A study on Thai exchange rate volatility model comparison: a nonparametric approach

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Agenda

- Introduction
  - Statement of problem
  - Objective
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- Introductory Nonparametric Econometrics
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Introduction

- Statement of problem
  - Basic assumption of conditional homoskedasticity is in appropriate
  - So the specification of a dynamic structure for conditional variance (heteroskedasticity) are introduced
  - There are ARCH, GARCH,… models estimated by NL-MLE
  - But sensitivity to model misspecification (ad hoc functional form determination)
  - Therefore, nonparametric econometrics is one of the solutions in corrective estimation

Introduction

- Objective
  - To study the differentiation of parametric and nonparametric estimation on exchange rate conditional volatility
  - To compare predictability performance among different approached competitive models

Introduction

- Scope of Study
  - Compare the nonparametric and parametric models prediction's performance
  - Volatility of weekly Baht/U.S. rate of return in 2000-2005
  - Rolling Mean squared prediction error (MSPE) as the measurement

Introductory Nonparametric Econometrics

- General Concept
  - Minimized set of assumptions
  - Minimum of structure imposed on the regression function
  - Only some degree of smoothness is necessary for nonparametric method
  - Continuity of function is enough to ensure the convergence of estimator as the size of data increases
  - Additional smoothness (existence of derivatives) allows more asymptotic efficient estimation

- Basic Model

$$y_t = m(x_t) + u_t \qquad ; t = 1, \dots, T$$

$$u_t \sim i.i.d.(0, \sigma^2)$$

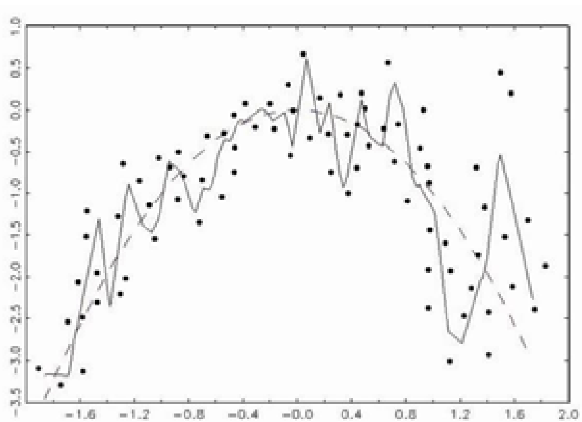
$$\hat{m}(x) = \sum_{t=1}^{T} w_t(x) y_t \qquad ; \sum_{t=1}^{T} w_t(x) = 1$$

"Weighted average" of y_t

- Kernel Function
  - Probability density function which is a piecewise continuous function, unimodal, integrating to one, symmetric about zero, and has first two moments finite
  - The examples of the kernel functions are
    - Uniform kernel: $K(z) = \frac{1}{2} I_{[-1,1]}(z)$
    - Standard normal kernel: $K(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}; -\infty < z < \infty$
    - Epanechnikov kernel: $K(z) = \frac{3}{4}(1-z^2) I_{[-1,1]}(z)$
    - Quartic kernel: $K(z) = \frac{15}{16}(1-z^2)^2 I_{[-1,1]}(z)$
  - As rolling windows to estimate curve at each sample point

- Different Bandwidths for Nadaraya-Watson estimators

Bandwidth = 0.04



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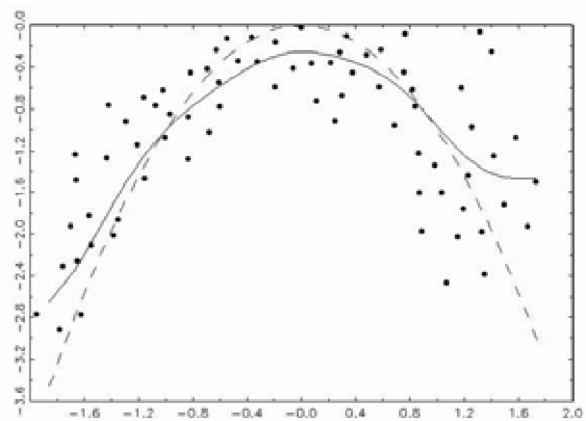

