wage will increase (decrease) but unskilled labor wage will decrease (increase).

This conclusion about the effect on real wages can be explained by the same intuition about the effect on the relative wage. Moreover, the volume of changes are determined by size of groups of labors. Equations (67) and (68) imply that, when skilled labors are minority group, i.e. $\Phi_2 < 0.5$, increasing (decreasing) rate of skilled labors wages is more than decreasing (increasing) rate of unskilled labors wages when labor proportion increases (decreases), on the other hand, when skilled labors are majority group, i.e. $\Phi_2 > 0.5$, increasing (decreasing) rate of skilled labors wages is less than decreasing (increasing) rate of unskilled labors wages when labor proportion increases (decreases).

Diminishing of marginal effectiveness of central planner's expenditure is the explanation for this phenomenon. For example, given initial expenditure ratio is high, if central planner transfers some $\overline{G}_H$ to $\overline{G}_L$, increasing rate of $\overline{A}_{L,2}$ will be more than decreasing rate of $\overline{A}_{H,2}$ because $0 < \delta < 1$. Finally, increasing rate of $\overline{w}_{L,2}$ will be more than increasing rate of $\overline{w}_{H,2}$.

Dynamic inequality under changing of the labor proportion

After integrating all implication in both previous and this section, the dynamic inequality under changing of the labor proportion can be conclusion as follows.

When the labor proportion slightingly increase (decrease) from Φ_1 in the first period to Φ_2 in the second period,

(1) if the labor proportion in the first period is more (less) than the critical value, inequality in the second period will increase (decrease) from the first period more extremely than the

case of unchanged labor proportion because, in the second period, skilled labors real wage are higher (lower) and unskilled labors real wages are lower (higher) than their wages in the case of unchanged labor proportion.

(2) if the labor proportion in the first period less (more) than the critical value

(2.1) if the labor proportion in the second period is still less (more) than the critical value, inequality in the second period will still decrease (increase) from the first period but decreasing (increasing) will be less extreme than the case of unchanged labor proportion because, in the second period, skilled labors real wage are lower (higher) and unskilled labors real wages are higher (lower) than their wages in the case of unchanged labor proportion.

(2.2) if the labor proportion in the second period is more (than) than the critical value, inequality in the second period will increase (decrease) from the first period because, in the second period, skilled labors real wage are much lower (higher) and unskilled labors real wages are much higher (lower) than their wages in the case of unchanged labor proportion.

Change in the relative world price and dynamic inequality

We have already discussed dynamic inequality when there are no changes in any exogeneous variables and when the labor proportion is changeable. Aligned with previous section, this section will analyze impact of change in the relative world price, $P_{X,t}^{W} \big/ P_{Y,t}^{W}$ on dynamic inequality.

Note that any changes in the relative world price mentioned in this section are the small changes. In more specifying, the relative world price after changing still lies in the range which satisfies equations (21) and (22).

The relative world price elasticity of the relative wage

Since Elasticity is necessary for analyzing both direction and size of impact, the elasticity is defined as follow.

$$\omega_{P,t} \equiv \frac{\partial\left(\overline{w}_{H,t} \big/ \overline{w}_{L,t}\right)}{\partial\left(P_{X,t}^{W} \big/ P_{Y,t}^{W}\right)} \times \frac{P_{X,t}^{W} \big/ P_{Y,t}^{W}}{\overline{w}_{H,t} \big/ \overline{w}_{L,t}} \tag{69}$$

where $\omega_{P,t}$ is the relative world price elasticity of relative wage in the t^{th} period, $t \in \{1,2\}$.

