OVERCONFIDENCE, RATIONAL BUBBLE, AND TRADING IN PROPERTY MARKET

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by
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A Thesis submitted in partial fulfillment of
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ABSTRACT

This study contributes to investigate on the effects of asset price bubble in the property market where agents agree to disagree about the available information. Toward such contribution, this study employs Preechametta (2005) model which is a new theoretical framework developed to find an optimal stopping time for a piece of vacant land with an option to construct irreversible building in the near future.

In this model, the source of heterogeneous beliefs among agents arises from overconfidence which is one of behavioral biases. The results of this model show that when agents are overconfident, they tend to overestimate their information which causes the aggressive trading among themselves. As a consequence, it can generate land price bubble and the higher value of resale option which particularly causes the land owner to tend to exercise his building option prior to its optimal date as a result of positive early exercise premium.

According to serious dynamic inefficiency problem posed by land price bubble, it is useful to identify and explore ways to deal with a possibility of future land price bubble appropriately. In light of these concerns, this study therefore analyzes the effects of five policies which are (1) an increase in real rate of interest, (2) an increase in resale cost, (3) an increase in overconfidence level, (4) an increase in long-run fundamental, and (5) an increase in information in signals on resale and building options by doing the policy simulations. The Finite Difference Method (FDM) and Monte Carlo simulation are two efficient techniques which are employed in this study.

Our core policy simulation results indicate that an increase in interest rate policy can generally decrease the size of bubble and, in turn, a delay in land development. This is so because during the period of rising interest rate, the reduction in the gain from investing the new development project immediately is much more significant than the reduction in building option value of the new development project. Besides, an increase in resale cost such as transfer fee can reduce the aggressive trading in property market as well.

This study also finds out that an increase in long-run fundamental caused by large investment in infrastructure and an increase in overconfidence level play a significant role in stimulating land owner to develop land to be building.

However, for the policy simulation on the effect of an increase in information in signals, this study shows that level of overconfidence is of crucial importance in determining this effect. In the case of low overconfidence level in this study, an increase in information brought about by a decrease in volatility of signals has insignificantly effect on the optimal time to develop land to be building. On the contrary, an increase in information caused by an increase in volatility of fundamentals drives significantly the land owner to develop land to be building prior the exercise date because of a positive value of early exercise premium.

The above distinct results generated from the different effects of a decrease in volatility of signals and an increase in volatility of fundamentals can be obtained because, in contrast to the case of a decrease in volatility of signals, when the volatility of fundamentals increases, it also increases the volatility of the difference in beliefs which causes the size of bubble to increase.

This study also tests the rational bubble in the stock market and sub-stock market (property stock market). In a nutshell, the results that come out from these tests demonstrate that both Thailand stock and property stock prices satisfy a sufficient condition for the absence of rational bubbles. In other words, rational bubble did not exist in stock and property stock prices for the period examined.

ACKNOWLEDGEMENT

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I now understand why family members are invariably mentioned in a preface. Without my family's love, support and understanding, this thesis would have remained a virtual commodity. I also would like to express my sincerest gratitude to my grandmother, Arma' Somsree. Although she passed away, her love, support, and care is still in my heart.

All these acknowledgments notwithstanding, responsibility for these materials is mine. If any of readers have any comments or suggestions, please bring them to my attention. Any comments will be deeply appreciated.

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CHAPTER 1

INTRODUCTION

1.1 Statement of the Problem

“ *I can calculate the motions of the heavenly bodies, but not the madness of people.*”

Sir Isaac Newton

South Sea Bubble in 1720

The issue of asset price bubbles is by no means the new topic in macroeconomics and financial economics theory. The well-known putative bubbles are the Dutch Tulip mania in 17th century and the South Sea bubble in 18th century. It means that these phenomena were discovered more than 300 years ago. Despite economists and policy makers have long been fascinated in such phenomena, asset price bubbles are still not well understood.

Asset price bubbles, therefore, represent a challenge to economists and policy makers because some fundamental questions have not been answered in a convincing manner: How does one define an asset price bubble in a practical way? How can we identify an asset price bubble?

Until now, identifying of asset price bubbles is the difficult task to a practical view as a policy maker. The ability to identify asset price, hence, would be critical if a policy maker were interested in pursuing a policy to deflate bubbles.

In order to identify asset price bubble, economist use different definitions of bubble in their analytic works. The common element is that asset or output prices which increase at a rate that is greater than another one explained by market fundamental (Kindleberger,1992) or, it is a price which is above its fundamental value today only because investors believe it will be higher tomorrow. The equilibrium of these models is a Rational Expectations Equilibrium (REE).

The typical rational bubble model, for instance, in Blanchard and Watson (1982) or Santos and Woodford (1997), agents have identical rational expectation, but prices include an extra bubble component that is always expected to grow at a rate equal to the risk free rate. However, models of rational bubbles are incapable of explaining the increase in trading volume that is typically observed in the historic bubble episodes.

Because rational bubble models can not explain the asset prices and their trading volumes, a complementary literature uses heterogeneous beliefs to study trading. Several papers have been written to emphasize the role of heterogeneous beliefs in generating higher levels of asset prices and trading volume.

Miller (1977) argued that if agents have heterogeneous beliefs about the asset's fundamentals and given that short sales are not allowed, equilibrium prices would, if opinions diverge enough, reflect the opinion of the more optimistic investor.

Harrison and Kreps (1978) explain the dynamic consequences of heterogeneous beliefs. Their concept is that if the investor knows that, in the future, there may be other investors that bid the asset more than he does, the investor is willing to pay more for an asset than he would pay if he was forced to hold the asset forever. A speculative motive is reflected by the difference between the investor's willingness to pay and his own discounts expected dividends.

Unfortunately, Harrison and Kreps do not discuss the source of heterogeneous beliefs. It, then, should identify that what is the main sources of heterogeneous beliefs among investors. One of possible natural sources of disagreement and of trading is the private information; however, a series of results known as " no-trade theorems" showing that if all agents are rational and share identical prior beliefs, heterogeneity of information can not generate trading or causes a price bubble.

To avoid these no-trade results, Scheinkman and Xiong (2004) set up the model by assuming that agents have the overconfidence behaviors which is one of behavioral biases that precludes "full rationality". They use overconfidence as a convenient way to generate a parameterized model of heterogeneous beliefs. In their equilibrium, an asset owner will sell the asset to agents in the other group, whenever his view of the fundamental is surpassed by the view of agents in the other group by a critical amount. They found out that if there are no trading costs, this critical amount is zero. It means that it is optimal to sell the asset immediately after the valuation of the fundamentals of the asset owner is “crossed” by the valuation of agents in the other group. The agents’ beliefs satisfy simple stochastic differential equations and it is a consequence of properties of Brownian motion, that once the beliefs of agents across, they will cross infinitely many times in any finite period of time right afterwards. Although agents’ profit from exercising the resale option is infinitesimal, the net value of the duration between trades is relatively large because of the high frequency of trades. The difference between the transaction price and the highest fundamental valuation can be legitimately called a bubble.

Besides the financial assets such as bonds, Treasury bills, and common stocks, house is the largest single asset of most households, and assets whose value is linked to residential real estate represent an important component of the aggregate portfolio of financial intermediaries. The behavior of house prices, therefore, influences not only business cycle dynamics, through their effects on aggregate expenditure, but also the performance of the financial system, through their effects on the profitability and soundness of financial institutions.

For Thailand, the real estate sector was dramatically boom in the late 1980s due to strong economics growth and ease of access to financing caused by the financial liberalization. Since the price of real estate asset depends on the future value of fundamentals, investors therefore may agree to disagree about the future value of fundamentals. As a consequence of this divergence, speculative demand occurred and pulled the property prices rose rapidly.

Optimistic expectations of growth, heavy capital inflows, inadequate corporate governance, and dependence on intermediation by underegulated banks and financial companies led almost inevitably to rapid growth in property prices or property price bubbles. When the property price bubbles had begun collapse in 1996, it made banking systems so weakening before went to experience an exchange rate crisis, a financial crisis, and a business cycle bust in 1997.

This study therefore explores ways to identify and deal with a possibility of future real estate bubble appropriately. We study the roles of behavioral biases in determining the volatility of property price by employing the model of property price with heterogeneous beliefs made by Preechametta (2005). Two simulation techniques which are Finite Difference Method (FDM) and Monte Carlo Simulation are applied to analyze the policy simulations from this model.

In this study, not only do we simulate the policy simulations, but we also identify whether there had a rational bubble in Thailand's property market in the past time or not. However, without complete property price data in Thailand, we can not directly test the rational bubble in property market. Given these deficiencies, it is useful to supplement these data with information from stock market index. Therefore, we test the rational bubble by using SET index and stock market index for the property subsector. In order to test the rational bubble, we based on the test of rational bubble by Fukuta (1996). In his study, he considered a sufficient condition for the absence of rational bubbles which is contrast to the earlier studies such as Campbell and Shiller (1987), Diba and Grossman (1988), Lim and Phoon(1991) and Craine (1993) that consider only necessary conditions for the absence of rational bubbles.

1.2 Objectives of the Study

This study is aimed at examining the rational bubble and exploring ways to identify and deal with a possibility of future real estate bubble appropriately by:

1. Investigating whether the rational bubble was occurred in stock market, especially in the property stock, or not.
2. Analyzing the policy simulations from the model of property price with heterogeneous beliefs.

1.3 Scope of the Study

Without complete property price data in Thailand, we can not directly test the rational bubble in property market. However, many evidences[1] show that property price bubble is typically procyclical with equity price bubble. Given these deficiencies, it is useful to supplement these data with information from stock market index. Therefore, we firstly test the rational bubble by using SET index and stock market index for the property subsector.

Secondly, we analyze the policy simulations on the model of property price with heterogeneous beliefs by using the Finite Difference Method (FDM) and the Monte Carlo simulation. For parameter values, we employ the U.S. housing database. For the U.S. Treasury bill rate and consumer price index, these variables are available in IFS database. All parameter values are deflated by consumer price index.

1.4 Organization of the Study

This thesis is composed of eight chapters, which are organized as follows. Chapter one proposes statement of the problem, objectives of the study, scope of the study and organization of the study. Chapter two presents the history of property market in Thailand. The related works of this study are presented in chapter three. Then, Chapter four explains two theoretical models which are the model of property price with heterogeneous beliefs and the model of rational bubble. Chapter five is devoted to the Finite Difference Method (FDM). After that, the econometric tests, numerical technique, and two simulation methods which are employed in this study are discussed and described in details in chapter six. The econometric and simulation results are given in chapter seven. Finally, chapter eight provides the conclusion and suggestions for further study.

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[1] For example, Aswin et.al (2003) find out that the peak in equity prices in Thailand tend to lead commercial and real estate prices by 1-2 years.

CHAPTER 2

THE PROPERTY BUBBLES IN THAILAND

This chapter intends to give a good understanding about the property boom and bust cycles in Thailand since the early 1970s. From this history, we can find out that the same forces of human psychology that driven the stock market over the years have the potential to affect property markets, especially in the third boom and bust (1986-1997).

2.1 The History of Property Market in Thailand: Boom and Bust Cycles[1]

Property market in Thailand has experienced three boom and bust cycles since the early 1970s. These phenomena can be summarized in following parts:

2.1.1. The First Boom and Bust in Property Market (1971-1994)

The first boom and bust in housing market in Bangkok was created from more defined property rights laws and the availability of loans for homebuyers from commercial banks and state-owned Government Housing Bank (GHB). In 1972, the Revolutionary Party Decree No. 286 (B.E. 2515) on land subdivision was enforced to help set standards for housing development. The purpose of this decree is to provide credibility to both developers and consumers in the purchase and sale of housing. Moreover, the establishing of the National Housing Authority in 1973 as a state enterprise under the Ministry of Interior also increases the property boom at that time. However, this boom was busted by the first oil shock causing prices of building materials and labor cost to rise. This oil shock was accompanied by an increase in inflation rate. As a consequence of these effects, in 1974 the new projects dramatically decreased approximately 3 thousand units compared to 1973 which had

[1] Some parts of this section are summarized from Nakornthab et.al (2004) .

the totally projects 35 thousand units.

2.1.2 The Second Boom and Bust in Property Market Subdivision (1976-late 1970's)

The second cycle began with the recovery of the housing market in 1976. The main causes came from both the government and the financial market policies. National Housing Authority announced a plan to build an average of 24,000 units or 3% of the total housing stock per year. Low interest rate to homebuyers caused by the expansion of the financial markets also speeded the demand for housing up. In 1977, GHB extended its loan services to housing developers and became to be a major housing bank in Thailand. Furthermore, townhouses began to emerge in Bangkok, giving homebuyers greater choices for home ownership. This boom was busted by the same cause, oil shock, like the first boom. Some developers abandoned their incomplete projects. This situation has shown the real fact that this market has highly volatile because it was easily affected by the economic blossoming.

2.1.3. The Greatest Boom and Bust in the Property Market (1986-1997)

The beginning of this boom came from the significantly changed from an agricultural-based economy into an industrial-based economy. This change landed Thailand in a remarkable growth journey. Figure 2.1 shows that since 1978, the proportion of GDP of manufacturing has significantly been higher than that of agriculture. Because of this transformation, wealth has accumulated resulting in the boom in housing and real estate for over a decade (1987-1997).

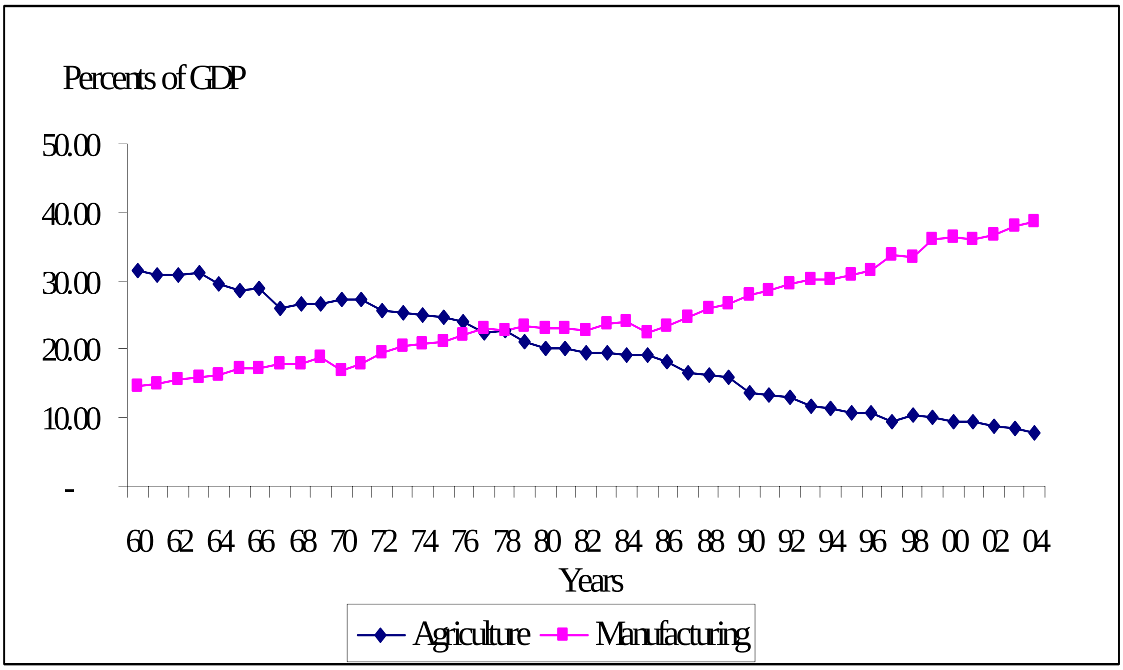
In the late 1980s, the blossoming Thai economy was fuelled by a large influx of foreign capital, particularly from Japan and Taiwan, as a result of currency realignments in the wake of the Plaza accords and relatively cheap labor and natural resources. The growth rate of the economy shapely increased, especially in the last three year of 1980s[2].

---

[2] The growth rates in 1988, 1989, and 1990 are 13.28%, 12.19%, and 11.17% respectively.

Figure 2.1.

Share of the Agriculture and Manufacturing Sectors of the GDP at 1988 Prices, 1960 -2004



Source: National Economic and Social Development Board: NESDB

The blooming economy together with large amounts of foreign direct capital investment and low interest rates for housing loans paved way for the property boom. Land particularly in the fringe area which formerly possessed a potential for agriculture uses, was converted to other alternative uses as factory sites and the like because the relatively higher returns.

Prior to 1986, most detached houses and townhouses were catered for middle and higher income groups. But since 1986, there was a down market trend to build cheaper housing units particularly in the form of low-income townhouses. Due to the economic growth and cheaper housing offers, people had relatively higher affordability.

During 1987 to 1990, a lot of symptoms of the boom emerged. Off-the-plan projects were sold very quickly. Offered prices of housing units were increased even on weekly or monthly basis. Many projects could close the sales within one day, one week or one month or a few months. In some quality projects, buyers queued up to book a house since 05.00 am in the morning.