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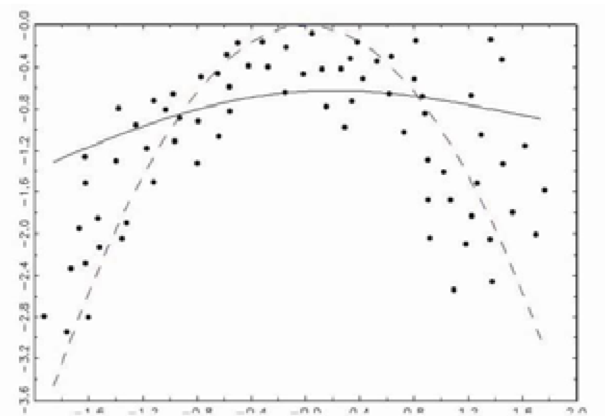


Introductory Nonparametric Econometrics

- Cross Validation Bandwidth
  - Optimal smoothing estimator
  - Automatic balancing the trade-off between biased and variance (smaller bandwidth >> smaller bias but higher variance)

$$h^* \subseteq \arg\min CV(h) = \frac{1}{T}\sum_{t=1}^{T}(Y_t - \hat{m}_{-t}(X_t, h))^2$$

(Cross-Validation criterion)

; $\hat{m}_{-t}$ = *"leave-one-out" estimator*

- Local Constant Estimator

$$m(x) \subseteq \arg\min \sum_{t=1}^{T} \{Y_t - m\}^2 K(\frac{X_t - x}{h})$$

$$\Rightarrow \quad \hat{m}(x) = (\sum_{t=1}^{T} K_t)^{-1} \sum_{t=1}^{T} K_t Y_t \qquad ; K_t = K(\frac{X_t - x}{h})$$

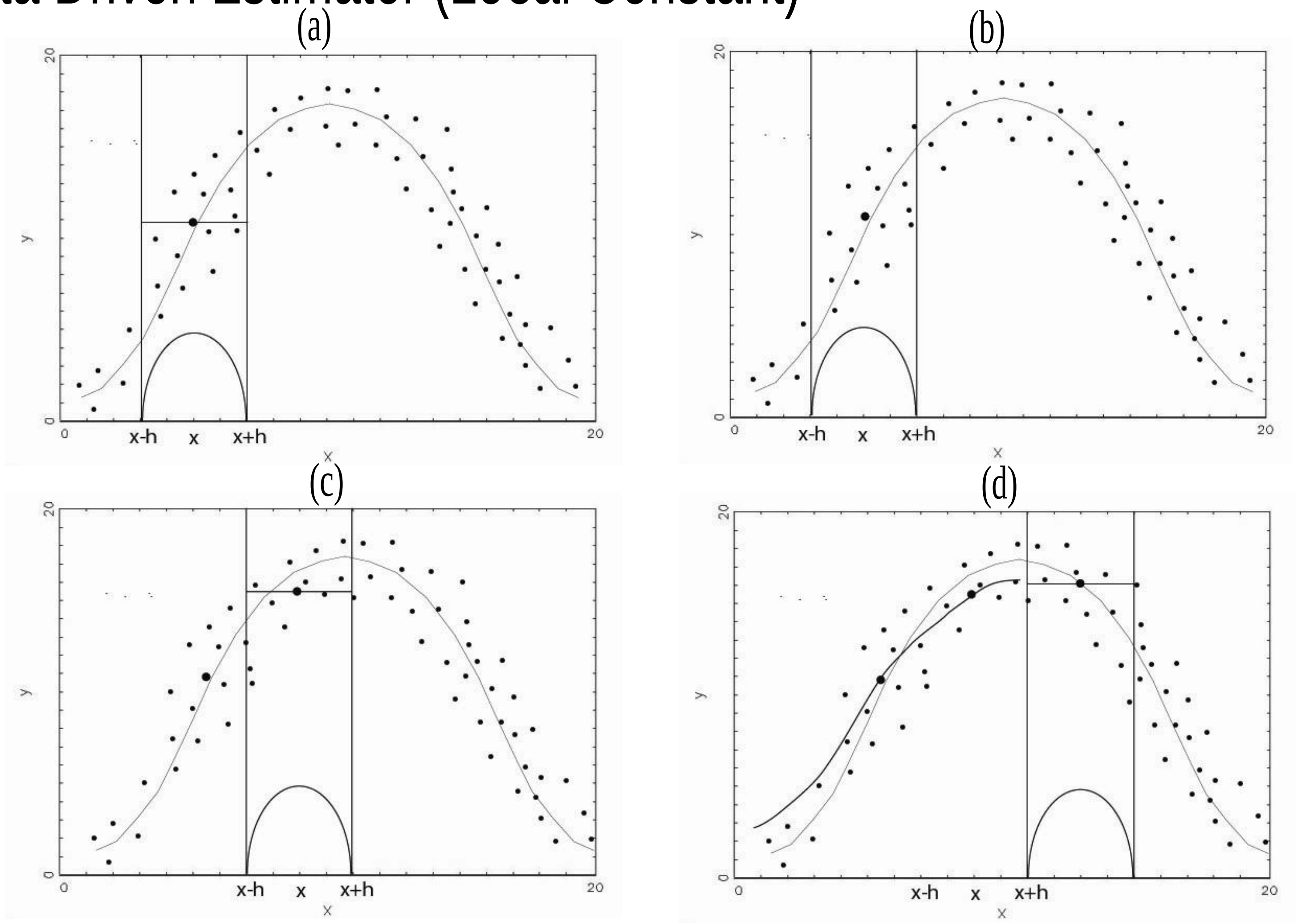
Matrix Notation:

$$\min_{\{m\}} (Y - mi)' \Omega \, (Y - mi) \qquad ; i = (1 \ldots 1)', \;\; \Omega = diag[K(\frac{X_t - x}{h})]$$

$$\Rightarrow \quad \hat{m} = (i'\Omega \, i)^{-1} \; i'\Omega \; y \equiv (\sum_{t=1}^{T} K_t)^{-1} \sum_{t=1}^{T} K_t Y_t$$

Introductory Nonparametric Econometrics

- Data Driven Estimator (Local Constant)



- Asymptotic Result of Local Constant Estimator

If $x_1,\dots,x_T$ be $i.i.d.$ observations with pdf $f(x)$ which are twice continuously differentiable, also the s^{th} order derivatives of $f(x)$, $f^{(s)}(x); s=0,1,2$, are bounded functions. And the kernel function $K(.)$ satisfies

$$\text{(i)} \int K(z)dz = 1 \qquad \text{(ii)} \int zK(z)dz = 0 \qquad \text{(iii)} \int z^2 K(z)dz = k_2 > 0$$

If as $T \to \infty$, $h \to 0$ and $Th \to \infty$, where

$$\hat{f}(x) = (Th)^{-1} \sum_{t=1}^{T} K\left(\frac{X_t - x}{h}\right),$$

then $\hat{f}(x) \to f(x)$ in MSE

$$\hat{m}(x) \xrightarrow{p} m(x)$$

$$; y_t = m(x_t) + u_t = E(y_t \mid x_t) + u_t$$

- Local Linear Estimator

$$m(x) \subseteq \arg\min_{\{m\}} \sum_{t=1}^{T} \{Y_t - m - (X_t - x)'\beta\}^2 K(\frac{X_t - x}{h})$$