To find impact of change in relative price on relative wage in the first period, we need to differentiate equation (48) with respect to the relative world price in the first period. Substituting in equation (69), the relative world price elasticity of relative wage in the first period is

$$\omega_{P,1} = -\frac{\left[\gamma_X^{\sigma}\left(1-\gamma_Y\right)^{\sigma} - \left(1-\gamma_X\right)^{\sigma}\gamma_Y^{\sigma}\right]\left(\frac{P_{X,1}^{W}}{P_{Y,1}^{W}}\right)^{1-\sigma}}{\left[\gamma_X^{\sigma} - \gamma_Y^{\sigma}\left(\frac{P_{X,1}^{W}}{P_{Y,1}^{W}}\right)^{1-\sigma}\right]\left[\left(1-\gamma_Y\right)^{\sigma}\left(\frac{P_{X,1}^{W}}{P_{Y,1}^{W}}\right)^{1-\sigma} - \left(1-\gamma_X\right)^{\sigma}\right]} \tag{70}$$

After rearranging equation (70), we can conclude that

$$\omega_{P,1} < -1 \tag{71}$$

Equation (71) means that, in the first period, the relative wage increases (decreases) more rapidly than small decreasing (increasing) of the relative world prices. This also implies that when the relative world price increases (decreases), skilled labors wages increase (decrease) by more than the price of good X, while unskilled labors wage decreases (increases) by more than the price of good Y. Therefore, this implication from the model is identical to the Stolper-Samuelson theorem. The proof for this is in next sub-section.

The mechanism of the change in relative wage in the first period is the same as explanation in Stolper-Samuelson theorem. Increasing of relative world price increases demand of goods X and decreases demand for goods Y. According to equations (25) and (26), economy produces more goods X but less goods Y when relative world price increases. Since goods X is unskilled labor intensive while goods Y is skilled labor intensive, producers need more unskilled labors but less skilled labors. Therefore relative inverse conditional demand for labor in the first period decreases and, finally, relative wage in the first period decreases in consequence. The case of decreasing relative world price can be explained by the same intuition.

To find the impact in the second period, consider equations (49), (50) and (51), the derivative of the relative wage with respective the relative world price in the second period is

$$\frac{d\left(\bar{w}_{H,2}/\bar{w}_{L,2}\right)}{d\left(P_{X,2}^{W}/P_{Y,2}^{W}\right)} = \frac{\partial\left(\bar{w}_{H,2}/\bar{w}_{L,2}\right)}{\partial\left(\bar{A}_{H,2}/\bar{A}_{L,2}\right)}\frac{\partial\left(\bar{A}_{H,2}/\bar{A}_{L,2}\right)}{\partial\left(\bar{G}_{H}/\bar{G}_{L}\right)}\frac{\partial\left(\bar{G}_{H}/\bar{G}_{L}\right)}{\partial\left(P_{X,2}^{W}/P_{Y,2}^{W}\right)} + \frac{\partial\left(\bar{w}_{H,2}/\bar{w}_{L,2}\right)}{\partial\left(P_{X,2}^{W}/P_{Y,2}^{W}\right)} \tag{72}$$

After solving above equation and substituting in equation (69), the relative world price elasticity of relative wage in the second period is

$$\omega_{P,2} = \frac{-1}{1-\delta}\frac{\left[\gamma_X^{\sigma}\left(1-\gamma_Y\right)^{\sigma} - \left(1-\gamma_X\right)^{\sigma}\gamma_Y^{\sigma}\right]\left(\frac{P_{X,2}^{W}}{P_{Y,2}^{W}}\right)^{1-\sigma}}{\left[\left(1-\gamma_Y\right)^{\sigma}\left(\frac{P_{X,2}^{W}}{P_{Y,2}^{W}}\right)^{1-\sigma} - \left(1-\gamma_X\right)^{\sigma}\right]\left[\gamma_X^{\sigma} - \gamma_Y^{\sigma}\left(\frac{P_{X,2}^{W}}{P_{Y,2}^{W}}\right)^{1-\sigma}\right]} \tag{73}$$

According to $0 < \delta < 1$, given $\frac{P_{X,1}^{W}}{P_{Y,1}^{W}} = \frac{P_{X,2}^{W}}{P_{Y,2}^{W}}$, we can conclude that

$$\omega_{P,2} < \omega_{P,1} < -1 \tag{74}$$

Equation (74) means that, if the relative world price increases (decreases) by the same small percentage, the relative wage in the second period will decreases (increases) more rapidly than the relative wage in the first period. This implies two things. First, Stolper-Samuelson theorem is still held in the second period of the model. In addition, a change in the relative world price affects the relative wage more extremely in the second period when technology is changeable.