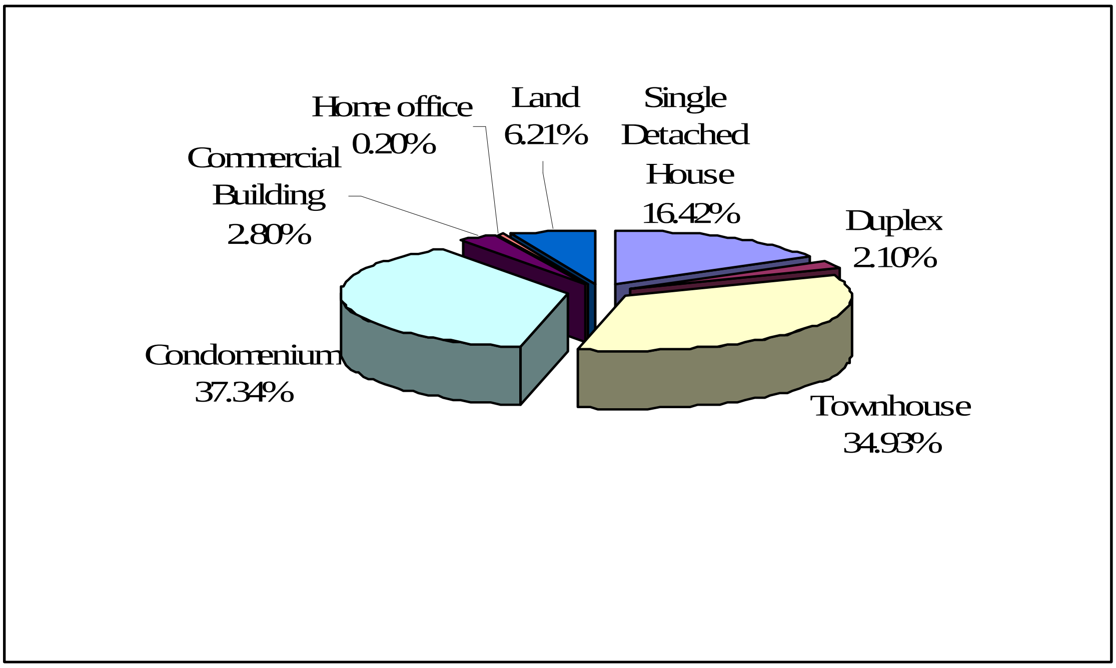
Other real estate products apart from housing were also boomed. These included hobby farm land subdivisions, golf courses, office buildings, and such like. Speculations prevailed on nationwide basis. As observed, a lot of foreigners came to speculate in properties in Thailand whereas real estate was not allowed to be owned by them. Therefore many foreigners found the solution to investment by paying for the booking fee and down payment. At the date of the completion and transfer, they could find another buyer to buy their units at a lot higher price than they originally booked.

The passionate real estate market slowed down temporary in 1990 due to the Gulf War. Speculative and extravagant real estate projects faced difficulties. While land and other luxurious projects became less popular for speculation due to the drops in prices, people began speculating low-income housing particularly low-income condominiums.

Moreover, the supporting from the Board of Investment (BOI) to encouraged housing developments by offering 5 years income tax exemption to developers who developed low-income housing units (under 600,000 baht/unit) also participated in the expanding of this boom. In 1994, approximately 114 projects of housing units (under 600,000 baht/unit) were in this program.

Figure 2.2

Percentages of Housing Completion Separated by Housing Types from 1994-1996



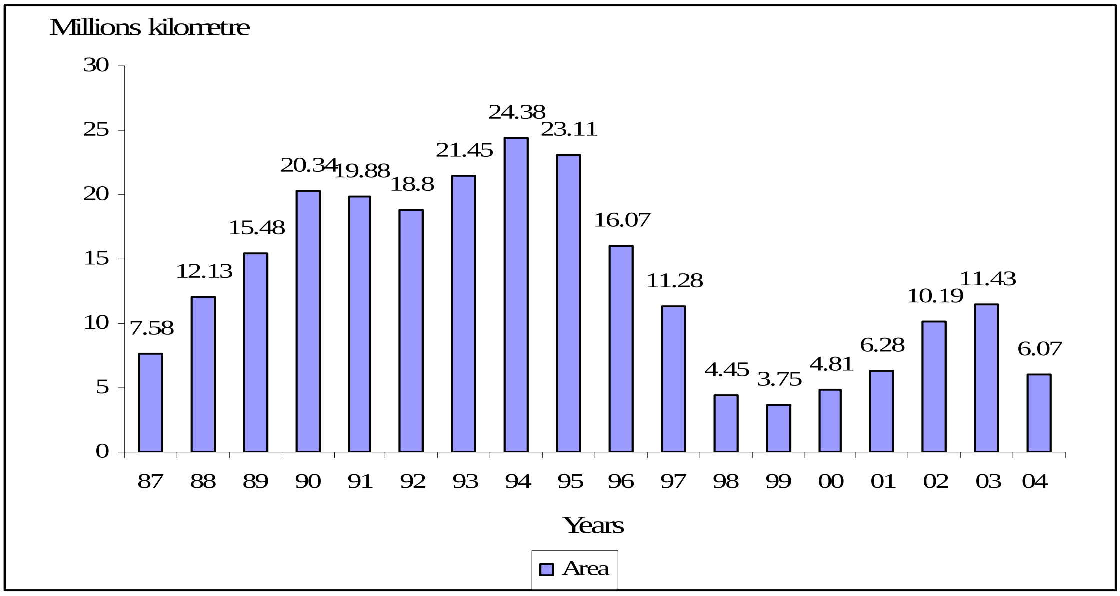
Source: GHB Collateral Database.

Figure 2.2 shows the percentages of housing completion separated by housing types from 1994 to 1996. Approximately 53.4% were the horizontal building such as single houses and townhouses. The left were the vertical building such as condominiums and flats. From this figure, it was found that most of the new projects came from condominiums and townhouses with the increasing growth rate from 3% in 1987 to 34 % in 1997. Due to the massive speculation in the housing sector, in 1995, half of the 300,000 units in Bangkok metropolitan regions were unoccupied condominiums.

Figure 2.3

Housing Construction Permit

in Bangkok Metropolitan Areas and Provinces in Thailand

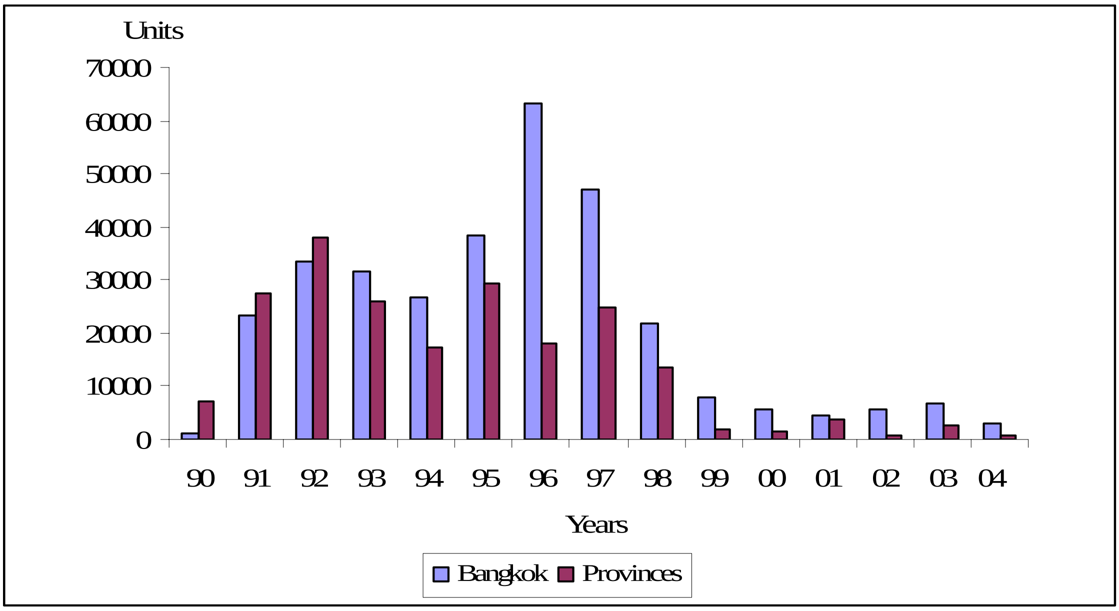


Source: GHB Collateral Database.

Figure 2.4

The New Registered of Condominium

in Bangkok Metropolitan Areas and Provinces in Thailand



Source: GHB Collateral Data Base.

By 1996, the supply of real estate exceeded the actual demand in almost all sectors of the property market. Many developers began to experience cash flow problems. In February 1997, one of the leading property developers, Somprasong Land, defaulted on its Euro-convertible debenture (ECD). Moreover, most developers abandoned many of their ongoing projects. Since approximately 70% of overall real estate developments in Thailand were housing units, the resulting crash in this sector was devastating to the rest of the economy. When the property price bubble had begun collapse in 1996, it made banking systems so weakening before went to experience an exchange rate crisis, a financial crisis, and a business cycle bust in 1997.

CHAPTER 3

REVIEW OF RELATED LITERATURE

Under the assumption of rational behavior and of rational expectation, economists usually believe that the price of an asset should simply reflect market fundamentals that come from information about current and future returns from this asset. Deviations from this are taken as prima facie evidence of irrationality. Some economists and market participants on the other hand, often believe that fundamentals are only part of what determines the prices of assets.

In this chapter, we intend to review the theoretical models of rational bubble and the role of heterogeneous beliefs in generating higher levels of asset prices and trading volume. We also summarize the empirical studies in asset price bubble which are the empirical studies in the effect of heterogeneous beliefs on asset price bubble and the empirical studies in asset price bubble in Thailand.

3.1 Theoretical Models of Rational Bubble

Blanchard and Watson (1982) argue that rationality of both behavior and of expectations often does not imply that the price of an asset be equal to its fundamental value. In other words, there can be rational deviations of the price from this value, which are rational bubbles. They begin by showing that arbitrage does not by itself prevent bubbles. From the price solution:

$$P_t = \sum_{i=0}^{\infty} \theta^{i+1} E(x_{t+i} \mid \Omega_t) + c_t = P_t^* + c_t , \tag{3.1}$$

with $$E(c_{t+1} \mid \Omega_t) = \theta^{-1} c_t , \quad \theta \equiv (1+r)^{-1} < 1, \tag{3.2}$$

where P_t is the price of the asset, x_t the direct return which is refer as the “dividend”. R_t is the rate of return on holding the asset, which is the sum of the dividend price ratio and the capital gain. Ω_t is the information set at time t, assumed common to all agents. P^* is the present value of expected dividends and thus can be called the "market fundamental" value of the asset. c_t embodies the popular notation of a "bubble".

Equation (3.1) shows that the market price can deviate from its market fundamental value without violating the arbitrage condition. The parameter θ^{-1} imply that if it greater than 1, this deviation c_t must be expected to grow over time.

After they showed that arbitrage does not by itself prevent bubbles, they found out that there exist some conditions such that bubbles can in fact be rule out. In the case that there are a finite number of infinitely lived players-market participants, the bubbles cannot emerge. If the price is below the market fundamental, then it will pay to buy the asset and to enjoy its returns. Thus there cannot be a negative bubble. In the case that the price is above market fundamentals, it will pay to sell the asset short forever and thus there cannot be a positive bubble. In the case absence of short selling, the only reason to hold an asset whose price is above its fundamental value is to resell it at some time and to realize the capital gain. But if all agents intend to sell in finite time, nobody will be holding the asset thereafter, and this can not be equilibrium. They conclude that the bubble condition like as Ponzi games, what is needed is the entry of new participants. If a market is composed of successive "generation" of participants, then the above argument does not hold and bubbles can emerge. Moreover bubbles are probably more likely in markets where fundamentals are difficult to assess, such as the gold market. By the same argument, bubbles are less likely for assets with clearly defined fundamentals such as blue chip stocks or perpetuities.

Tirole (1985) presents his model to investigate whether the fundamentalist view of asset pricing extends to overlapping generations economies. In order to study the existence and characterization of bubble, he constructs his model based on Diamond's classic contribution[1]. In his model each consumer has 2 lived periods and the population grows at rate $n > 0$. All consumers work in the first period and get the

[1] See Diamond(1965).

real return equals $w_t = \phi(r_t)$. They save some of their income to the next period. Their saving is divided into 2 parts; (1) productive saving and (2) non productive saving.

Each consumer uses productive saving to invest in capital stock which generates the real return equals $r_t = f'(k_t)$, where k_t is the capital stock per worker. In the equilibrium, given r_{t+1}, firms invest at time t so as to equalize the marginal productivity of capital and the interest rate. Let a_t be the difference between savings per capita and the level of capital stock per capita in the constant returns to scale sector. Thus we have

$$r_{t+1} = f'\left(\frac{s(w_t, r_{t+1}) - a_t}{1+n}\right), \tag{3.3}$$

where $s\left(w_t, r_{t+1}\right)$ is represented the individual saving function.

Equation (3.3) can be defining followed Diamond as

$$r_{t+1} = \psi(w_t, a_t), \tag{3.4}$$

and that : $\psi_w < 0, \psi_a > 0$.

Let $\bar{r}$ be defined by

$$\bar{r} = \psi\left(\phi(\bar{r}), 0\right), \tag{3.5}$$

In the case where $\forall_t a_t = 0$, Diamond has shown that there exists a unique competitive equilibrium. In this equilibrium, the interest rate converges to $\bar{r}$. The equilibrium path is efficient if $\bar{r} > n$ and inefficient if $\bar{r} < n$.

For his paper, he extends Diamond's model to include rents and bubbles. First, a real rent (dividend), such as a natural resource, land, paintings, and jewels or decreasing returns to scale technologies, has the market fundamental per capita, f_t defined by

$$f_t = \frac{F_t}{(1+n)^t}\,, \tag{3.6}$$

where F_t is defined as the market fundamental of the corresponding assets. For a sequence of real interest rates, its value is equal to

$$F_t = R\left[\sum_{s=t+1}^{\infty} \frac{1}{(1+r_{t+1})\ldots(1+r_s)}\right], \tag{3.7}$$

where R is represented the total rent in the economy.

Second, consumers can invest in asset with a zero market fundamental and are called bubbles. The aggregate bubble per capita is denoted by b_t. Under perfect foresight the bubble must bear the same yield as capital as

$$b_{t+1} = \frac{1+r_{t+1}}{1+n} b_t\,, \tag{3.8}$$

where b_t must be positive[2]

The difference between savings per capita and capital stock per capita is then equal to

$$a_t = f_t + b_t\,, \tag{3.9}$$

which is called nonproductive savings.

In the case where Diamond's (bubbleless and rentless) equilibrium is inefficient $(\bar{r} < n)$, define $\hat{b}$ by

$$n = f'\left(\frac{s(\phi(n), n) - \hat{b}}{1+n}\right), \tag{3.10}$$

[2] See the proof in Tirole(1985).

where $\hat{b}$ is well-defined and is unique. We summarize the results from three conditions of $\bar{r}$ in table 3.1.

Table 3.1

The Equilibrium Solutions from Three Conditions on $\bar{r}$

The condition on $\bar{r}$	**Equilibrium solution**
1. $\bar{r} > n$	There exists a unique equilibrium. This equilibrium is bubble less and the interest rate converges to $\bar{r}$.
2. $0 < \bar{r} < n$	There exists a maximum feasible bubble $\hat{b}_0 > 0$, such that (i) for any b_0 in $[0, \hat{b}_0)$, there exists a unique equilibrium with initial bubbles b_0 where equilibrium is asymptotically bubbleless and the interest rate converges to $\bar{r}$. (ii) there exists a unique equilibrium with initial bubble $\hat{b}_0$. The bubble per capita converges to $\hat{b}$ and the interest rate converges to n. $\hat{b}_0$ and the initial level of nonproductive savings $\hat{a}_0$ both increase with k_0.
3. $\bar{r} < 0$	There exists no bubbleless equilibrium. There exists a unique bubbly equilibrium. It is asymptotically bubbly and the interest rate converges to n.

He also gives a rough intuition for the case that there are no rents in the inefficient economy. The dynamic system can then be simply described by two difference equations:

$$b_{t+1} = \frac{1 + f'(k_{t+1})}{1+n} b_t \text{, and} \tag{3.11}$$

$$(1+n)k_{t+1} + b_t = s\left(f(k_t) - k_t f'(k_t), f'(k_{t+1}) \right). \tag{3.12}$$

From Figure 3.1 , it is the corresponding phase diagram which give an heuristic description of the behavior of this system. The constant-capital-per-capita locus $(k_{t+1} = k_t)$ slopes negatively at the Diamond bubbleless steady state from the stability assumption. From (3.11), (3.12) and the assumption that savings increase with income, the constant-bubble-per-capita locus $(b_{t+1} = b_t)$ always slopes up. The asymptotically bubbly path is the saddle path converging to the golden rule steady state in the figure 3.1. Along this path, the per capita levels of capital and bubble converge monotonically to their steady state values. Moreover, higher capital levels allow higher bubbles. If the system starts under the saddle path, the equilibrium is asymptotically bubbleless which converges to the Diamond steady state. The system cannot start above the saddle path; if it did the bubble would inflate too fast, so that capital would become negative in finite time.

When he reintroduces rents, the long run rate of interest in the bubbleless and rentless economy can be negative if capital depreciates. Then, the interaction between rents and bubbles becomes important. First, there exists no bubbleless equilibrium. He explains that when if the market fundamental of rents per capita f_t converges to zero, almost comsumers will use their saving for capital accumulation. Thus the economy will behave asymptotically like the Diamond rentless and bubbleless economy and the interest rate converges to $\bar{r} < 0$. However, there is a contradiction because the market fundamental of rents will come up to infinite. Therefore if $\bar{r} < 0$, bubbles are necessary for the existence of an equilibrium in an economy in which there exists an rent.

Second, in the case that the rents per period grow at the rate of population growth, a perfect foresight equilibrium must be efficient. Otherwise the rent per period would grow at a rate exceeding the rate of interest and its market fundamental would be infinite.

He argues that because rents are created over time, bubbles are not necessarily inconsistent with rents per period growing as fast as the economy. An important feature of rent creation is that most rents are not capitalized before their “creation”. To formalize this idea, he assumes the each consumer is born with R units of real rents that are increase at the population rate. The rest of the model is the same as before.