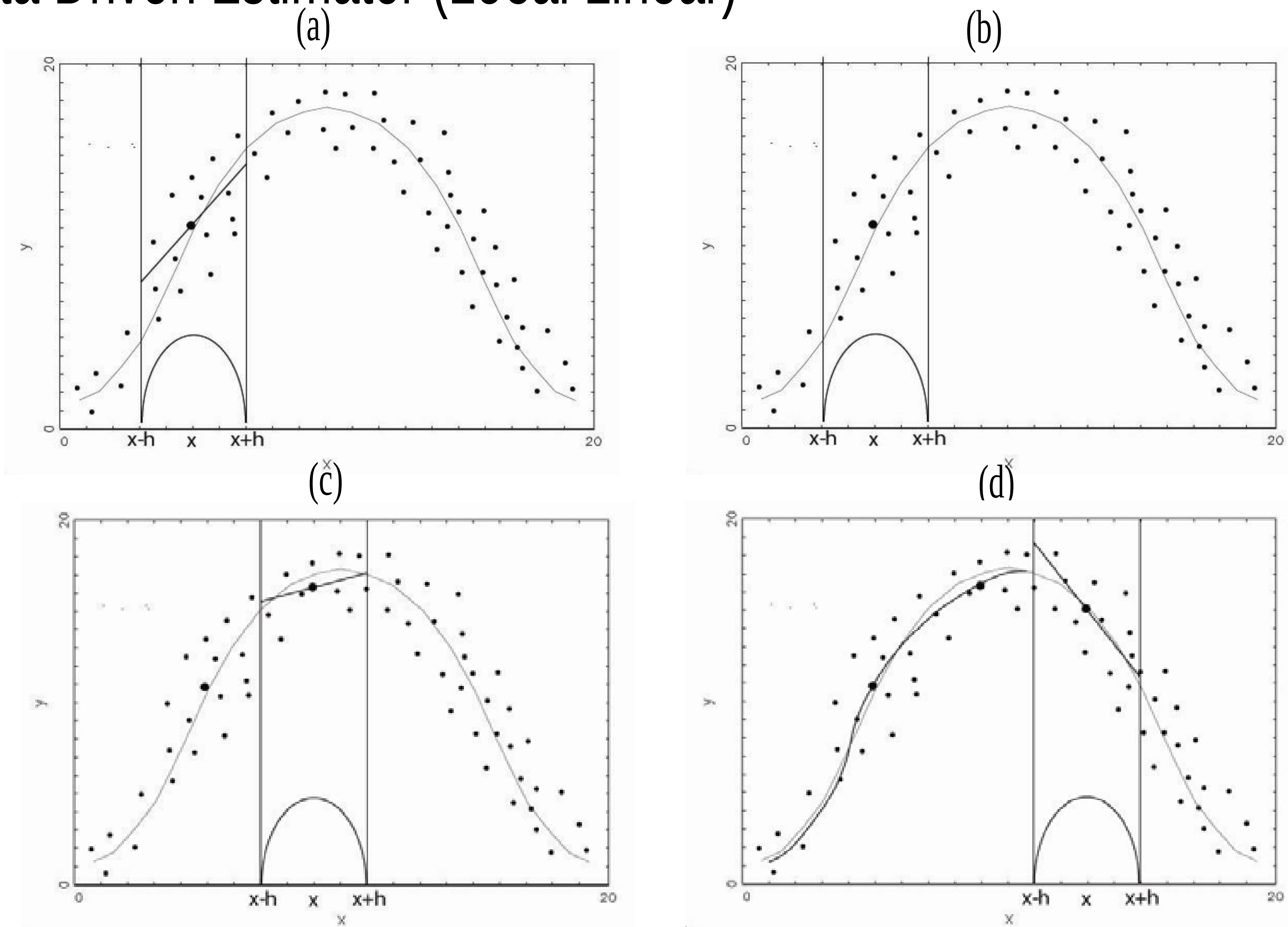
$$\Rightarrow \quad \hat{\delta}(x) = (\hat{m}(x), \quad \hat{\beta}(x)')' \qquad ; \beta(x) = \partial m(x) / \partial x$$