For explanation, in the first period when the technology level is fixed, mechanism of impact on relative wage can be explained by the mechanism in Stolper-Samuelson theorem which has been already explained.

But, in the second period when there is technology progress, Stolper-Samuelson's mechanism is reinforced through skilled-biased technological progress. Increasing of the relative world price could be previously expected by central planner in the first period. Remind that $GDP_t^O \equiv p_X^W \bar{X}_t^P + p_Y^W \bar{Y}_t^P$. To increase output of X in the second period for maximizing pDNI, central planner expends more on promoting efficiency of unskilled labors in the first period, then technology is biased to unskilled labors in consequence. Unskilled biased technology finally makes the relative wage decrease. Since impact from Stolper-Samuelson's mechanism and impact skilled biased technological progress have the same direction, the relative world price affects relative wage in the second period more extremely than the first period, when technology is fixed and Stolper-Samuelson's mechanism works alone.

Change in the relative world price and real wages

To know how change in the relative world price affects real wages, we must sure that the change in nominal wages are more than the change in relative world price. Let us start with the case of the first period. Since equilibrium wages are determined by zero-profit conditions alone, we find the total differential of zero-profit conditions in the first period. According to equation (11),

$$dP_{X,1}^W = d\left[\gamma_X^\sigma A_{L,1}^{\sigma-1}\left(\bar{w}_{L,1}^M\right)^{1-6} + \left(1-\gamma_X\right)^\sigma A_{H,1}^{\sigma-1}\left(\bar{w}_{H,1}^M\right)^{1-6}\right]^{\frac{1}{1-\sigma}}$$

Rearranging above equation, then yield

$$\left(P_{X,1}^W\right)^{1-\sigma}\frac{dP_{X,1}^W}{P_{X,1}^W} = \left[\gamma_X^\sigma A_{L,1}^{\sigma-1}\left(\bar{w}_{L,1}^M\right)^{1-6}\frac{d\bar{w}_{L,1}^M}{\bar{w}_{L,1}^M} + \left(1-\gamma_X\right)^\sigma A_{H,1}^{\sigma-1}\left(\bar{w}_{H,1}^M\right)^{1-6}\frac{d\bar{w}_{H,1}^M}{\bar{w}_{H,1}^M}\right] \tag{75}$$

Do the same pattern with equation (12) and yield

$$\left(P_{Y,1}^W\right)^{1-\sigma}\frac{dP_{Y,1}^W}{P_{Y,1}^W} = \left[\gamma_Y^\sigma A_{L,1}^{\sigma-1}\left(\bar{w}_{L,1}^M\right)^{1-6}\frac{d\bar{w}_{L,1}^M}{\bar{w}_{L,1}^M} + \left(1-\gamma_Y\right)^\sigma A_{H,1}^{\sigma-1}\left(\bar{w}_{H,1}^M\right)^{1-6}\frac{d\bar{w}_{H,1}^M}{\bar{w}_{H,1}^M}\right] \tag{76}$$