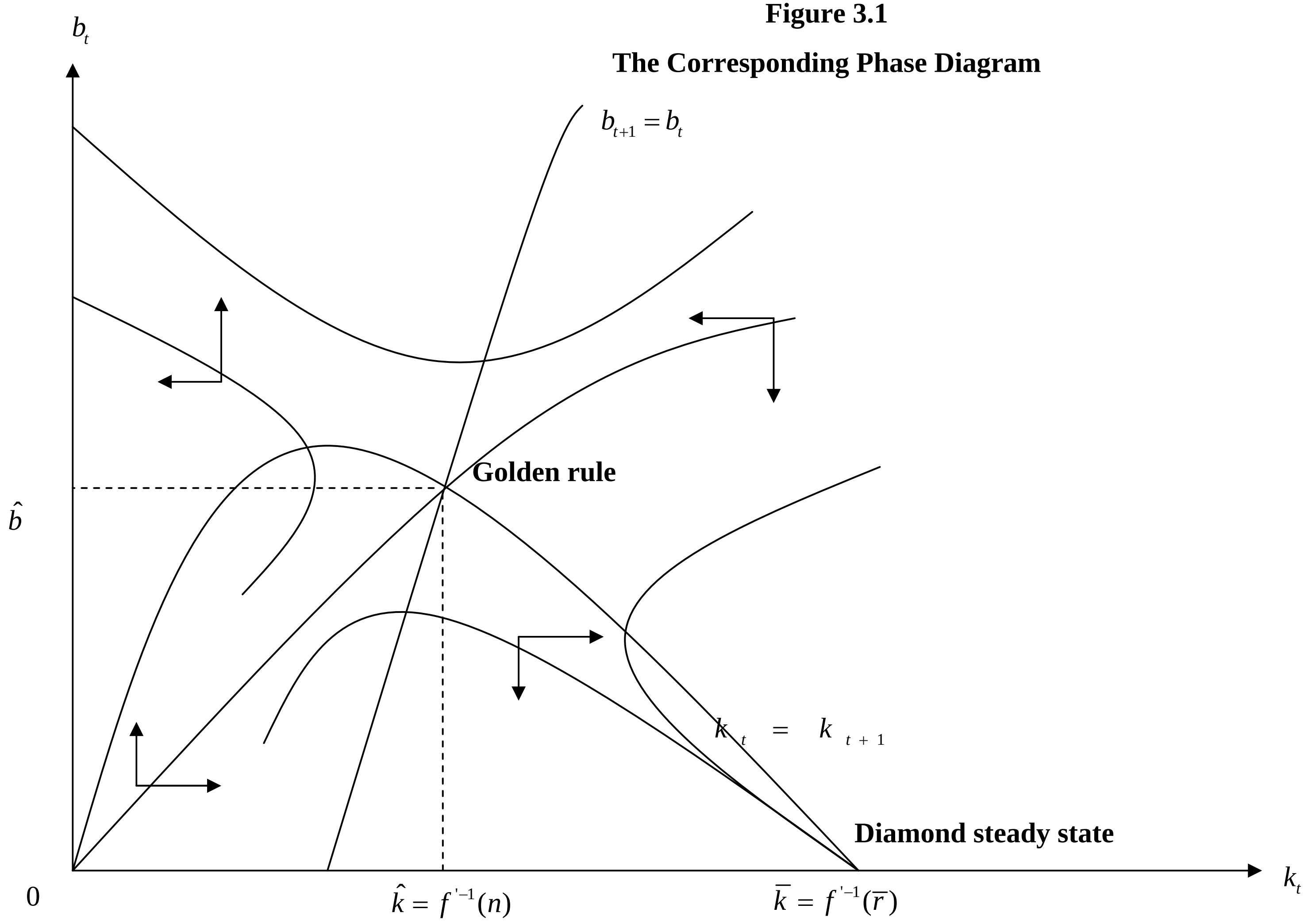
He finds out that there exists an (asymptotically) bubbly steady state of the economy with rent creation if and only if $r^* < n$. The steady bubble per capita, $\hat{b}$, is then given by

$$n = f'\left(\frac{s\left(\phi(n) + \frac{R}{n}, n\right) - \frac{1+n}{n}\frac{R}{n} - \hat{b}}{1+n} \right), \tag{3.13}$$

where $\hat{b}$ decreases with R because intensive rent creation crowds out bubbles. In this model, the absence of ex-ante capitalization is crucial to this conclusion: At any moment of time most rents still remain to be created and thus do not necessarily crowd out current bubbles fully.

Figure 3.1

The Corresponding Phase Diagram



3.2 Heterogeneous Beliefs in Asset Price

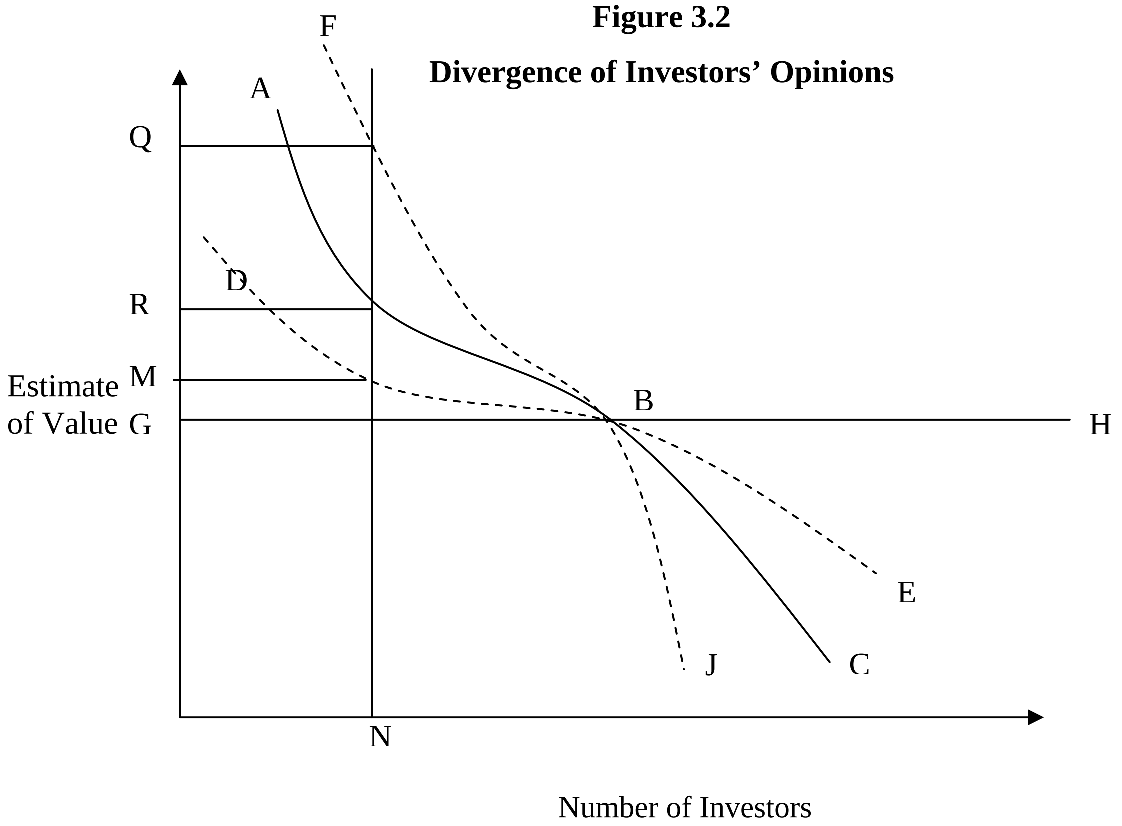
There are several papers that have been written to emphasize the role of heterogeneous beliefs in generating higher levels of asset prices and trading volume. In this part, we present a selective survey of this literature.

Miller (1977) argues that if agents have heterogeneous beliefs about an asset's fundamentals and short sales are not allowed, equilibrium prices would, if opinions diverge enough, reflect the opinion of the more optimistic investor.

He starts with a relative simple financial market in which securities are risk free one year government bonds and assumes a single common stock company organized to carry out a one year project. At the end of the year, the company will be liquidated and the assets will be divided among investors. The investors seek only to maximize the present value of their investment. Given the uncertainty about the true return to the investment in the security, potential investors make different estimates of expected returns from the investment. Let any single investor is able to purchase only one share and there are N shares available. N investors who bid the highest price therefore will end up being owned stocks. The equilibrium can be presented in the figure 3.2 . Curve ABC in figure 3.2 is plotted showing the cumulative distribution of the number of investors with estimate above a certain value for the amount received at liquidation of investment. From curve ABC, it can be seen that there are N investors who estimate the final value to be R or above. The selling price of the stock will therefore be R. As long as a minority of potential investors can absorb the issue, an increase in the divergence of opinions will increase the market clearing price. This can be seen by noting that if curve ABC is replaced with curve FBJ, the market clearing price rises from R to Q. On the other hand, if the divergence of opinion decreases, causing curve ABC to be replaced with curve DBE, the market clearing price falls from R to M. In the limit, where there is no disagreement about the return from the security, curve ABC becomes the straight line GBH, and the market price falls to G.

Figure 3.2

Divergence of Investors' Opinions



Scheinkman and Xiong (2004) illustrate Miller's argument using a version of Lintner (1969)[3] model by adding short-sales constraints. They begin with one period but two dates: t = 0,1. There is one risky asset which will be liquidated at t = 1. The liquidation value is assumed to be

$$\tilde{f} = \mu + \varepsilon, \tag{3.14}$$

where μ is a constant unknown to investors and ε is normally distribution with mean zero. Investors have heterogeneous beliefs about the distribution of liquidation values. Each investor believes that $\tilde{f}$ has a normal distribution with mean μ_i and variance σ^2.

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[3] Related models can be found in Jarrow (1980), Varian (1989), Chen, Hong and Stein (2002), and Gallmeyer and Hollifield (2004).

Since all investors share the same views concerning the variance, they index investors by their mean beliefs μ_i, and assume that μ_i is uniformly distribution around μ in the interval $[\mu - k, \mu + k]$. The parameter k measures the heterogeneity of beliefs. The total supply of the asset is Q. Investors, moreover, can borrow or lend at a risk-free interest rate of zero, whereas short-sales of the asset are prohibited.

At t = 0, each investor chooses his asset demand to maximize his expected utility at t = 1:

$$\max E\left[-e^{-\gamma(W_0 + x_i(\bar{f} - p_0))}\right], \tag{3.15}$$

where γ is the investor's risk aversion parameter. $\bar{f}$ is the long run mean of liquidation value. W_0 is the initial wealth, p_0 is the market price of the asset, and x_i is the investor's asset demand, subject to $x_i \geq 0$. It is immediate that

$$x_i = \max\left[\frac{\mu_i - p_0}{\gamma\sigma^2}, 0\right]. \tag{3.16}$$

Investors with mean beliefs μ_i below the market price stay out the market. The market clearing condition implies that $\int_i x_i = Q$, thus implies that

$$\int_{\max\{p_0, \mu - K\}}^{\mu + K} \frac{\mu_i - p_0}{\gamma\sigma^2} \frac{d\mu_i}{2K} = Q, \tag{3.17}$$

and the equilibrium price is defined as

$$p_0 = \begin{cases} \mu - \gamma\sigma^2 Q & if \quad K < \gamma\sigma^2 Q \\ \\ \mu + K - 2\sqrt{K\gamma\sigma^2}Q & if \quad k \geq \gamma\sigma^2 Q \end{cases} \tag{3.18}$$

In the absence of short–sales constraints, the equilibrium price would be $\mu - \gamma\sigma^2 Q$. Therefore, the short-sales constraints cause the asset price to become higher when the heterogeneity of investor's beliefs K is greater than $\gamma\sigma^2 Q$.

From the above model, it simply shows that short-sales constraints combined with heterogeneous beliefs can cause asset prices to become higher than they would be in the absence of the short-sales constraints.

The model in the previous part, even though it can explain the price effect when beliefs are sufficiently heterogeneous, and short-sale constraints exist, it has no prediction concerning the dynamics of trading. Harrison and Kreps (1978), therefore, study a dynamic model in discrete time with short-sales constraints.

They argue that investors exhibit speculative behavior if the right to resell a stock makes them willing to pay more for it than they would pay if obliged to hold it forever. This can not occur in a world with one period remaining (as in the capital-asset-pricing model), in a world where all investors are identical, or in a world with complete and perfect contingency claims markets.

Harrison and Kreps divide investors into 2 groups $\{A, B\}$, each with an infinite number of agents. Investors are risk-neutral, and discount future revenues at a constant rate $\gamma < 0$ or equivalently can borrow and lend at a rate $r = \frac{1-\gamma}{\gamma}$.

In the economy, there exists one unit of an asset that pays a non-negative random dividend d_t at each time t. Agents are divided into two groups $\{A, B\}$, each with an infinite number of agents that differ on their views on the distribution of the stochastic process d_t.

For simplicity, they assume that each group $C \in \{A, B\}$ views d_t as a stochastic process defined on a probability space $\{\Omega, \mathcal{F}, P^C\}$ [4] and that $P^A \sim P^B$ [5]. They write E^C for the expected value with respect to the probability distribution shared by all agents in group C $\in \{A, B\}$. $\mathcal{F}_t$, $t \geq 0$ for the σ-algebra generated by the realizations of $d^t \equiv (d_1, ..., d_t)$. A price process is an $\mathcal{F}_t$ adapted non-negative process.

The owner of an asset at t must decide on a strategy to sell all or part of his holdings in the future. Because each group has an infinite number of agents and there is a single unit of the asset, competition among buyers will lead to a price that equals the reservation price of the buyers. The value of the asset at t for an agent in group $C \in \{A, B\}$ is then given by

$$\sup_{T>t} E^C \left[\sum_{k=t+1}^{T} \gamma^{k-t} d_k + \gamma^{T-t} p_T \,|\mathcal{F}_t \right], \tag{3.19}$$

where $\sum_{k=t+1}^{T} \gamma^{k-t} d_k$ presents the value of the discounted dividend streams receive from t to stopping period T. $\gamma^{T-t} p_T$ represents the discounted value from selling the asset at the prevailing market price at T. The buyer will belong to the group that bid the highest price on the asset. An equilibrium price process therefore has to satisfy:

$$p_t = \max_{C \in \{A,B\}} \sup_{T>t} E^C \left[\sum_{k=t+1}^{T} \gamma^{k-t} d_k + \gamma^{T-t} p_T \,|\mathcal{F}_t \right], \tag{3.20}$$

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[4] Ω is represented an abstract set (or space), $\mathcal{F}$ is represented a class of subsets of Ω, and P^c is represented the probability of d_t given the information set $\mathcal{F}$.

[5] $P^A \sim P^B$ means agents in group A and B have the same cumulative distribution function (CDF) of d_t.

for all t = 0,1,…and $t+1 \le T \le \infty$. This equation is a natural condition for equilibrium in the market. Suppose that a price scheme is to be followed (3.20), then for each class $C \in \{A, B\}$, we can represent the maximum expected present worth that an investor from that class can realize from stock held at time t when he follows a legitimate strategy for subsequent sale, given the economic information which is available at time t as

$$\sup_T E^c \left[\sum_{k=t+1}^{T} \gamma^{k-t} d_k + \gamma^{T-t} p_T \,\middle|\, \mathcal{F}_t \right]. \tag{3.21}$$

Therefore, the right-hand side of (3.20) is the maximum amount that the stock is worth to any investor at time t. If this amount was strictly larger than the price p_t , then members of the maximizing class(es) would complete among themselves to drive the price up. If it were smaller, then whoever held the stock at time t would want to sell but would find no buyer, so the price would have to fall.

From this study, speculative behavior arises, because the owner of the asset retains in addition to the flow of future dividends an option to resell the asset to other investors. This option will become to be valuable when there are investors that have a relative more optimistic view of future dividends than the current owner.

Nevertheless, Harrison and Kreps do not explicitly address the source of heterogeneous beliefs among investors. Up to this point, we have already known that heterogeneous beliefs can cause the price diverts from its fundamental but what are the sources that make the belief of one person/ group of investors clearly difference from the other person/group. On that account, we will examine specific mechanisms to generate beliefs' heterogeneity.

One may argue that private information is a possible source of heterogeneous beliefs. The presence of private information suggests that investors could use their information to trade and realize a profit.

However, Tirole (1982) and Milgrom and Stokey (1982) prove that this can not happen when all agents are rational and share identical prior beliefs, the conditions that are imposed in the standard rational expectation models. Therefore, private information cannot be the source of speculative trading. We call this situation as " no-trade-theorem".

The main idea of "no-trade-theorem" can be explained by using a static setup from Tirole (1982). He employs the rational expectation equilibrium (REE) concept into a dynamic speculation. The idea behind rational expectations equilibrium (REE) is that each trader is able to make inferences from the market price about the profitability of his trade.