Matrix Notation:

$$\min_{\{m,\beta\}} (Y - \chi\delta)' \Omega \, (Y - \chi\delta) \qquad ; \Omega = diag[K(\frac{X_t - x}{h})]$$

$$\Rightarrow \quad \hat{\delta}(x) = (\chi'\Omega \, \chi)^{-1} \, \chi'\Omega \, y$$

- Data Driven Estimator (Local Linear)



- Asymptotic Result of Local Linear Estimator

Given assumptions

(i) $\{X_t, Y_t\}_{t=1}^{T} \sim i.i.d.$; $X_t \in \mathbb{R}^p$. Both X_t and $u_t = Y_t - m - (X_t - x)'\beta$ have finite fourth moment

(ii) $m(x)$ is twice differentiable, and its second order derivative is bounded. $\sigma^2(x) = E(u^2 \mid X = x)$ is continuous in x

and (iii) K is a second order kernel, as $T \to \infty$, $h \to 0$, and $Th^{p+2} \to \infty$. Also assumes that K is a compact supported bounded function, such that $K > 0$ on a set of positive Lebesgue measure, then

$$\Rightarrow \quad \hat{\delta}(x) \xrightarrow{p} \delta(x)$$

Nonparametric Volatility Model

- Generalization for any time series smoothed conditional mean and conditional variance functions

$$y_t = m(y_{t-1}) + \sigma(y_{t-1})u_t \qquad ; u_t \sim i.i.d.(0,1)$$

$$\sigma^2(x) = E((Y_t - m(x))^2 \mid Y_{t-1} = x)$$
$$= E(Y_t^2 \mid Y_{t-1} = x) - \left(E(Y_t \mid Y_{t-1} = x)\right)^2$$

$$\Rightarrow \hat{\sigma}^2(x) = \hat{g}(x) - \hat{m}^2(x) \qquad ; m(x) = E(Y_t \mid Y_{t-1} = x)$$
$$g(x) = E(Y_t^2 \mid Y_{t-1} = x)$$

Parametric Volatility Models

- Autoregressive Conditional Heteroskedasticity Processes

ARCH: $\sigma_t^2 = \zeta + \alpha u_{t-1}^2$

GARCH: $\sigma_t^2 = \zeta + \alpha u_{t-1}^2 + \alpha u_{t-1}^2$

I-GARCH: $\sigma_t^2 = \zeta + \alpha u_{t-1}^2 + \alpha u_{t-1}^2 \qquad ; \delta + \alpha = 1$

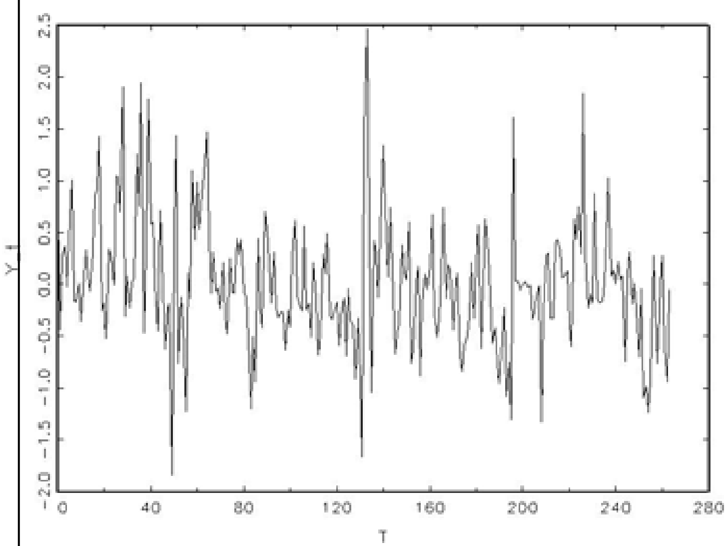
EGARCH: $\ln \sigma_t^2 = \omega + \delta \ln \sigma_{t-1}^2 + \alpha \left| u_{t-1} / \sigma_{t-1} \right| + \gamma (u_{t-1} / \sigma_{t-1})$