Solving equations (75) and (76) simultaneously, finally we yield

$$\frac{d\overline{w}_{L,1}^{M}}{\overline{w}_{L,1}^{M}}=\frac{dP_{X,1}^{W}}{P_{X,1}^{W}}+\left[\frac{\left(1-\gamma_{X}\right)^{\sigma}\left(P_{Y,1}^{W}\right)^{1-\sigma}}{\left(1-\gamma_{Y}\right)^{\sigma}\left(P_{X,1}^{W}\right)^{1-\sigma}-\left(1-\gamma_{X}\right)^{\sigma}\left(P_{Y,1}^{W}\right)^{1-\sigma}}\right]\left(\frac{dP_{X,1}^{W}}{P_{X,1}^{W}}-\frac{dP_{Y,1}^{W}}{P_{Y,1}^{W}}\right) \tag{77}$$

$$\frac{d\overline{w}_{H,1}^{M}}{\overline{w}_{H,1}^{M}}=\frac{dP_{Y,1}^{W}}{P_{Y,1}^{W}}-\left[\frac{\gamma_{Y}^{\sigma}\left(P_{X,1}^{W}\right)^{1-\sigma}}{\gamma_{X}^{\sigma}\left(P_{Y,1}^{W}\right)^{1-\sigma}-\gamma_{Y}^{\sigma}\left(P_{X,1}^{W}\right)^{1-\sigma}}\right]\left(\frac{dP_{X,1}^{W}}{P_{X,1}^{W}}-\frac{dP_{Y,1}^{W}}{P_{Y,1}^{W}}\right) \tag{78}$$

According to equations (21) and (22), the coefficients of $\left(\frac{dP_{X,1}^{W}}{P_{X,1}^{W}}-\frac{dP_{Y,1}^{W}}{P_{Y,1}^{W}}\right)$ in equations (77) and (78) are positive. An increase of relative world price is mathematical expressed as $\frac{dP_{X,1}^{W}}{P_{X,1}^{W}}>\frac{dP_{Y,1}^{W}}{P_{Y,1}^{W}}$ or $\frac{dP_{X,1}^{W}}{P_{X,1}^{W}}-\frac{dP_{Y,1}^{W}}{P_{Y,1}^{W}}>0$, while a decrease of relative world price is mathematical expressed as $\frac{dP_{X,1}^{W}}{P_{X,1}^{W}}<\frac{dP_{Y,1}^{W}}{P_{Y,1}^{W}}$ or $\frac{dP_{X,1}^{W}}{P_{X,1}^{W}}-\frac{dP_{Y,1}^{W}}{P_{Y,1}^{W}}<0$.

Therefore, we can conclude from equations (77) and (78) that, when relative world price in the first period increases, i.e. $\frac{dP_{X,1}^{W}}{P_{X,1}^{W}}-\frac{dP_{Y,1}^{W}}{P_{Y,1}^{W}}>0$,

$$\frac{d\overline{w}_{L,1}^{M}}{\overline{w}_{L,1}^{M}}>\frac{dP_{X,1}^{W}}{P_{X,1}^{W}}>\frac{dP_{Y,1}^{W}}{P_{Y,1}^{W}}>\frac{d\overline{w}_{H,1}^{M}}{\overline{w}_{H,1}^{M}} \tag{79}$$

And when relative world price in the first period decreases, i.e. $\frac{dP_{X,1}^{W}}{P_{X,1}^{W}}-\frac{dP_{Y,1}^{W}}{P_{Y,1}^{W}}<0$,

$$\frac{d\overline{w}_{L,1}^{M}}{\overline{w}_{L,1}^{M}}<\frac{dP_{X,1}^{W}}{P_{X,1}^{W}}<\frac{dP_{Y,1}^{W}}{P_{Y,1}^{W}}<\frac{d\overline{w}_{H,1}^{M}}{\overline{w}_{H,1}^{M}} \tag{80}$$

Above conclusion implies that Stolper-Samuelson theorem is held in the first period of the model. For the second period, since the range of relative world price elasticity in the second period is $\omega_{P,2}<\omega_{P,1}<-1$, we can claim that equation (79) and (80) are also true for the case of the second period. Finally, we can conclude the effect of a change in the relative world price on real wages that, when the relative world price in the first and the second period increases (decreases) by the same small percentage, skilled labors real wages will decrease (increase) and unskilled labors real wages will increase (decrease). Moreover, increasing or decreasing of real wages in the second period are more extreme than the first period.