Consider a market with I risk averse or risk neutral traders:1,…, I . The traders exchange claims for an asset with random value $\tilde{p} \in E$, which will be realized after the trading. The set $E \subset \mathbb{R}$ is the set of all possible payoff-relevant environments. In the equilibrium, claims are traded at an equilibrium market price p . If a trader buys x_i units of the asset and he receives $\tilde{p}$. Trader i's ex-post (realized) gain is

$$G_i = (\tilde{p} - p)x_i \,. \tag{3.22}$$

Each trader i receives a private signal s^i belonging to a possible set of signals S^i . The vector of all signals is $s = \left(..., s^i, ...\right)$ belonging to a set S. Then $\Omega \equiv E \times S$ is the set of states of nature. Assume that all traders have a common prior ν on Ω. Let T be a set contained in S; $v^i\left(s^i \middle| T\right)$ be the marginal probability of signal s^i conditional on $\{s \in T\}$. $v^i\left(s^i\right)$ is denoted the prior probability of signal s^i. All signals have a positive probability:

$$\forall i, \forall s^i \in S^i : \qquad v^i\left(s^i\right) > 0 \,. \tag{3.23}$$

Trader i = 1,…,I chooses an amount x^i to maximize his conditional expected value of G. Each trader uses all information at his disposal, including the observed market price p in a REE. Moreover, a REE does not involve only an equilibrium price, but a forecast function that maps each vector of all signals $s \in S$ into a price that establishes equilibrium in the market if the state s obtains. Trader i is not able to identify the full vector of signals s by investigating the price p as well as his own signal s^i. However, a forecast function $\Phi : S \to \mathbb{R}$, an observed p, and a signal s^i induce a conditional distribution on $E \times S, \Gamma^i_{\phi,p,s^i}$.

Definition 1: A REE is the forecast function ϕ which associates with each set of signals s a price $p = \phi(s)$, and a set of trades $x^i\left(p, s^i, S\left(p\right)\right)$ for each agent i, relative to information s^i and $s \in S(p) \equiv \phi^{-1}(p)$ such that:

1. $x^i(\phi(s), s^i, S(p))$ maximizes i's expected utility conditional on i's private information s^i, and the information conveyed by the price S(p), $E_\phi(G^i \left| p, s^i\right.) \equiv \int G d\Gamma^i_{\phi,p,s^i}$
2. The market clears for each $s \in S : \sum_i x^i\left(\phi\left(s\right), s^i, S\left(p\right)\right) = 0$.

Because the total monetary gain in such a market is zero: $\sum G^i = 0$, the market is purely speculative if moreover the participants' initial positions are uncorrelated with the return on the asset.[6]

Since trader i has a concave utility function, has no insurance motive in the market, and has the option not to trade, he must expect a nonnegative gain:

$$E_\phi\left(G^i \left| s^i, S\left(p\right)\right.\right) \geq 0. \tag{3.24}$$

It should be true for any single s^i belonging to the projection $S^i\left(p\right)$ of $S\left(p\right)$ on S^i. This implies:

$$\begin{aligned} E_\phi\left(G^i \left| S\left(p\right)\right.\right) &= \sum_{s^i \in S^i(p)} E_\phi\left(G^i \left| s^i, S\left(p\right)\right.\right) v^i\left(s^i \left| S\left(p\right)\right.\right), \\ &= E_\phi\left(E_\phi\left(G^i \left| s^i, S\left(p\right)\right.\right) \left| S\left(p\right)\right.\right) \geq 0. \end{aligned} \tag{3.25}$$

[6] "Uncorrelated" is relative to the information of the trader.

From the market clearing condition, $\sum_i G^i = 0,$. Then,

$$\sum_i E_\phi\left(G^i \middle| S(p)\right) = 0\,. \tag{3.26}$$

This implies that $\forall i: \; E_\phi\left(G^i \middle| S(p)\right) = 0$, and consequently

$$\forall i: \qquad E_\phi\left(G^i \middle| s^i, S(p)\right) = 0\,. \tag{3.27}$$

From equation (3.27), it means that in a REE no trader can expect a gain. Therefore, we can conclude that in a REE of purely speculative market with risk-averse or risk-neutral traders, risk-averse traders do not trade; risk-neutral traders may trade, but they do not expect any gain from their trade.

3.3 The Empirical Studies in Asset Price Bubble

3.3.1 The Empirical Studies in the Effect of Heterogeneous Beliefs on Asset Prices Bubble

Mei, Scheinkman , and Xiong (2003) provide direct evidence in support the role of heterogeneous beliefs on asset price. To analyze non-fundamental components in stock prices, they test their model using unique data from the Chinese stock market during of 1994-2000. Chinese stock markets are suitable to study in this paper because 2 main reasons. First, Chinese stock markets were only recently re-opened in early 1990s, therefore, most domestic investors are new to stock trading, and are more likely to be behavioral biases such as overconfidence. Second, short-sale and equity issuance[7] are not allow in China.

[7] Equity issuance by firms is a common practice that firms use to “arbitrage” the over-valuation of their own stocks.

The problem arises from two classes of shares, class A and class B, with identical rights. Domestic investors could only buy A shares while foreign investors could only hold B shares. Despite their identical payoffs, class A shares traded on average at 400% more than the corresponding B shares. In addition, A share shares turn over at much higher rate 500% versus 100% per year for B shares. The striking price difference and big share turnover are often attributed to speculative bubbles by commentators.

Accordingly, they firstly propose a formal regression analysis to test a speculative bubble. According to the theory of speculative bubbles described in Scheinkman and Xiong (2003), the stock price should move in the same way of its turnover. Their results find out a positive and significant cross-sectional relationship between A-share turnover and A-B premium (ρ_{it}) [8]. It supports the main hypothesis of this study that A-share investors' speculative motives contributed a speculative component to A-share prices. In this paper, they also study the relationship between the liquidity effect and A-share turnover. They start with the argument that the relationship between A-share turnover and A-B premium was related to the market liquidity of A-shares. If a firm's A-shares were relatively more liquid, investors would have traded more and been willing to pay more for these shares, because of the smaller transaction cost. As such, cross-sectional difference in liquidity could also generate a positive relationship between A-share turnover and A-B premium. Nevertheless, they find out a negative and significant cross-sectional relationship between share turnover and asset float (the publicly tradeable shares) in A-share market but a positive and significant relationship in B-share markets. Since their model predicts a negative correlation between turnover and float, and liquidity usually improves with larger float, these results suggest that trading in A-shares is driven by speculation, while trading in B-shares is more consistent with liquidity. Moreover, they find out that the asset float affects share premium. The asset float or market cap

[8] A-B premium in their paper is defined as $\rho_{it} = \dfrac{P_{it}^{A} - P_{it}^{B}}{P_{it}^{B}}$ where P_{it}^{A} and P_{it}^{B} are represented A-share price and B-share price of firm i at time t, respectively.

of A-shares has the a negative and highly significant effect on A-B share premium (ρ_{it})-higher asset float of A-shares controlling for a host of contemporaneous variables including turnover leads to lower prices of A-share relative to its B counterpart. On the contrary, the market cap of B-shares has a negative and highly significant effect on A-B premium, consistent with higher float leading to more liquid B shares and higher B prices.

Scheinkman, Hong, and Xiong (2005) develop a discrete-time, multi-period model to investigate the relationship between the float of an asset (the publicly tradeable shares) and the propensity for speculative bubble to form. Due to the insider lock-up restrictions, investors trade a stock that initially has a limited float. However, the tradeable shares of which increase over time as these restrictions expire. In this paper, investors are assumed to have heterogeneous beliefs due to overconfidence and are short-sales constrained. As a result, they pay prices that exceed their own valuation of future dividends because they believe that they can find a buyer who is willing to pay even more in the future. This is called “resale option effect” which imparts a bubble component in the asset price. Based on the limited risk absorption capacity, this resale option depends on float as investors anticipate the change in asset supply over time and speculate over the degree of insider selling. Their empirical implications are consistent with stylized accounts of the importance of float for the behavior of internet stock prices during the late nineties. These implications are: 1) a stock price bubble dramatically decreases with float; 2) share turnover and return volatility also decline with float; and 3) the stock price tends to decline on the lock-up expiration date even though it is known to all in advance.

The above empirical studies are based on the asset type “stock”. Now we turn to Wong(2005) paper which is the study how it had the upswing in the Hong Kong residential housing prices during the mid-1990s by employing a unified framework, an application of a speculative model (Scheinkman and Xiong (2003)). She begins by showing that the market-wide index (CentaCity Index) experienced a real increase of 50 percent from 1995 to 1997, followed by a real decrease of 57 percent from 1997 to 2002. The main objective in her study is to test whether Hong Kong residential has a positive cross-sectional relationship between the size of the speculative price component and the turnover rate or not. In order to test this, she controls for liquidity, following the approach in Mei, Scheinkman and Xiong (2004). Moreover, the correlation is assumed both in and out of the “speculative period”, which is defined as the period during which at least 100 estates[9] were at the point between their trough and peak prices. If the positive correlation is mainly due to speculation, one expects to see a stronger and more significant relationship during the speculative period. On the other hand, if the positive correlation is caused by liquidity premium and other non-speculative factors, it should remain more or less constant in and out of the speculative.

Based on her results, it shows a stronger and more robust correlation between price movements and turnover rate within the speculative as compared to the non-speculative period. Hence, it gives a strong support for speculative activities fuelling the price upswing. In her study, she also investigates the relationship between the price upswing and the media reporting because it has been suggested that the hype generated by media reporting of home price movements has a positive impact on the spread of speculative activities (e.g., Shiller). The results show that media reporting does not seem to have promoted the price upswing. In the last section, she studies the effect of uncertainties about political future of Hong Kong around the Handover and finds out that it is related to the price upswing too.

3.3.2 The Empirical Studies in Asset Price Bubble in Thailand

We firstly begin with Weerapon (2000) paper which is the study about the effect of asset price bubbles on the economy and the impacts of monetary policy on the asset price bubbles. His study is based on Thai’s experience which had the aggressive rose in asset prices since the late 1980s, and subsequently went collapsed in the second half of the year 1996. This dramatic rise and fall in the asset prices were

[9] Estate is a large-scale housing in which a large proportion of the Hong Kong population lives. These estates consist of many blocks of almost identical units, and are spread across different geographical areas in the territory.

obviously said to originate from the equities and the real estate assets or “asset price bubble”.

As of data availability limitation, the empirical investigation can be done only in the area of stock market. The first objective in his study is to measure the bubble size of the asset prices. In order to achieve this objective, he begins the study by identifying the fundamental asset prices based on long-run relationship between stock price index (p_t) and earnings per share(EPS_t). The stock price bubble is verified as the deviated of the stock prices from its fundamental prices. The result from the regression shows that the long-run relationship between stock index and earnings per share performed well during 1886 till October 1989; however, he finds out that the bubble sizes for the stock market were large in two periods. One of two periods started from the beginning period of 1989 and lasted until the third quarter of 1990. This boom was driven from a good expectation about the prosperity of the Thai economy. The optimistic expectations upon the growth of the economy certainly influence the stock prices to rise. Moreover, during this period, the economy was growing at a high growth rate with the abrupt development in infrastructures. The second period began in the forth quarter of 1993 and ended up in the mid-1996. A significant adjustment in Thai economic environment within this period was due to the financial liberalization.

In the second and third objectives, he aims to find out the impacts of monetary factors on asset price bubbles and the effect of asset price bubbles on the economy by employing a Vector Autoregression model (VAR) and its interpretation methods which are variance decomposition and impulse response function. The main results show that the credit control policy is the effective policy in order to control the size of the bubble in the stock market when comparing with interest rate policy and the control of money supply. For the effects of asset price bubbles, he finds out that the innovation of the bubble size in the stock market can explain a variation of private investment only 4 % but it grows continually over the forecasted horizon. Furthermore, the innovation of the bubble size in the stock market can cause an immediate negative effect on investment, but later the effect will turn to be positive. He explains that this immediate effect of investment to the shock of the bubble size is probably due to competitions between the group of inventors in the stock market and business sectors in attracting funds from financial market to finance their activities. However, the appreciations of the asst prices increase the wealth of economic agents, which in turn encourages more consumption and aggregate investment activity in the later period.

He also finds out that the stock market bubble has the positive effect on the price level. This result supports the argument that the asset price appreciation in this period has a positive pressure on the future price level.

Finally, he focuses on the effect of the bubble on the aggregate credit. The result shows that an increase in asset price bubbles also affects activities of financial sector by inducing expansion of credits. When the credit is expanded, it will encourage more investment and consumption, which in effect pressure price level to increase.

In the last section of his work, he argues that even though the role of credits in controlling the emergence of the bubbles in asset prices seems to be an effective tool, it also risks inducing a negative impact on the real economy. Similarly, preventing asset price bubbles by increasing the interest rate will also have the negative effect on the real economy. For these reasons, the appropriate action at the micro levels is to strengthen prudential regulation and supervision of financial intermediaries.

After the burst of the tech-stock "bubble" and amidst growing concern on house price bubble in U.S., U.K., and Australia, a debate on the role of the monetary policy to deal with a possibility of asset price bubble has become more intense in the U.S. and Europe. The main question is whether the monetary authorities should only be gunning for low inflation or doing more to temper episodes of asset price boom and bust. In 2003, Ashvin et.al. from bank of Thailand present their study on asset price bubble and monetary policy by exploring ways to identify and deal with a possibility of future financial instability appropriately under the framework that target low and stable inflation.

They firstly provide a set of stylized fact observed in Thailand's data based on three basic aspects of the cyclical behavior of the aggregates:

- The amplitude of fluctuations.
- The degree of comovements with cyclical real SET index.
- The phase shift of variable relative to the cyclical real SET index.

The following table 3.2 presents some stylized facts found for Thailand:

Table 3.2

Some of the Stylized Facts Found for Thailand

The economic asset price issues	Stylized facts
1.Asset price cycles	Increases in asset price are relatively slow and decrease are abrupt.
2.Average magnitude of asset price	The amplitude of the equity price cycles appears to be roughly 9 times as large as the business cycle and 2.5 times as large as that of investment.
3.Asset price and real output and its components	• Thailand's real equity price usually is procyclical with the business cycle (output) and leads it by 1 year. • Private and total investment, consumption, export and import are all pro-cyclical with equity price, each lagging equity price by 1 year.
4.Relationship between asset price and private credit	• Real private credit is procyclical with real equity price, with real equity price leading it by 1-2 years. • Real private credit and output are highly correlated, with the former lagging the latter slightly by about 0-1

The economic asset price issues	Stylized facts
	year.
5.Relationship between asset price and monetary aggregates	Both the monetary base (M_0) and (M_1) are procyclical with real equity price; but neither leads equity price. M_1 and M_2 lag real equity price by roughly 1 and 4 years , respectively. The components of M_2 not in M_1 (time and saving deposits) lag real equity price by 4 years.
6. Relationship among asset price and other asset classes	Peak in equity prices tend to lead those in commercial and real estate prices by 1-2 years. Condomenium and commercial price tend to move together.
7. Equity price and inflation	Both CPI and core CPI are counter-cyclical to real equity price. Both lag real equity price by roughly 3-4 years. The amplitude of the equity price cycles appear to be roughly 10 times larger than cyclical CPI and 20 times larger than of core inflation.

In this study, they also point out the difference among asset classes by emphasizing the characteristics of the two asset classes; (1) equities and (2) property. The following characteristics of the two asset classes should be taken into account when we consider its valuation.

First is liquidity. Equities are liquid and related to a tradable sector but properties are less liquid and related mostly to a non-tradable sector. Second is the financial structure. They argue that the proportions of equity and property in total wealth depended on the financial structure in an economy, whether it is more capital-market based or bank based. For the bank-based such as Thailand, households tend to hold more wealth in property than equity. The next is transparency which is the one characteristic that differ across these assets. By comparison, equity market quit has more information and more good governance than property market. Forth is the credit dependency. Different financing methods to acquire equity and property are observed, while most households and businesses are using their savings to buy equity, they are contrary borrowing money to purchases property. Lastly, tax and subsidies are found to be different in every country and also different across countries.

To identify of asset price bubble, they firstly examine two traditional indicators commonly used for direct identification of equity and house price bubble, namely the price-to-earning ratio (P/E) and the Gordon's formula. For equity price, when they apply the P/E, it averages 19.6, peaking above 25 in 1989 and 1993. It shows that the high P/E ratios during 1993-1995 are associated with rapid credit growth as a result of financial liberalization. Using this historical benchmark, it can be suggested that cases of equity price bubble is witnessed in 1989 and 1993.

To confirm the identified equity price overvaluation using the P/E ratio in 1989 and 1993, a test based on the Gordon'formula is performed. Using this method, they find out that equity price is somewhat overvalued in 1989 and 1993; however, it is undervalued compared with historical averages using the average risk premium during 1980-2002.

For property (house) price, they apply two indicators that are commonly used to gauge whether houses are properly priced, the P/E ratio and the house-price-to-income ratio. The available condominium price and rent data reveal that the price-to-rent ratio increases during 1994-1996, declines in 1997-1998 before becoming more or less constant afterward. While price-to-rent increases, the price-to-per-capita-GDP ratio is constant and price-to-income declines sharply up until 1998 and slightly afterward. However, these time series are too short and not be a representative of the housing market to be used to forming a judgment based on historical benchmark.

Next, they discuss an indirect approach that focuses on the symptoms of bubble or "financial imbalances", rather than directly identifying a bubble per *se.* This approach relies on key financial variables such as asset price, credit-to-GDP ratio, and the real exchange rate. They also test the indirect approach on past episodes of suspected equity price bubble in Thailand and find that it is useful as an early warning method. Specifically, they find that through the use of only *ex ante* data, this method warns of a financial crisis two years before it happens.

Nevertheless, early warning indicators can not be substituted for analysis, so they explore implications of changes in fundamental determinants on asset-price trend and cycles, using a model developed by McGrattan and Prescott (2000, 2002).

In the last section, they address the issue of appropriate policy response to asset price cycle. In summary, similar to Weerapon (2000), they do not recommend using interest rate or credit policy to burst the bubble because it is difficult to be confident about the existence of a bubble and the timing of the burst *ex ante,* and it is almost impossible to calibrate a correct magnitude of policy interest rate movement that will be sufficient to pop the bubble without harming the economy. On the other hand, they emphasize preventive measure such as good corporate governance, a strong regulation and supervisory regime, the improvement and disclosure of information useful for asset pricing to help minimize the size of the bubble and make the economy more resilient to shocks generated in the asset market.