TGARCH: $\sigma_t^2 = \kappa + \alpha u_{t-1}^2 + \alpha^- u_{t-1}^2 d_{t-1} + \delta \sigma_{t-1}^2 \qquad ; d_t = 1 \ (u_t < 0) \text{ or } 0 \text{ (otherwise)}$

ARCH-M : $\sigma_t^2 = \zeta + \alpha u_{t-1}^2 \qquad ; Y_t = Y_{t-1}{}' \beta + \xi \sigma_t^2 + u_t$

Data and Descriptive Statistics

Weekly Baht/U.S. Return (264 observations)



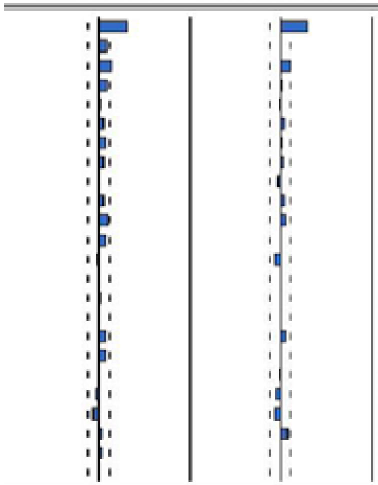
Descriptive Statistics

Mean	0.012550
Median	-0.044315
Maximum	2.463100
Minimum	-1.845640
Std. Dev.	0.624152
Skewness	0.561577
Kurtosis	4.478065
Jarque-Bera	37.90766
Probability	0.000000

Number of observations = 265

ACF and PACF functions

Autocorrelation Partial Correlation



	AC	PAC	Q-Stat	Prob
1	0.310	0.310	25.687	0.000
2	0.094	-0.003	28.037	0.000
3	0.146	0.130	33.785	0.000
4	0.095	0.014	36.208	0.000
5	0.023	-0.018	36.353	0.000
6	0.057	0.044	37.246	0.000
7	0.061	0.021	38.260	0.000
8	0.050	0.026	38.946	0.000
9	0.002	-0.034	38.946	0.000
10	0.054	0.056	39.760	0.000
11	0.113	0.082	43.323	0.000
12	0.058	0.000	44.265	0.000
13	-0.024	-0.061	44.431	0.000
14	0.002	-0.004	44.433	0.000
15	0.012	0.000	44.471	0.000
16	-0.002	0.003	44.473	0.000
17	0.068	0.076	45.787	0.000
18	0.058	0.004	46.734	0.000
19	0.001	-0.024	46.736	0.000
20	-0.033	-0.042	47.047	0.001
21	-0.070	-0.074	48.469	0.001
22	0.040	0.084	48.927	0.001
23	0.034	0.006	49.261	0.001
24	-0.007	0.002	49.274	0.002

Statistical Tests

OLS AR(1) and its statistics

$$Y_t = 0.3104 Y_{t-1}$$

s.d. = (0.078640)

p-value = [0.000051]

$$Y_t = 0.310179 + 0.000059 Y_{t-1}$$

s.d.= (0.000366) (0.058688)

p-value = [0.8721] [0.000000]

Unit Root test and ARCH-LM test

Unit Root Test		Statistic	Probability
Augmented Dickey-Fuller t-test		-12.14690	0.000000
Test Critical Values :	1% level	-3.993335	
	5% level	-3.427004	

ARCH Test	Statistic	Probability
T*R-squared	14.56590	0.000135
F-statistic	15.30563	0.000117

Empirical Results

- For in-sample estimation and test of unbiasedness, there are no supports to indicate the favor between nonparametric and parametric models
- For out-of-sample performance, there is inconclusive evidence to justify whether nonparametric model outperform parametric models to forecast return
- However, predictive efficiency measured by rolling MSPE can find grounds for choosing nonparametric model to forecast volatility
- An asymmetric U-shaped "smiling face" form of the nonparametric volatility function is found.