Dynamic inequality under changing of the relative world price

After integrating all of conclusion from this and previous sections, the dynamic inequality under changing of the relative world price can be conclusion as follows.

(1) When the relative world price in the first period slightly decreases (increases), inequality the first period increases (decreases) but inequality in the second period is the same as the case of unchanged relative world price.

(2) When the relative world price in the second period slightly decreases (increases),

(2.1) if the labor proportion in the second period is more (less) than the critical value, inequality in the second period will increase (decrease) from the first period more extremely than the case of unchanged relative world price because, in the second period, skilled labors real wage are higher (lower) and unskilled labors real wages are lower (higher) than their wages in the case of unchanged relative world price.

(2.2) if the labor proportion in the second period is less (more) than the critical value, inequality in the second period will still decrease (increase) from the first period but decreasing (increasing) will be less extreme than the case of unchanged relative world price because, in the second period, skilled labors real wage are lower (higher) and unskilled labors real wages are higher (lower) than their wages in the case of unchanged relative world price. Moreover, if relative price decreases (increases) much enough, inequality will increase (decrease).

Implication of changing in inequality

Implication 1 **Without government as a technology promoter, inequality arises from skilled biased technological change, i.e. increasing of the efficiency ratio, and increasing (decreasing) of price of goods which is skilled(unskilled)-labor intensive.**

Explanation An increase in the efficiency ratio endowment or a decrease in the relative world price lead to an increase in the equilibrium relative wage, according to equations (48) and (49). In addition, increasing of relative world price may comes from increasing of price of goods X which is unskilled-labor intensive or decreasing of price of goods Y which is skilled-labor intensive.

Implication 2 **Given amount of labors and world prices being equal throughout two periods, under actions of a pDNI maximizing government as a technology promoter , government is "inequality creator" if labor proportion is more than critical value and is "inequality reducer" if labor proportion is less than critical value.**

Explanation The relative wage in the second period is higher than the first period if the labor proportion in the second period is more than the critical value and is lower than the first period if the labor proportion in the second period is less than the critical value.

Implication 3 **Under actions of a pDNI maximizing government as a technology promoter, any external factors which increases relative supply of labors in the second period can decrease inequality in the second period from the first period.**

- Government policy which creates more unskilled labors in the second period can be interpreted as "Inequality reduction Policy". For example, assume that all immigrants are accepted as citizen by local government. Immigrant permission policy in the long-run will reduce inequality in the second period if there most of immigrants are unskilled labors.

Explanation The labor proportion elasticity of the relative wage in the second period is always positive.

Implication 4 **Under actions of a pDNI maximizing government as a technology promoter, any external shocks through swing in the relative world price lead to more extreme fluctuation of the relative wage, comparing to the case of without government as a technology promoter.**

Explanation Since the absolute value of the labor proportion elasticity of the relative wage in the second period is more than the first period, a change in the relative world price will change the relative wage rather extremely than the case without central planner.

Implication 5 **Both an Increase (decrease) in skilled (unskilled) labors and an increase (decrease) of price of goods which is skilled(unskilled)-labor intensive in the second period will retard government's inequality reduction but reinforce government's inequality creation.**

Explanation Since, in the second period, the labor proportion elasticity is positive but the relative world price elasticity is negative, a decrease in the relative world price or an increase in the labor proportion in the second period leads to an increase in the relative wage in the second period, comparing to the case of unchanged exogeneous variables. Integrating these results with conclusion 2, it is the case in conclusion 5.

Implication 6 **Under actions of a pDNI maximizing government as a technology promoter, any external factors which decrease (increase) of price of goods which is skilled(unskilled)-labor intensive will reduce inequality, while any external factors which increase (decrease) of price of goods which is skilled(unskilled)-labor intensive will create inequality.**

- For example, if the government decreases (increases) import tariff rate on skilled(unskilled)-labor intensive goods, inequality will be reduced. If the government acts oppositely, inequality will be created.
- if government decreases (increases) commercial tax rate on skilled(unskilled)-labor intensive goods, inequality will be reduced. If government acts oppositely, inequality will be created.