Based on the financial structure in Thailand, developments in the property and the banking sectors have a direct bearing on financial stability. Recent trends in the residential segment of the property market and related bank lending have caused concerns among observers about the risks involved. Nakornthab et.al (2004) therefore examine the interwoven nature of the two sectors and systematically accesses the degree of financial fragility associated with the property market and bank lending in the current rising interest rate environment using a Structural Vector Autoregression (SVAR). The results show evidence of a strong causal link from monetary policy to property prices. The transmission mechanism of this process is as follows: An increase in the policy rate first causes short-term market interest rates to rise which in turn increase the cost of loans and reduce the demand for credit by both developers and consumers. This consequently decreases house buying and planned fixed investment, two components of real output, which in turn contributes to fall in property prices. In their study, they also seek to assess the degree of financial fragility associated with bank lending and the property market in the current environment by calibrating the risks facing banks. The core results of the paper indicate that, as far as the stability of the banking sector is concerned, there is no cause for undue alarm. They find out that all banks have priced in future interest rate increase in their mortgage plans, which should help banks withstand the upturn in the interest rate cycle for a while. Moreover, Thai banks as a group have enough capital and loan-loss reserves to withstand the impact of an isolated 30%-decline in property prices. At individual bank level, they find out that some banks are more vulnerable to interest rate increases or a property market downturn than the others. Banks with low monthly payment plays and heavy exposures to the property sectors are generally more at risk. Although it shows the positive results, they also suggest the authorities to continue to closely monitor developments in the property market and banks' lending practices and to stand ready to adopt appropriate measures when necessary. Finally, they advise banks to find out their own way to protect themselves from fluctuations in the property market. The effective risk management and routine stress testing will help banks optimize risk, return, and shareholder's wealth. Careful analysis of borrowers' risk profiles together with effective internal credit rating systems will help banks withstand future property market fluctuations with relative ease.

CHAPTER 4

THEORETICAL FRAMEWORK

This chapter intends to present 2 models. First model is a continuous time equilibrium model of property price where overconfidence generates disagreements among agents about property fundamental. With short-sale constraint, the land owner has an option to develop a piece of vacant land to be an irreversible building and also has resale option to sell this building to other overconfident agents who have more optimistic beliefs. This resale option has a recursive structure which is the main causes of significant bubble component in property market.

Second model is the model of rational bubbles which is employed to test the existence of rational bubbles. In this model, we consider a sufficient condition for the absence of rational bubbles which is contrast to the earlier studies such as Campbell and shiller (1987), Diba and Grossman (1988), Lim and Phoon(1991) and Craine (1993) that consider only necessary conditions for the absence of rational bubbles.

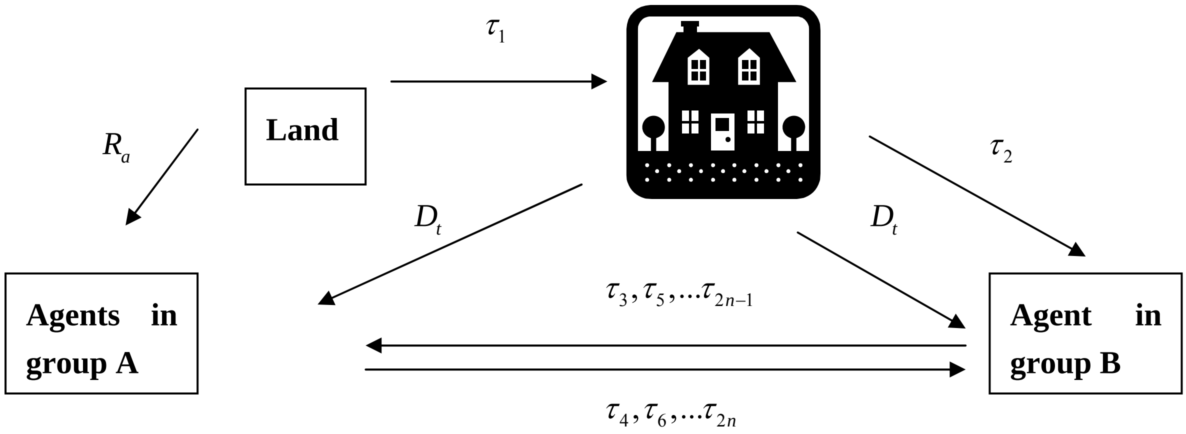
A sufficient condition for the absence of rational bubbles is derived on the assumption that the risk premium and the real interest rate are stationary (Fukuta (1996)).

4.1 The Model of Property Price with Heterogeneous Beliefs

We employ Preechametta (2005) model which bases on the idea of asset pricing function introduced by Lucas (1978) and the idea of speculative bubbles by Scheinkman & Xiong (2003) to study the asset price bubble in property market. This new theoretical framework is developed to find an optimal development time for a piece of vacant land with an option to construct permanent buildings in the near future. The key concept can be explained by figure 4.1.

Figure 4.1

The Concept of Model of Property Price with Heterogeneous Beliefs



From figure 4.1, agents are divided into two groups (group A and group B). Each agent in group A is the owner of a piece of vacant land which can be developed to be permanent building (risky asset) at time τ_1 . Before the transformation of land, agents in group A receive a constant return equal R_a .When land is developed to be building, it can not reverse to be vacant land again. This type of investment is called irreversible investment.

After each agent in group A develops land to be building, he now receives the return equal D_t and decides to resale his building to agent in group B at time τ_2. When each agent in group B becomes to be the building owner, he also gets the returns equal D_t and decides to resell his building turn back to agents in group A. These processes will continue between agents in group A and group B and stimulate bubble in property market.

From the model, we assume the return from building (D_t) is the sum of two components. One of two components is a fundamental variable that determines future dividends or the predicted movement during the infinitesimal time interval dt . The second is "noise" or unpredictable random shock. The cumulative dividend process D_t satisfies equation (4.1):

$$dD_t = f_t dt + \sigma_D Z_t^D \tag{4.1}$$

where Z^D is a standard Brownian Motion[1] and σ_D is the volatility parameter. The stochastic process of fundamentals f that determines future dividends of the asset is unobservable to all agents but it satisfies equation(4.2):

$$df_t = -\lambda(f_t - \bar{f})dt + \sigma_t dZ_t^f \tag{4.2}$$

where $\lambda \geq 0$ is the mean reversion parameter[2], $\bar{f}$ is the long-run mean of f, $\sigma_f > 0$ is a volatility parameter and Z^f is a standard Brownian motion, uncorrelated to Z^D. Equation (4.2) is built in a term that describes the long term behavior of fundamental price to drift back to a long-term level. This process is known as mean reversion process. When $\bar{f}$ is greater than f_t, it will be pulled back toward $\bar{f}$, although random shocks generated by dZ_t^f will delay this process. When $\bar{f}$ is below f_t, it will be pulled up towards $\bar{f}$. The presence of dividends noise makes it impossible to infer f perfectly from observations of the cumulative dividend process.

There are two sets of risk-neutral agents[3], who use the observations of D and any other signals that are correlated with f to infer current f and to value the asset. All agents observe a vector of signals s^A and s^B that satisfy equation(4.3):

$$\begin{aligned} ds_t^A &= f_t dt + \sigma_s dZ_t^A \\ ds_t^B &= f_t dt + \sigma_s dZ_t^B \end{aligned} \tag{4.3}$$

---

[1] More details about Brownian motion see "the Mathematics of Financial Modeling & Investment Management" by Fabozzi, Frank j. and Focardi, Sergio M. "Investment Under Uncertainty" by Pindyck, Robert S. and Dixit, Avinash K.

[2] It is the speed of adjustment.

[3] The assumption of risk neutrality not only simplifies many calculations but also serves to highlight the role of information in the model.

where Z^A and Z^B are standard Brownian motions, and $\sigma_s > 0$ is the common volatility of the signals. We assume that all four processes Z^D, Z^f, Z^A, Z^B are mutually independent.

From equation (4.3) , there are two sets of information available at each instant. These information sets are available to all agents, but agents are divided in two groups which differ on their interpretation of the signals.

This difference is a result of agents' overconfidence, a behavioral bias that has been observed in psychological experiments[4]. The consequence of overconfidence makes agents in group A(B) think of $s^A\left(s^B\right)$ as their own signal, although they can also observe $s^B\left(s^A\right)$. Heterogeneous beliefs arise because each agent believes that the informativeness of his own signal is larger than its true informativeness. Agents in group A(B) believe that innovations $dZ^A(dZ^B)$ in the signal $s^A(s^B)$ are correlated with the innovation dZ^f in the fundamental process, with $\phi(0 \le \phi \le 1)$ as the correlation parameter.

Thus, agents in group A believe that process for s^A [5] is

$$ds_t^A = f_t dt + \sigma_s \phi dZ_t^f + \sigma_s \sqrt{1-\phi^2}\, dZ_t^A . \tag{4.4}$$

---

[4] Psychological studies suggest that people are overconfident. For the extensive reviews of the literature, see Hirshleifer (2001) and Barber and Odean (2002).

[5] Agents in group A believe that innovations dZ^A in the signal s^A are correlated with the innovations dZ^f in the fundamental process, with $\phi(0 < \phi < 1)$ as the correlation parameter. Therefore, the volatility of ds^A come from 2 parts as following:

1. dZ_t^f with the fraction equals ϕ and
2. dZ_t^A with the fraction equal $\sqrt{1-\phi^2}$.

Similarly, agents in group B believe that the process for s^B is

$$ds_t^B = f_t dt + \sigma_s \phi dZ_t^f + \sigma_s \sqrt{1-\phi^2}\, dZ_t^B. \tag{4.5}$$

On the other hand, agents in group A(B) believe (correctly) that innovations to $s^B(s^A)$ are uncorrelated with innovations to $Z^B(Z^A)$ where the joint dynamics of the processes D, f, s^A, s^B in the mind of agents of each group is public information.

The larger ϕ increases the precision that agents attribute to their own forecast of the current level of fundamentals. Agents in both groups are "irrational" in the sense that they do not infer the precision of their signals through the observations of the signals, even though they could do it[6]. For this reason, we can refer to ϕ as the overconfidence parameter.

Since all variables are Gaussian[7], the filtering problem of the agents is standard. The variance[8] of this stationary solution is the same for both groups of agents and equals

$$\gamma \equiv \frac{\sqrt{\left[\lambda + (\phi\sigma_f / \sigma_s\right]^2 + (1-\phi^2)(\left[2\sigma_f^2 / \sigma_s^2\right] + (\sigma_f^2 / \sigma_D^2))} - \left[\lambda + (\phi\sigma_f / \sigma_s\right]}{(1/\sigma_D^2) + (2/\sigma_s^2)}. \tag{4.6}$$

---

[6] This is a behavioral assumption that is well supported by experimental studies.

[7] Gaussian random variables are extremely important in probability theory and statistics. Their importance stems from the fact that any phenomena made up of a large number of independent or weakly dependent variables has a Gaussian distribution. Gaussian distributions are also known as normal distribution. See Fabozzi and Focardi (2004) p.194.

[8] Section VI.9 in Rogers and Williams (1987) and Theorem 12.7 in Liptser and Shiryayev (1977), can be used to compute the variance of the stationary solution and the evolution of the conditional mean of beliefs.q

From equation (4.6), It can verify that the stationary variance γ decreases with ϕ[9]. When $\phi > 0$, agents have an exaggerate view of the precision of their estimates of f . A larger ϕ leads to more overstatement of this precision. ϕ is therefore the "overconfidence" parameter.

In addition, the conditional mean of the beliefs of agents in group A satisfies

$$d\hat{f}_t^A = -\lambda\left(\hat{f}_t^A - \overline{f}\right)dt + \frac{\phi\sigma_s\sigma_f + \gamma}{\sigma_s^2}\left(ds_t^A - \hat{f}_t^A dt\right) + \frac{\gamma}{\sigma_s^2}\left(ds_t^B - \hat{f}_t^A dt\right) + \frac{\gamma}{\sigma_D^2}\left(dD_t - \hat{f}_t^A dt\right). \qquad (4.7)$$

Since f mean-reverts, the conditional beliefs also mean-revert. The other three terms represent the effect of "surprises". These surprises can be represented as standard mutually independent Brownian motions for agents in group A where

$$dW_t^{A,A} = \frac{1}{\sigma_s}(ds_t^A - \hat{f}_t^A dt), \qquad (4.8)$$

$$dW_t^{A,B} = \frac{1}{\sigma_s}(ds_t^B - \hat{f}_t^A dt), \qquad (4.9)$$

$$dW^{A,D} = \frac{1}{\sigma_D}\left(dD_t - \hat{f}_t^A dt\right). \qquad (4.10)$$

Note that these processes are only Wiener processes in the mind of group A agents. Due to overconfidence (ϕ>0), agents in group A over-react to surprises in s^A. Similarly, the conditional mean of the beliefs of agents in group B satisfies

[9] See poof of equation 4.6 in appendix A.

$$d\hat{f}_t^B = -\lambda(\hat{f}_t^B - \overline{f})dt + \frac{\gamma}{\sigma_s^2}(ds_t^A - \hat{f}_t^B dt) + \frac{\phi\sigma_s\sigma_f + \gamma}{\sigma_s^2}(ds_t^B - \hat{f}_t^B dt) + \frac{\gamma}{\sigma_D^2}(dD_t - \hat{f}_t^B dt) \tag{4.11}$$

and the surprise terms can be represented as mutually independent Wiener processes as

$$dW_t^{B,A} = \frac{1}{\sigma_s}(ds_t^A - \hat{f}_t^B dt), \tag{4.12}$$

$$dW_t^{B,B} = \frac{1}{\sigma_s}(ds_t^B - \hat{f}_t^B dt), \tag{4.13}$$

$$dW^{B,D} = \frac{1}{\sigma_D}\left(dD_t - \hat{f}_t^B dt\right). \tag{4.14}$$

These processes form a standard 3-d Brownian only for agents in group B. Let g_A and g_B denote the differences in beliefs where

$$\begin{aligned} g^A &= \hat{f}^B - \hat{f}^A, \\ g^B &= \hat{f}^A - \hat{f}^B. \end{aligned} \tag{4.15}$$

We can apply proposition 1 (Scheinkman and Xiong (2003))[10] to find the change in the difference in beliefs by following equation:

$$dg_t^A = -\rho g_t^A dt + \sigma_g dW_t^{A,g}, \tag{4.16}$$

where

[10] See proof of equation 4.16 in appendix A.

$$\rho = \sqrt{\left(\lambda + \phi\frac{\sigma_f}{\sigma_s}\right)^2 + (1-\phi^2)\sigma_f^2\left(\frac{2}{\sigma_s^2} + \frac{1}{\sigma_D^2}\right)}, \tag{4.17}$$

$$\sigma_g = \sqrt{2}\phi\sigma_f\,, \tag{4.18}$$

and $W^{A,g}$ is a standard Wiener process for agents in group A. Equation 4.16 implies that the difference in beliefs g^A follows a simple mean reverting diffusion process in the mind of group A agents. If the volatility of the difference in beliefs is zero, the overconfidence will absent. In particular, a larger ϕ leads to greater volatility and causes an increase in fluctuations of opinions and a slower mean-reversion.

In an analogous fashion, for agents in group B, g^B satisfies

$$dg_t^B = -\rho g_t^B dt + \sigma_g dW_t^{B,g}\,, \tag{4.19}$$

where $W^{B,g}$ is a standard Wiener process.

Equation (4.16) and (4.19) state that, in each group's mind, the difference of opinions follows a mean-reverting diffusion process. The coefficients of this process are linked to the parameters describing the original uncertainty and the degree of overconfidence.

4.1.1 Land Price

It is similar to the Harrison and Kreps model described in chapter 3, we assume that each group of agents is large and there is no short selling[11] of the risky asset. To value future cash flows, we also assume that every agent can borrow and lend at the same rate of interest r.

At each t, agents in group $C = \{A, B\}$ are willing to pay p_t^C for a unit of the

[11] See the effect of short sale constraint in the asset pricing in chapter 3.

asset. As in the Harrison and Kreps model, the presence of the short-sale constraint, a finite supply of the asset (land), and an infinite number of prospective buyers guarantee that any successful bidder will pay his reservation price. The amount that an agent is willing to pay reflects the agent's fundamental valuation and the fact that he may be able to sell his holdings at a later date at the demand price of agents in the other group for a profit.

In Preechametta (2005), when deciding the value of the asset, agents consider their own view of the fundamental as well as the fact that the owner of the asset has an option to sell the asset in the future to the agent in the other group. If $o \in \{A, B\}$ denotes the group of the current owner, $\bar{o}$ the other group, and E_t^o the expectation of members of group o, conditional on the information they have at t, then

$$P_t^{L,o} = \sup_{\tau_1 \geq 0} \left[\int_t^{t+\tau_1} e^{-r(s-t)} R_a ds + e^{-r\tau_1} \left\{ \sup_{\tau_2 \geq 0} E_{t+\tau_1}^o \left[\left\{ \int_{t+\tau_1}^{t+\tau_1+\tau_2} e^{-r(s-(t+\tau_1))} dD_s - c_1 \right\} + e^{-r\tau_2} (P_{t+\tau_1+\tau_2}^{h,\tilde{o}} - c_2) \right] \right\} \right], \tag{4.20}$$

where τ_1 is an optimal stopping time which land owner decides to develop land to be building,

τ_2 is an optimal stopping time to sell building to agent group $\tilde{o}$,

$P_{t+\tau_1+\tau_2}^{h,\tilde{o}}$ is the reservation value of the buyer at the time $t+\tau_1+\tau_2$,

c_1, c_2 are building and resale costs, respectively, and

r is the constant discount rate.

The equation (4.20) shows that the reservation value of land at time t is composed of three expected values. First is the present value of constant returns from vacant land. Second is the expected present value of returns from building by assuming that land is developed to be the building at the time $t+\tau_1$ where $\tau_1 \geq 0$ and the building cost is c_1. Last is the net expected value of building resale option at time $t+\tau_1+\tau_2$.

After agents in group $\tilde{o} \in \{A, B\}$ bought the building from agents in group

$o \in \{A, B\}$ at time $t+\tau_1+\tau_2$, they will hold and resell it again at the time τ_3. The reservation value of the building from the agents in group $\tilde{o}$ view point can be rewritten in the following equation:

$$P_{t+\tau_1+\tau_2}^{h,\tilde{o}} = \sup_{\tau_3} E_{t+\tau_1+\tau_2}^{\tilde{o}} \left[\left\{ \int_{t+\tau_1+\tau_2}^{t+\tau_1+\tau_2+\tau_3} e^{-r(s-(t+\tau_1+\tau_2))} dD_s \right\} + e^{-r\tau_3} \left(P_{t+\tau_1+\tau_2+\tau_3}^{h,o} - c_2 \right) \right]. \quad (4.21)$$

Using the equations (4.1), (4.7), and (4.11) for the evolution of dividends and for the conditional mean of beliefs, the present value of total rent from the building from period $t+\tau_1$ to $t+\tau_1+\tau_2$ can be presented in the following form[12]

$$\int_{t+\tau_1}^{t+\tau_1+\tau_2} e^{-r(s-(t+\tau_1))} dD_s = \int_{t+\tau_1}^{t+\tau_1+\tau_2} e^{-r(s-(t+\tau_1))} \left[\overline{f} + e^{-\lambda(s-(t+\tau_1))} \left(\hat{f}_{t+\tau_1}^{o} - \overline{f} \right) \right] ds + M_{t+\tau_1+\tau_2}$$

(4.22)

where $E_{t+\tau_1}^{o} M_{t+\tau_1+\tau_2} = 0$.

Equation (4.20) can be rewritten by using equations (4.21) and (4.22). Thus,

$$\begin{aligned} P_t^{L,o} = \sup_{\tau_1 \geq 0} \Bigg[\int_t^{t+\tau_1} e^{-r(s-t)} R_a ds + e^{-r\tau_1} \Bigg\{ \sup_{\tau_2 \geq 0} E_{t+\tau_1}^{o} \Bigg[\Bigg\{ \int_{t+\tau_1}^{t+\tau_1+\tau_2} e^{-r(s-(t+\tau_1))} \left[\overline{f} + e^{-\lambda(s-(t+\tau_1))} \left(\hat{f}_{t+\tau_1}^{o} - \overline{f} \right) \right] ds - c_1 \Bigg\} \\ + e^{-r\tau_2} (P_{t+\tau_1+\tau_2}^{h,\tilde{o}} - c_2) \Bigg] \Bigg\} \Bigg], \end{aligned}$$

(4.23)

where

[12] See Fwu-Ranq Chang (2004) p.89.

$$P^{h,\tilde{o}}_{t+\tau_1+\tau_2} = \sup_{\tau_3} E^{\tilde{o}}_{t+\tau_1+\tau_2}\left[\left\{\int_{t+\tau_1+\tau_2}^{t+\tau_1+\tau_2+\tau_3} e^{-r(s-(t+\tau_1+\tau_2))}\left[\overline{f}+e^{-\lambda(s-(t+\tau_1+\tau_2))}\left(\hat{f}^{\tilde{o}}_{t+\tau_1+\tau_2}-\overline{f}\right)\right]ds\right\}\right.$$
$$\left. +e^{-r\tau_3}\left(P^{h,o}_{t+\tau_1+\tau_2+\tau_3}-c_2\right)\right].$$

(4.24)

If we assume that $E^o_t\left(\int_t^\infty e^{-r(s-t)}R_a ds\right)=\dfrac{R_a}{r}$, we can rewrite equation (4.23) as

$$P^{L,o}_t = \sup_{\tau_1\geq 0}\left[\int_t^{t+\tau_1} e^{-r(s-t)}R_a ds + e^{-r\tau_1}\left\{\sup_{\tau_2\geq 0} E^o_{t+\tau_1}\left[\left\{\int_{t+\tau_1}^{t+\tau_1+\tau_2} e^{-r(s-(t+\tau_1))}\left[\overline{f}+e^{-\lambda(s-(t+\tau_1))}\left(\hat{f}^o_{t+\tau_1}-\overline{f}\right)\right]ds - c_1\right\}\right.\right.\right.$$
$$\left.\left.\left. +e^{-r\tau_2}(P^{h,\tilde{o}}_{t+\tau_1+\tau_2}-c_2)\right]\right\}\right] + E^o_t\left[\int_{t+\tau_1}^{t+\tau_1+\tau_2} e^{-r(s-(t+\tau_1))}R_a ds + \int_{t+\tau_1+\tau_2}^{t+\tau_1+\tau_2+\tau_3} e^{-r(s-(t+\tau_1+\tau_2))}R_a ds + ...\right]$$
$$-E^o_t\left[\int_{t+\tau_1}^{t+\tau_1+\tau_2} e^{-r(s-(t+\tau_1))}R_a ds + \int_{t+\tau_1+\tau_2}^{t+\tau_1+\tau_2+\tau_3} e^{-r(s-(t+\tau_1+\tau_2))}R_a ds + ...\right],$$

or

$$P^{L,o}_t = \frac{R_a}{r} + \sup_{\tau_1\geq 0} E^o_t\left[e^{-r\tau_1}\left\{\sup_{\tau_2\geq 0} E^o_{t+\tau_1}\left[\left\{\int_{t+\tau_1}^{t+\tau_1+\tau_2} e^{-r(s-(t+\tau_1))}\left[\overline{f}+e^{-\lambda(s-(t+\tau_1))}\left(\hat{f}^o_{t+\tau_1}-\overline{f}\right)-R_a\right]ds - c_1\right\}\right.\right.\right.$$
$$\left.\left.\left. +e^{-r\tau_2}(P^{h,\tilde{o}}_{t+\tau_1+\tau_2}-c_2)\right]\right\}\right].$$

(4.25)

Equation (4.25) shows that the total present value of land at time t composes of two components as in following equation:

$$P^{L,o}_t = \frac{R_a}{r} + P^{h,o}\left(\hat{f}^o_t, g^o_t\right), \tag{4.26}$$

where

$$P_t^{h,o} = \sup_{\tau_1 \ge 0} E_t^o \left[e^{-r\tau_1} \left\{ \sup_{\tau_2 \ge 0} E_{t+\tau_1}^o \left[\left\{ \int_{t+\tau_1}^{t+\tau_1+\tau_2} e^{-r(s-(t+\tau_1))} \left[\overline{f} + e^{-\lambda(s-(t+\tau_1))} \left(\hat{f}_{t+\tau_1}^o - \overline{f} \right) - R_a \right] ds - c_1 \right\} \right. \right. \right.$$
$$\left. \left. \left. + e^{-r\tau_2} (P_{t+\tau_1+\tau_2}^{h,\tilde{o}} - c_2) \right] \right\} \right].$$

(4.27)

Therefore, the equilibrium of land price depends on the equilibrium of building price as shown in equation (4.27). In order to identify the optimal stopping time to develop land to be building, the equation (4.27) can be rewritten by using equation (4.24) as

$$P_t^{h,o} = \sup_{\tau_1 \ge 0} E_t^o \left[\left\{ \sup_{\tau_2 \ge 0} E_{t+\tau_1}^o \left[e^{-r\tau_1} \left\{ \int_{t+\tau_1}^{t+\tau_1+\tau_2} e^{-r(s-(t+\tau_1))} \left[\overline{f} + e^{-\lambda(s-(t+\tau_1))} \left(\hat{f}_{t+\tau_1}^o - \overline{f} \right) - R_a \right] ds - c_1 \right\} \right. \right. \right.$$
$$+ e^{-r(\tau_1+\tau_2)} \left(\sup_{\tau_3} E_{t+\tau_1+\tau_2}^{\tilde{o}} \left[\left\{ \int_{t+\tau_1+\tau_2}^{t+\tau_1+\tau_2+\tau_3} e^{-r(s-(t+\tau_1+\tau_2))} \left[\overline{f} + e^{-\lambda(s-(t+\tau_1+\tau_2))} \left(\hat{f}_{t+\tau_1+\tau_2}^{\tilde{o}} - \overline{f} \right) - R_a \right] ds \right\} \right. \right.$$
$$\left. \left. \left. \left. \left. + e^{-r\tau_3} \left(P_{t+\tau_1+\tau_2+\tau_3}^{h,o} - c_2 \right) \right] - c_2 \right) \right] \right\} \right].$$

(4.28)

From equation (4.28), we can simplify it to be equation 4.29 by using the iterating process. Thus,

$$P^{h,o}\left(\hat{f}_t^o, g_t^o \right) = \sup_{\tau_1 \ge 0} E_t^o \left[\left[\left\{ \frac{\overline{f} - R_a}{r} + \frac{\hat{f}_{t+\tau_1}^o - \overline{f}}{r+\lambda} - c_1 \right\} e^{-r\tau_1} + \sup_{\tau_2 \ge 0} E_{t+\tau_1}^o \left[\left\{ \frac{g_{t+\tau_1+\tau_2}^o}{r+\lambda} \right\} + q^h \left(g_{t+\tau_1+\tau_2}^{\tilde{o}} \right) - c_2 \right] e^{-r(\tau_1+\tau_2)} \right] \right]$$

(4.29)

where

$$g_t^o = f_t^{\tilde{o}} - f_t^o, g_t^{\tilde{o}} = f_t^o - f_t^{\tilde{o}}, \tag{4.30}$$

$$q^h(g_t^o) = \sup_{\tau_2 \geq 0} E_{t+\tau_1}^o \left[\left(\frac{g_{t+\tau_1+\tau_2}^o}{r+\lambda} \right) + q^h\left(g_{t+\tau_1+\tau_2}^{\tilde{o}} \right) - c_2 \right] e^{-r(\tau_1+\tau_2)}, \tag{4.31}$$

where $q^h(g_t^o)$ in equation (4.31) is the value of resale option. Therefore, building option is the function of resale option which is called "compound option". $q^h(g_t^o)$ is the value of resale option which depends on the current difference between the beliefs of the other group's agents and the belief of the current owner.

Then, equation (4.25) can be rewritten as equation(4.32):

$$P_t^{L,o} = \frac{R_a}{r} + P^{h,o}\left(\hat{f}_t^o, g_t^o \right). \tag{4.32}$$

From equation (4.32), the value of building option is the function of two variables (1) $\hat{f}_t^o$, and (2) g_t^o where both of these variables are the mean-reverting processes.

4.1.2. The Valuation of Resale Option

As Preechametta (2005) states that the resale option is the implicit function of the building option, it is known as "compound option". In general, the value of resale option can be divided into two cases.

4.1.2.1 Case1: the Value of Resale Option When It Is Not the Optimal Time to Develop Land to Be Building

In this case, we can specify equation (4.29) such that $\hat{f}_t^o$ and g_t^o are in the continuation region $\mathbb{C}$. Therefore, we will yield the following condition:

$$P^{h,o}\left(\hat{f}_t^o, g_t^o \right) > \left[\left[\frac{\overline{f} - R_a}{r} + \frac{\hat{f}_t^o - \overline{f}}{r+\lambda} - c_1 \right] + q^h\left(g_t^o \middle| \left(\hat{f}_t^o, g_t^o \right) \in \mathbb{C} \right) \right]. \tag{4.33}$$

We can apply the study of Scheinkman and Xiong (2003)[13] to equation (4.33). In their study, they show the explicit solution for the following conditions:

$$q^h\left(x\middle|\left(\hat{f}_t^{o},x\right)\in\mathbb{C}\right)>\left[\frac{x}{r-\lambda}+q^h\left(-x\middle|\left(\hat{f}_t^{\tilde{o}},-x\right)\in\mathbb{S}\right)-c_2\right], \tag{4.34}$$

and

$$\frac{1}{2}\sigma_g^2 q^{h''}-\rho x q^{h'}-rq^h=0 \tag{4.35}$$

where $\mathbb{C}$ is the continuation region, $\mathbb{S}$ is the stopping region, and x is represented the difference in belief which is defined as $\hat{f}^B-\hat{f}^A$.

When they combine equation (4.34) and (4.35), it will yield the explicit solution[14] which can be represented by following equation:

$$q^h\left(x\middle|\left(\hat{f}_t^{o},x\right)\in\mathbb{C}\right)=\begin{cases}\beta_1(\hat{f}_t^{o}).h(x) & \mathit{for}\ \left(\hat{f}_t^{o},x\right)\in\mathbb{C}\\ \\ q^h\left(x\middle|\left(\hat{f}_t^{o},x\right)\in\mathbb{S}\right) & \mathit{for}\ \left(\hat{f}_t^{o},x\right)\in \mathit{boundary\ of}\ \mathbb{S}\end{cases}, \tag{4.36}$$

where

$$\beta_1(\hat{f}_t^{o})=\frac{1}{\left[h'\left(\bar{k}\left(\hat{f}_t^{o}\right)\right)+h'\left(-\bar{k}\left(\hat{f}_t^{o}\right)\right)\right]\left(r+\lambda\right)}, \tag{4.37}$$

and function $h(x)$ is equal to

---

[13] See Scheinkman and Xiong (2003) p.1195 equation (14) and (15).

[14] See proof of equation 4.36 in appendix A.

$$h(x)=\begin{cases} U\left(\dfrac{r}{2\rho},\dfrac{1}{2},\dfrac{\rho}{\sigma_g^2}\right) & if \quad x\le 0 \\ \dfrac{2\pi}{\Gamma\left(\dfrac{1}{2}+\left(\dfrac{r}{2\rho}\right)\right)\Gamma\left(\dfrac{1}{2}\right)}M\left(\dfrac{r}{2\rho},\dfrac{1}{2},\dfrac{\rho}{\sigma_g^2}x^2\right)-U\left(\dfrac{r}{2\rho},\dfrac{1}{2},\dfrac{\rho}{\sigma_g^2}\right) & if \quad x>0 \end{cases}$$

(4.38)

4.1.2.2 Case2: the Value of Resale Option When It Is the Optimal Time to Develop Land to Be Building

Now let's consider the value of resale option when it is the optimal time to develop land to be building. In this case, we also employ the explicit solution from the study of Scheinkman and Xiong (2003). Thus,

$$q^h\left(x\middle|\left(\hat{f}_t^o,x\right)\in\mathbb{S}\right)=\begin{cases} \dfrac{b}{h(-k^*)}h(x) & for \ \ (\hat{f}_t^o,x)\in\mathbb{S}, and \ \ x<k^* \\ \dfrac{x}{r+\lambda}+\dfrac{b}{h(-k^*)}h(-x)-c_2 & for(\hat{f}_t^o,x)\in\mathbb{S}, and \ \ x>k^* \end{cases},$$

(4.39)

where

$$b\equiv q(-k^*)=\frac{1}{r+\lambda}\frac{h(-k^*)}{h'(k^*)+h'(-k^*)}, \tag{4.40}$$

and k^* in equation (4.39) must satisfy

$$\left[k^*-c\left(r+\lambda\right)\right]\left[h'(k^*)+h'(-k^*)\right]-h(k^*)+h(-k^*)=0 \ . \tag{4.41}$$

From equation (4.41) , for each trading cost $c>0$, there exists a unique k^* that solves this equation. If $c=0$, then $k^*=0$. If $c>0$, $k^*>c(r+\lambda)$.

Equation (4.39) is represented the value of resale option which is divided into two cases: (1) It is not the optimal time to resell building $\left(g_t^o<k^*\right)$ and (2) It is the optimal time to resell building$\left(g_t^o\geq k^*\right)$.

4.1.3 The Valuation of Building Option

At any time t, building option, $P^{h,o}(\hat{f}_t^o,g_t^o)$, should be at least as large as the immediate gain from developing land to be building at time t. In other words, it has the relation as the following equation:

$$P^{h,o}\left(\hat{f}_t^o,g_t^o\right)\geq\left[\left\{\frac{\overline{f}-R_a}{r}+\frac{\hat{f}_t^o-\overline{f}}{r+\lambda}-c_1\right\}+q^h\left(g_t^o\middle|\left(\hat{f}_t^o,g_t^o\right)\in\mathbb{C}\right)\right]. \qquad (4.42)$$

Using the mathematical method, we can divide equation (4.42) into two regions. The first region is called "stopping region" which is the region that the value of developing land to be building is equal to the value of immediate gain from building. Complement of stopping region is called "continuation region".

Therefore, equation (4.42) satisfies three following conditions as illustrated in proposition1:

Proposition 1

1. The value of $P^{h,o}\left(\hat{f}_t^o,g_t^o\right)$ will be continuous value in $\mathbb{R}^+\times\mathbb{R}^+$,
2. $P^{h,o}\left(\bullet,g_t^o\right),P^{h,o}\left(\hat{f}_t^o,\bullet\right)$ are nondecreasing on $\mathbb{R}^+$ for all $\hat{f}_t^o$ and g_t^o in $\mathbb{R}^+$,
3. $P^{h,o}\left(\bullet,\bullet\right)$ is convex in $\mathbb{R}^+\times\mathbb{R}^+$.