Explanation Since the relative world price elasticity of the relative wage in the second period is negative, an increase in the relative world price will decrease the relative wage, comparing to the case of unchanged relative world price.

Summary

This study arises from the hypothesis that chronic wage gap between skilled and unskilled labors may come from skilled-biased technological progress. Unlike many studies which consider skilled-biased technological progress as exogeneous variable, this study points that government policy, a change in skilled to unskilled labors proportion and a change in relative world price may be principals of this phenomenon.

Following some theoretical studies, inequality in this study refers to the relative wage which is skilled to unskilled labors wages ratio. There is perfect equality when the relative wage is equal to one. This study assumes that the initial relative wage is more than one. Therefore, an increase of inequality refers to an increase of the relative wage while a decrease of inequality refers to a decrease of the relative wage.

Many studies point that inequality in each country has risen for an interval of time. There is an argument among economists for the causes of inequality which is called trade-and-wage debate; some believe that skilled-biased technological change is the major cause while the others claim that international trade has an account. For disaggregating the efficiency of skilled and unskilled labors

This study constructs the international traded-model which includes the role of government in technological promotion. There is technological progress by the central planner who promotes efficiency of skilled and unskilled labor. The economy in this model is set up to last for two periods.

The model in this study includes three main sides: the production side, the consumer side and the central planner side. The production side includes zero-profit and full employment conditions and yields equilibrium the equilibrium skilled and unskilled labors wages and the equilibrium productions of outputs. The consumer side yields the equilibrium consumptions of outputs which are derived from utility maximizing problem. For the central planner side, this study sets up the per Capita disposable national income (pDNI) maximizing central planner who imposts taxation and expenses to

promote efficiency of skilled and unskilled labors in the first period. Therefore, the central planner side yield the optimal tax rate and expenditure for promoting efficiency of skilled and unskilled labors. The model in this study is shown that its trade pattern aligns with the prediction in the Hecksher-Ohlin model.

For explaining the mechanism of the change in the relative wage in the first period, the relative supply for labor and the relative inverse conditional demand for labor in the first period are derived. For the second period, the equilibrium relative wages are on the long-run equilibrium expansion path which is constructed from the movement of equilibrium points when the relative supply in the second period changes.

While the change in inequality in the first period directly comes from exogeneous variables, the change in inequality in the second period can be explained by “3 links of chain reaction”: change in expenditure ratio, change in efficiency ratio and change in relative inverse conditional demand.

This study explains three causes of inequality as follows. The first cause of inequality is the central planner’s expenditure. Given amount of labors and world prices being equal throughout two periods, under actions of national income maximizing government as technology promoter, government increases inequality if labor proportion is more than critical value but decreases inequality if labor proportion is less than critical value.

The second cause is the increasing of skilled to unskilled labor proportion in the second period. While a small change in labor proportion does not affect any wages in the first period, it increases skilled labors real wages and decreases unskilled labors real wages in the second period, comparing to the case of unchanged labor proportion.

The third cause is the decreasing of relative world price. The result from this study aligns with the prediction in the Stolper-Samuelson theorem; When the relative world price in the first and the second period increases (decreases) by the same small percentage, skilled labors real wages will decrease (increase) and unskilled labors real wages will increase (decrease). Moreover, the magnitude of increasing or decreasing of real wages in the second period is more extreme than the first period.

Integrating all of these conclusions, an increase (a decrease) of skilled (unskilled) labors and an increase (a decrease) of price of goods which is skilled(unskilled)-labor intensive in the second period will retard government’s inequality reduction if the labor proportion is more than critical value, but reinforce government’s inequality creation if the labor proportion is more than critical value.

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