Expanding $P^{h,o}\left(\hat{f}_t^{o}, g_t^{o}\right)$ by using Ito's lemma ,we will yield the partial differential equation which is presented the proposition 2 (Variation inequality characterization for American option):

Proposition 2[15]

1. The values of $\dfrac{\partial P^{h,o}\left(\hat{f}_t^{o}, g_t^{o}\right)}{\partial \hat{f}_t^{o}}$ and $\dfrac{\partial P^{h,o}\left(\hat{f}_t^{o}, g_t^{o}\right)}{\partial g_t^{o}}$ are uniformly bounded , and
2. $\dfrac{\partial^2 P^{h,o}\left(\hat{f}_t^{o}, g_t^{o}\right)}{\partial \hat{f}_t^{o}\partial g_t^{o}}$ is locally bounded on $\mathbb{R}^+ \times \mathbb{R}^+$.

If we define $L\left(P^{h,o}\left(\hat{f}_t^{o}, g_t^{o}\right)\right)$ is equal to

$$
\begin{aligned}
L\left(P^{h,o}\left(\hat{f}_t^{o}, g_t^{o}\right)\right) = \Bigg[& \left[r-\left(\delta_f - \frac{\lambda\left(\bar{f}-\hat{f}_t^{o}\right)}{\hat{f}_t^{o}}\right)\right]\hat{f}_t^{o}\,\frac{\partial P^{h,o}\left(\hat{f}_t^{o}, g_t^{o}\right)}{\partial \hat{f}_t^{o}} \\
& +\left(r-\left[\delta_g-\rho\right]\right) g_t^{o}\,\frac{\partial P^{h,o}\left(\hat{f}_t^{o}, g_t^{o}\right)}{\partial g_t^{o}} \\
& +\left(\frac{1}{2}\right)\left[\left[\left[\frac{\phi\sigma_s\sigma_f}{\sigma_s}\right]^2+\left[\frac{\gamma}{\sigma_s}\right]^2+\left[\frac{\gamma}{\sigma_D}\right]^2\right]\frac{\partial^2 P^{h,o}\left(\hat{f}_t^{o}, g_t^{o}\right)}{\left(\partial \hat{f}_t^{o}\right)^2}\right. \\
& +2\rho_{\hat{f},g}\left(\left[\frac{\phi\sigma_s\sigma_f}{\sigma_s}\right]+\left[\frac{\gamma}{\sigma_s}\right]+\left[\frac{\gamma}{\sigma_D}\right]\right)\sigma_g g_t^{o}\,\frac{\partial^2 P^{h,o}\left(\hat{f}_t^{o}, g_t^{o}\right)}{\partial \hat{f}_t^{o}\partial g_t^{o}} \\
& \left. +\sigma_g^2\left(g_t^{o}\right)^2\frac{\partial^2 P^{h,o}\left(\hat{f}_t^{o}, g_t^{o}\right)}{\left(\partial g_t^{o}\right)^2}\right]
\end{aligned}
\qquad (4.43)
$$

[15] See Proposition 4 and 5 in Broadie Mark and Detemple (1994) for the value of an American option on the maximum of two assets at time t with asset prices.

where δ_f is the risk-adjusted discount rate for $\hat{f}_t^o$, δ_g is the risk-adjusted discount rate for g_t^o, and $\rho_{\hat{f}g}$ is the correlation coefficient between $\hat{f}_t^o$ and g_t^o. Therefore, the value of $P^{h,o}\left(\hat{f}_t^o, g_t^o\right)$ satisfies the following conditions:

1. $P^{h,o}\left(\hat{f}_t^o, g_t^o\right) \geq \left[\left\{\dfrac{\bar{f}-R_a}{r}+\dfrac{\hat{f}_t^o-\bar{f}}{r+\lambda}-c_1\right\}+q^h\left(g_t^o \middle| \left(\hat{f}_t^o, g_t^o\right)\in\mathbb{C}\right)\right]$,

2. $\dfrac{\partial P^{h,o}\left(\hat{f}_t^o, g_t^o\right)}{\partial t}+L\left(P^{h,o}\left(\hat{f}_t^o, g_t^o\right)\right)\leq 0$, and

3. $\left[L\left(P^{h,o}\left(\hat{f}_t^o, g_t^o\right)\right)\right]\left[P^{h,o}\left(\hat{f}_t^o, g_t^o\right)-\left\{\dfrac{\bar{f}-R_a}{r}+\dfrac{\hat{f}_t^o-\bar{f}}{r+\lambda}-c_1\right\}+q^h\left(g_t^o \middle| \left(\hat{f}_t^o, g_t^o\right)\in\mathbb{C}\right)\right]=0$

almost everywhere on $\left[0,T\right]\times\mathbb{R}^+\times\mathbb{R}^+$.

From proposition 2, we can divide the continuation and stopping regions for $P^{h,o}\left(\hat{f}_t^o, g_t^o\right)$ into 2 cases where:

Case 1: It is not the optimal time to develop land to be building ($\hat{f}_t^o, g_t^o$ are in the continuation region). Then, it yields

$$P^{h,o}\left(\hat{f}_t^o, g_t^o\right) \geq \left[\left\{\frac{\bar{f}-R_a}{r}+\frac{\hat{f}_t^o-\bar{f}}{r+\lambda}-c_1\right\}+q^h\left(g_t^o \middle| \left(\hat{f}_t^o, g_t^o\right)\in\mathbb{C}\right)\right], \tag{4.44}$$

$$q^h\left(x \middle| \left(\hat{f}_t^o, x\right)\in\mathbb{C}\right)=\begin{cases}\beta_1(\hat{f}_t^o).h(x) & \text{for } \left(\hat{f}_t^o, x\right)\in\mathbb{C}\\ \left[\dfrac{x}{r-\lambda}+q^h\left(-x \middle| \left(\hat{f}_t^{\tilde{o}}, -x\right)\in\mathbb{S}\right)-c_2\right] & \text{for } \left(\hat{f}_t^o, x\right)\in \text{boundary of } \mathbb{S}\end{cases}$$

(4.45)

, and

$$\begin{aligned}\frac{\partial P^{h,o}\left(\hat{f}_t^o, g_t^o\right)}{\partial t} + L\left(P^{h,o}\left(\hat{f}_t^o, g_t^o\right)\right) = \frac{\partial P^{h,o}\left(\hat{f}_t^o, g_t^o\right)}{\partial t} + &\left[r - \left(\delta_f - \frac{\lambda\left(\overline{f} - \hat{f}_t^o\right)}{\hat{f}_t^o} \right) \right] \hat{f}_t^o \frac{\partial P^{h,o}\left(\hat{f}_t^o, g_t^o\right)}{\partial \hat{f}_t^o} \\ &+ \left(r - \left[\delta_g - \rho \right] \right) g_t^o \frac{\partial P^{h,o}\left(\hat{f}_t^o, g_t^o\right)}{\partial g_t^o} \\ &+ \left(\frac{1}{2}\right) \left[\left[\left[\frac{\phi \sigma_s \sigma_f}{\sigma_s} \right]^2 + \left[\frac{\gamma}{\sigma_s} \right]^2 + \left[\frac{\gamma}{\sigma_D} \right]^2 \right] \frac{\partial^2 P^{h,o}\left(\hat{f}_t^o, g_t^o\right)}{\left(\partial \hat{f}_t^o\right)^2} \right. \\ &+ 2\rho_{\hat{f},g} \left(\left[\frac{\phi \sigma_s \sigma_f}{\sigma_s} \right] + \left[\frac{\gamma}{\sigma_s} \right] + \left[\frac{\gamma}{\sigma_D} \right] \right) \sigma_g g_t^o \frac{\partial^2 P^{h,o}\left(\hat{f}_t^o, g_t^o\right)}{\partial \hat{f}_t^o \partial g_t^o} \\ &\left. + \sigma_g^2 \left(g_t^o\right)^2 \frac{\partial^2 P^{h,o}\left(\hat{f}_t^o, g_t^o\right)}{\left(\partial g_t^o\right)^2} \right] = 0. \end{aligned} \tag{4.46}$$

Because the distribution of g_t^o is orthogonal with the distribution of $\hat{f}^o$, therefore , $\rho_{\hat{f}g} = 0$. Equation (4.46) can be rewritten as

$$\begin{aligned}\frac{\partial P^{h,o}\left(\hat{f}_t^o, g_t^o\right)}{\partial t} + L\left(P^{h,o}\left(\hat{f}_t^o, g_t^o\right)\right) = \frac{\partial P^{h,o}\left(\hat{f}_t^o, g_t^o\right)}{\partial t} + &\left[r - \left(\delta_f - \frac{\lambda\left(\overline{f} - \hat{f}_t^o\right)}{\hat{f}_t^o} \right) \right] \hat{f}_t^o \frac{\partial P^{h,o}\left(\hat{f}_t^o, g_t^o\right)}{\partial \hat{f}_t^o} \\ &+ \left(r - \left[\delta_g - \rho \right] \right) g_t^o \frac{\partial P^{h,o}\left(\hat{f}_t^o, g_t^o\right)}{\partial g_t^o} \\ &+ \left(\frac{1}{2}\right) \left[\left[\left[\frac{\phi \sigma_s \sigma_f}{\sigma_s} \right]^2 + \left[\frac{\gamma}{\sigma_s} \right]^2 + \left[\frac{\gamma}{\sigma_D} \right]^2 \right] \frac{\partial^2 P^{h,o}\left(\hat{f}_t^o, g_t^o\right)}{\left(\partial \hat{f}_t^o\right)^2} \right. \\ &\left. + \sigma_g^2 \left(g_t^o\right)^2 \frac{\partial^2 P^{h,o}\left(\hat{f}_t^o, g_t^o\right)}{\left(\partial g_t^o\right)^2} \right] = 0. \end{aligned} \tag{4.47}$$

Case 2: It is the optimal time to develop land to be building ($\hat{f}_t^o$ and g_t^o are in the stopping region). We will yield two sub-cases:

- It is the optimal time to develop land to be building but not to resell. Then, we yield

$$P^{h,o}\left(\hat{f}_t^o, g_t^o\right) = \left[\left\{\frac{\overline{f} - R_a}{r} + \frac{\hat{f}_t^o - \overline{f}}{r+\lambda} - c_1\right\} + q^h\left(g_t^o \middle| \left(\hat{f}_t^o, g_t^o\right) \in \mathbb{S}\right)\right], \tag{4.48}$$

$$q^h\left(x \middle| \left(\hat{f}_t^o, x\right) \in \mathbb{S}\right) = \frac{b}{h(-k^*)} h(x), \qquad for\left(\hat{f}_t^o, x\right) \in \mathbb{S}, and\, x < k^* \tag{4.49}$$

, and

$$\frac{\partial P^{h,o}\left(\hat{f}_t^o, g_t^o\right)}{\partial t} + L\left(P^{h,o}\left(\hat{f}_t^o, g_t^o\right)\right) = \frac{\partial P^{h,o}\left(\hat{f}_t^o, g_t^o\right)}{\partial t} + \left[r - \left(\delta_f - \frac{\lambda\left(\overline{f} - \hat{f}_t^o\right)}{\hat{f}_t^o}\right)\right] \hat{f}_t^o \frac{\partial P^{h,o}\left(\hat{f}_t^o, g_t^o\right)}{\partial \hat{f}_t^o}$$
$$+ \left(r - \left[\delta_g - \rho\right]\right) g_t^o \frac{\partial P^{h,o}\left(\hat{f}_t^o, g_t^o\right)}{\partial g_t^o}$$
$$+ \left(\frac{1}{2}\right)\left[\left[\left[\frac{\phi\sigma_s\sigma_f}{\sigma_s}\right]^2 + \left[\frac{\gamma}{\sigma_s}\right]^2 + \left[\frac{\gamma}{\sigma_D}\right]^2\right] \frac{\partial^2 P^{h,o}\left(\hat{f}_t^o, g_t^o\right)}{\left(\partial \hat{f}_t^o\right)^2}\right.$$
$$\left. + \sigma_g^2 \left(g_t^o\right)^2 \frac{\partial^2 P^{h,o}\left(\hat{f}_t^o, g_t^o\right)}{\left(\partial g_t^o\right)^2}\right] \le 0. \tag{4.50}$$

- It is the optimal time to develop land to be building and to resell. Then, we yield

$$P^{h,o}\left(\hat{f}_t^o, g_t^o\right) = \left[\left\{\frac{\overline{f} - R_a}{r} + \frac{\hat{f}_t^o - \overline{f}}{r+\lambda} - c_1\right\} + q^h\left(g_t^o \middle| \left(\hat{f}_t^o, g_t^o\right) \in \mathbb{S}\right)\right], \tag{4.51}$$

$$q^h\left(x \middle| \left(\hat{f}_t^o, x\right) \in \mathbb{S}\right) = \frac{x}{r+\lambda} + \frac{b}{h(-k^*)} h(-x) - c_2 \qquad for\left(\hat{f}_t^o, x\right) \in \mathbb{S}, and, x \ge k^*, \tag{4.52}$$

, and

$$\frac{\partial P^{h,o}\left(\hat{f}_t^o, g_t^o\right)}{\partial t} + L\left(P^{h,o}\left(\hat{f}_t^o, g_t^o\right)\right) = \frac{\partial P^{h,o}\left(\hat{f}_t^o, g_t^o\right)}{\partial t} + \left[r - \left(\delta_f - \frac{\lambda\left(\overline{f} - \hat{f}_t^o\right)}{\hat{f}_t^o}\right)\right]\hat{f}_t^o \frac{\partial P^{h,o}\left(\hat{f}_t^o, g_t^o\right)}{\partial \hat{f}_t^o}$$
$$+\left(r - \left[\delta_g - \rho\right]\right) g_t^o \frac{\partial P^{h,o}\left(\hat{f}_t^o, g_t^o\right)}{\partial g_t^o}$$
$$+\left(\frac{1}{2}\right)\left[\left[\left[\frac{\phi\sigma_s\sigma_f}{\sigma_s}\right]^2 + \left[\frac{\gamma}{\sigma_s}\right]^2 + \left[\frac{\gamma}{\sigma_D}\right]^2\right]\frac{\partial^2 P^{h,o}\left(\hat{f}_t^o, g_t^o\right)}{\left(\partial \hat{f}_t^o\right)^2}\right.$$
$$\left. +\sigma_g^2\left(g_t^o\right)^2 \frac{\partial^2 P^{h,o}\left(\hat{f}_t^o, g_t^o\right)}{\left(\partial g_t^o\right)^2}\right] \leq 0. \tag{4.53}$$

4.1.4 The Effect of Speculative Bubble on the Optimal Time to Develop Vacant Land to be Building

Preechametta (2005) applies the concept of European option and early exercise premium to explain the effect of speculative bubble. Assuming that the building option has same characteristics as European option which has the exactly exercise date is equal to time T where $0 \leq t \leq T < \infty$, the value of building option is then equal to

$$P_{EU}^{h,o}\left(\hat{f}_t^o, g_t^o\right) = E_t^o\left[\sup_{\tau_2 \geq 0} E_T^o\left[\left\{\int_T^{T+\tau_2} e^{-r(s-T)}\left[\overline{f} + e^{-\lambda(s-T)}\left(\hat{f}_T^o - \overline{f}\right) - R_a\right]ds - c_1\right\}\right.\right.$$
$$\left.\left. + e^{-r\tau_2}\left(\hat{P}_{T+\tau 2}^{h,\tilde{o}} - c_2\right)\right]\right] e^{-r(T-t)} \tag{4.54}$$

where T is represented the exercise date.

When comparing between equation (4.54) and(4.29), the value of building option in equation(4.29) , $P^{h,o}(\hat{f}_t^o, g_t^o)$, has the same characteristics as American option which has the right to exercise its option before the exercise date, T. Therefore, the optimal time to develop land to be building can be divided into 2 cases.

Case 1: If $0 \le t+\tau_1 < T$, the land owner decides to develop land to be building before time T because of the positive early exercise premium.

We can define the positive early exercise premium as

$$eP^o\left(\hat{f}^o_{t+\tau_1}, g^o_{t+\tau_1}\right)$$
$$= \int_{t+\tau_1}^{T} e^{-r(s-t-\tau_1)} E^o_t \left[\left(\delta_f \hat{f}^o_{t+\tau_1} - rc_1 + q^h(g_{t+\tau_1}\right) 1_{\left\{P^{h,o}\left(\hat{f}^o_{t+\tau_1}, g^o_{t+\tau 1}\right) \le \left[\left\{\frac{\bar{f}-R_a}{r} + \frac{\hat{f}^o_{t+\tau_1}-\bar{f}}{r+\lambda} - c_1\right\} + q^h(g^o_{t+\tau_1})\right]\right\}} \right] ds > 0$$

(4.55)

where the value of $1_{\left\{P^{h,o}\left(\hat{f}^o_{t+\tau_1}, g^o_{t+\tau_1}\right) \le \left[\left\{\frac{\bar{f}-R_a}{r} + \frac{\hat{f}^o_{t+\tau_1}-\bar{f}}{r+\lambda}\right\} - c_1\right] + q^h\left(g^o_{t+\tau 1} \middle| \left(\hat{f}^o_{t+\tau_1}, g^o_{t+\tau_1}\right) \in \mathbb{S}\right)\right\}}$ is equal to 1 if $P^{h,o}\left(\hat{f}^o_{t+\tau_1}, g^o_{t+\tau_1}\right) \le \left[\left\{\frac{\bar{f}-R_a}{r} + \frac{\hat{f}^o_{t+\tau_1}-\bar{f}}{r+\lambda} - c_1\right\} + q^h\left(g^o_{t+\tau_1} \middle| \left(\hat{f}^o_{t+\tau_1}, g^o_{t+\tau_1}\right) \in \mathbb{S}\right)\right]$. Otherwise, it is equal to 0.

From equation (4.55), $eP^o\left(\hat{f}^o_{t+\tau_1}, g^o_{t+\tau_1}\right)$ or early exercise premium is the net positive returns from period $t+\tau_1$ until T. These returns compose of two components. First is the total net return from rent $\left(\delta_f \hat{f}^o_{t+\tau_1} - rc_1\right)$ from period $t+\tau_1$ to T . Second is the value of resale option from period $t+\tau_1$ to T .

We can then write the total value of $P^{h,o}\left(\hat{f}^o_t, g^o_t\right)$ as

$$P^{h,o}\left(\hat{f}^o_t, g^o_t\right) = P^{h,o}_{EU}\left(\hat{f}^o_t, g^o_t\right) + ep^o\left(\hat{f}^o_{t+\tau_1}, g^o_{t+\tau 1}\right). \qquad (4.56)$$

Case 2: If $T \le t+\tau_1$, even though the land owner does not exercise at time T, the value of $P^{h,o}\left(\hat{f}^o_t, g^o_t\right)$ is still greater than zero.

In summary, speculative bubble is one important factor which causes the land owner to develop land to be building earlier. Therefore, it is interesting to analyze the policies which affect to the size of speculative bubble and to the optimal time to develop land to be building.

4.1.5 Policy Implications

In this section, we present the effects of five policies on the optimal time to develop land to be building based on Preechametta (2005) model. The core purpose of this section generally aims to identify ways to handle all possible asset price bubble in property market in the future.

4.1.5.1 The Effect of Real Interest Rate (r) on Resale and Building Options

If central bank or Federal Reserve Bank announces to increase the interest rate policy, this policy will affect both resale and building option values.

For the resale option, an increase in the interest rate policy causes the real rate of interest in the economy goes up. Once the real rate of interest goes up, it then causes an decrease in immediate gain from sale and causes the resale option value to decline.

For building option, an increase in interest rate policy affects the optimal stopping time to develop land to be building in two ways:

1. An increase in interest rate policy leads the real interest rate in the economy to be higher. Therefore, the value of building option $P^{h,o}\left(\hat{f}_t^o, g_t^o\right)$ in equation (4.29) decreases because of the higher discount rate.
2. Moreover, the higher real interest rate also causes the fundamental value of building in the right hand side of equation (4.42) to decrease. This effect decreases striking price or exercise price.

According to these two effects, it can either expand or shrink the optimal time to develop the land to be building.

4.1.5.2 The Effect of Resale Cost (c_2) on Resale and Building Options

The main cause of larger speculative bubble comes from an increase in trading volume. In order to reduce trading volume, we should identify the policy such that it can reduce the size of bubble. One policy that satisfies this condition is an increase in resale cost. When the policymaker increases the resale cost, the agents then suffer from the higher transaction cost to trade with the other. Because of an increase in transaction cost, they decide to reduce the frequency to trade with other agents.

In other words, when the resale cost increases, it causes the value of immediate gain from sale to decrease by reducing the size of bubble. Since the immediate gain decrease, from (4.34), the resale option has continued its value greater than the value of immediate gain. The agents therefore decide to delay their right to exercise their resale option. Hence, it implies the decrease in trading frequency.

Not only can resale cost decrease the trading frequency in resale process, it also partially help to delay the optimal stopping time to develop land to be building.

Based on the value of building option in equation (4.42), when resale cost increases, it typically reduces the value of resale option which is the one component of the immediate gain from developing land to be building. This causes the value of building option greater than the value of immediate gain from developing land to be building more than before. Thus, an increase in resale cost, if its magnitude is large enough, can also delay the optimal stopping time to develop land to be building.

4.1.5.3 The Effect of Overconfidence Level (ϕ) on Resale and Building Options

Overconfidence level is also one important factor causing the speculative bubble. An increase in overconfidence level affects both resale and building option values. For resale option, as ϕ increases, the volatility parameter σ_g in the difference of beliefs increases, whereas the mean reversion parameter ρ decreases. As a result, the resale becomes more valuable to the asset owner and the size of bubble becomes larger.

According to the higher value of resale option, it makes the immediate gain from developing land to be building in right hand side of equation (4.42) to increase. At the same time, an increase in value of resale option also pulls up the value of building option which is represented in equation (4.29). Therefore, an increase in overconfidence level can either increase or decrease the optimal stopping time to develop land to be building as well as an increase in the interest rate policy.

4.1.5.4 The Effect of Long-Run Fundamental Policy ($\bar{f}$) on Resale and Building Options

The investment in transportation, especially in the central business district (CBD), will generally increase the long-run fundamental of average returns on the building or $\bar{f}$. When the long-run fundamental increases, it then causes the present value of building fundamental in the right hand side of equation (4.42) to increase. Consequently, the immediate gain from developing land to be building increases which will stimulate the land owners to quickly use their right to exercise the building option.

4.1.5.5 The Effect of Information in Signals (i_s) on Resale and Building Options

As we have argued about the role of information, it can generate disagreement among agents and causes agents to be overconfident. Therefore, an increase in information in signals typically affects both resale and building option values. In this study, we measure information in each of two signals by $i_s = \frac{\sigma_f}{\sigma_s}$. When i_s increases, it means that there is more information for agents to disagree. However, because we measure the information in term $i_s = \frac{\sigma_f}{\sigma_s}$ therefore the information in signals can be increased by 2 reasons. First, holding the volatility of fundamentals σ_f constant, a decrease in σ_s is equivalent to an increase in the information. And second, holding the volatility of signals σ_s, an increase in σ_f is also equivalent to an increase in the information.

For the case a decrease in volatility of signals σ_s, it is equivalent to an increase in the information. When information in signals increases, the mean reversion parameter ρ of the difference in beliefs increases, and the volatility parameter σ_g is unchanged. An increase in ρ then causes the trading barrier and the duration between trades to drop. The lower trading barrier then causes the size of bubble to increase. In summary, an increase in information in signals caused by a decrease in volatility of signals normally causes the value of resale option to goes up.

As illustrated in the right hand side of equation (4.42), an increase in value of resale option caused by rising of information in signals, it additionally causes the value of immediate gain from developing land to be building to increase. However, the value of building option in equation (4.29) also increases from an increase in resale option.

Due to these two offsetting effects as the resale option increases, the optimal stopping time to develop land to be building therefore can either shorter or longer than before when information in signals is increased by a decrease in volatility of signals.

Another case is an increase in information caused by an increase in volatility of fundamentals. When the volatility of fundamentals increases, the volatility parameter σ_g in the difference of beliefs and the mean reversion parameter also increase. An increase in the volatility parameter σ_g causes the resale option becomes more valuable to the asset owner. On the other hand, an increase in the mean reversion parameter causes the resale option to become less valuable to the asset owner. Therefore, an increase in volatility of fundamentals can either increase or decrease the optimal trading barrier. For the size of bubble, when the volatility of fundamentals increases, it means that there is more information for agents to disagree. Therefore, agents tend to trade more aggressively. The result of higher volume in trade causes the size of bubble to increase. Therefore, an increase in information in signals caused by an increase in volatility of fundamentals typically causes the value of resale option to rise.

Moreover, an increase in value of resale option caused by rising of information in signals also bring about the value of immediate gain from developing land to be building to increase. As same as the value of immediate gain, the value of building option in equation (4.29) also increases from an increase in resale option.

Based on these two offsetting effects, the optimal stopping time to develop land to be building therefore can either shorter or longer than before when information in signals is increased by an increase in volatility of fundamentals.

However, it should be noted that the effect of information in signals also depends on the level of overconfidence level. In the case that the overconfidence level is relatively low, an increase in information in signals may instead reduce the disagreement among agents.

4.2 The Model of Rational Bubble

In this study, not only do we simulate the policy simulations, but we also identify whether there had a rational bubble in Thailand's property market in the past time or not. However, without complete property price data in Thailand, we can not directly test the rational bubble in property market. Given these deficiencies, it is useful to supplement these data with information from stock market index. Therefore, we test the rational bubble by using SET index and stock market index for the property subsector. However, it should be noted that such data must be interpreted with care and can only be the rough proxy for property price.

In order to test the rational bubble, we base on the test of rational bubble by Fukuta (1996). In his study, he considered a sufficient condition for the absence of rational bubbles which is contrast to the earlier studies such as Campbell and Shiller (1987), Diba and Grossman (1988), Lim and Phoon(1991) and Craine (1993) that consider only necessary conditions for the absence of rational bubbles.

A sufficient condition for the absence of rational bubble is derived on the assumption that the risk premium and the real interest rate are stationary (Fukuta (1996)).

Let p_t be the real price of a share at the beginning of period t, d_{t+1} be the real dividend paid to the owner at the end of period t, r_t be the real interest rate at period t and ρ_t be the risk premium at period t. From standard no-arbitrage condition:

$$p_t(1+r_t+\rho_t) = E_t(d_{t+1}+p_{t+1}), \tag{4.57}$$

where $E_t(\bullet)$ denotes the market's expectations conditional on information available at period t. It is assumed that the real interest rate and the risk premium are stationary.

Therefore, both variables have unconditional means. Let r and ρ are unconditional mean of r_t and ρ_t respectively. The equation (4.57) can be rewrite as

$$x_{1t} - x_{2t} + (1+r+\rho)p_t = E_t(p_{t+1}+d_{t+1}), \tag{4.58}$$

where $x_{1t} = p_t(r_t+\rho_t)$ and $x_{2t} = p_t(r+\rho)$. Thus, equation (4.58) can be rewritten as

$$p_t = \beta\left\{E_t\left(d_{t+1}+p_{t+1}\right) - x_{1t} + x_{2t}\right\}, \tag{4.59}$$

where $\beta = (1+r+\rho)^{-1}$. Assuming $0 < \beta < 1$. Applying recursive forward substitution to equation (4.59) yields

$$p_t = E_t\sum_{j=0}^{\infty}\beta^{j+1}(d_{t+j+1} - x_{1t+j} + x_{2t+j}) + \lim_{j\to\infty}\beta^{j+1}E_t p_{t+j+1}. \tag{4.60}$$

The first term on the right hand side of equation (4.60) is the fundamental component of the stock price. In the ordinary present value model with a constant discount rate, the fundamental component is the discount value of future dividends. The last term on the right hand side of equation (4.60) is the rational bubble component. If a rational bubble does not exist, then

$$B_t = \lim_{j\to\infty}\beta^{j+1}E_t p_{t+j+1} = 0. \tag{4.61}$$

If the first difference of a real stock price is stationary, then we find

$$\Delta p_t = \mu + a_t, \tag{4.62}$$

where $\Delta p_t, \mu\, and\, a_t$ denote the first difference of a real stock price, the drift term and the stationary error term, respectively. We can rewrite (4.62) as

$$p_t = p_{t-1} + \mu + a_t. \tag{4.63}$$

Equation (4.63) implies

$$p_{t+j+1} = p_t + \mu(j+1) + \sum_{i=t+1}^{t+j+1} a_i. \tag{4.64}$$

Substituting equation (4.64) in equation (4.61), we find that

$$B_t = \lim_{j\to\infty} \frac{p_t + \mu(j+1) + \nu_t}{(1+r+\rho)^{j+1}}, \tag{4.65}$$

where $\upsilon_t = E_t \sum_{i=t+1}^{t+j+1} a_i$. The first and third term on the right hand side of equation (4.65) converge to zero as j goes to infinity. Since $\mu(j+1)$ grows more slowly than $(1+r+\rho)^{j+1}$ as j becomes large, the second term on the right hand side of equation (4.65) converges to zero as j goes to infinity. Therefore, we find that rational bubble behavior is excludes.

The conclusion of this model, therefore, is that "given the real interest rate and the risk premium are stationary and that the sum of unconditional means of them is strictly larger than zero. If the first difference of a real stock prices movement is stationary, then the stock price behavior does not contain rational bubbles".

CHAPTER 5

FINITE DIFFERENCE METHOD

In order to study the policy simulations from the model of property price with heterogeneous beliefs described in the previous chapter, the main problem is to identify the optimal stopping time to develop land to be building. This problem is the real option problem which has the characteristics remarkably similar to American option. Due to the lack of closed form solution, we therefore begin with the essential concept of the mathematic technique applied widely in financial engineering which is called Finite Difference Method (FDM) to identify the value of building option from the partial differential equation.

5.1 Introduction and Classification of Partial Differential Equations

Partial differential equations (PDEs) play the major role in engineering, physics, financial engineering, and economics. For financial engineering and economics, it turns out that PDEs have become an important tool in option valuation because it provides a powerful and consistent framework for pricing rather complex derivatives. However, analytical solutions like Black-Scholes formula are not available in general. Thus, one must often resort to numerical methods.

A classification of PDEs is relevant in that the choice of a numerical method to cope with PDEs generally depend on its characteristics. To classify PDEs, we should know the order of a PDE and the types of each PDEs-linear or non linear equation. In general, the order of a PDE is come from the highest order of the derivatives involved. For example, if we have an unknown function $\phi(x, y)$, depending on variables x and y, a generic first-order equation has the form

$$a(x, y)\frac{\partial \phi}{\partial x} + b(x, y)\frac{\partial \phi}{\partial y} + c(x, y)\phi + d(x, y) = 0\,, \tag{5.1}$$

where a, b, c, d are given functions of the independent variables. The highest order of derivative in (5.1) is one. Moreover, it is linear, since the functions a,b,c, and d depend only on the independent variables x and y and not on ϕ itself. Therefore we can conclude that equation (5.1) is the generic form of a linear first-order equation.

By the same idea, the general form of a linear second-order equation is

$$a\frac{\partial^2\phi}{\partial x^2} + b\frac{\partial^2\phi}{\partial x\partial y} + c\frac{\partial^2\phi}{\partial y^2} + d\frac{\partial\phi}{\partial x} + e\frac{\partial\phi}{\partial y} + f\phi = 0\,, \tag{5.2}$$

where again all the given functions depend only on x and y.

Other types of PDEs are non linear equation which their parameters are not only depending on the independent variables but also its function. Nonetheless, in our study, we deal only with the linear equations[1].

In order to illustrate step by step how to solve partial differential equation by employing Finite Difference Method (FDM), we therefore start with the price of American option which depends on three variables x, y, and t ,where two variables x and y have the Brownian processes.

Due to the early exercise possibility of the American option, its price can be obtained by solving a time dependent complementarity problem which is

$$U(x, y, t) \geq \max(E - x, 0) =: g(x), \quad (x, y, t) \in \Omega \times \left[0, T\right]$$

$$\begin{aligned} &LU \geq 0 \\ &U \geq g \\ &(U - g)LU = 0. \end{aligned} \tag{5.3}$$

[1] Even though most of the models in financial engineering and economics are linear, non linear equations may be obtained when relaxing some of assumption; for example in the Black-Scholes model ,a nonlinear equation can be occurred when introducing transaction cost.

A general form of two dimensional parabolic partial differential[2] inequality can be derived for the price of this option using Ito'lemma. We define a generalized operator as

$$LU = \frac{\partial U}{\partial t} + a\frac{\partial^2 U}{\partial x^2} + b\frac{\partial^2 U}{\partial x \partial y} + c\frac{\partial^2 U}{\partial y^2} + d\frac{\partial U}{\partial x} + e\frac{\partial U}{\partial y} + fU = 0\,. \qquad (5.4)$$

The price of option can be obtained by solving (5.4). Due to the early exercise possibility of American option, the arising problems are free boundary problems. In order to value its price, we therefore employ the financial technique which is called "Finite Difference Method (FDM)".

5.2 Finite Difference Method (FDM)

In order to find the value of American option, the strategy is to apply the Finite Difference Method (FDM) to equation (5.4). Finite Difference Method (FDM) solving partial differential equation is based on the simple idea of approximating each partial derivative by a difference quotient. This method is widely applied in many academic fields such as physics, chemical science, and financial economics. In financial economics, FDM is applied for valuating value of option which cannot be solved analytically. As in many numerical algorithms, the starting point is a finite series approximation.

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[2] LU equation is called parabolic equation in two space dimensions if it is satisfied these conditions

Let $\frac{\partial U}{\partial t} = LU$,

where

$$L \equiv \sum_{i=1}^{2}\left[\frac{\partial}{\partial x_i}\left(a_i(x_1, x_2, t)\frac{\partial}{\partial x_i}\right) + b_i\left(x_1, x_2, t\right)\frac{\partial}{\partial x_i}\right] - c(x_1, x_2, t),$$

with

(i) a_1, a_2 strictly positive,and

(ii) c non-negative.

Under suitable continuity and differentiability hypotheses, a function $U(x)$ using Taylor's theorem may be represented as:

1. The forward approximation

By using the Taylor's series approximation, a function $U(x)$ can be represented as

$$U(x+h) = U(x) + hU'(x) + \frac{1}{2}h^2U''\left(x\right) + \frac{1}{6}h^3U'''(x) + \dots . \tag{5.5}$$

If we omit the terms of second order and higher orders in (5.5) , we then yield the first-order forward approximation which is

$$U'(x) = \frac{U(x+h) - U(x)}{h} + O(h)\,, \tag{5.6}$$

where $O\left(h\right)$ is represented a truncation error from this approximation.

2. The backward approximation

It is similar to the forward approximation, we may write the function $U(x)$ as

$$U(x-h) = U(x) - hU'(x) + \frac{1}{2}h^2U''\left(x\right) - \frac{1}{6}h^3U'''(x) + \dots, \tag{5.7}$$

from which we obtain the backward approximation,

$$U'(x) = \frac{U(x) - U(x-h)}{h} + O(h)\,, \tag{5.8}$$

where $O\left(h\right)$ is also represented a truncation error from this approximation.

Based on these two cases, we get a truncation error order $O(h)$. However, we can reduce a truncation error by using the following approximation

3. The central approximation

In the previous cases, we get a truncation error of order $O(h)$. The central method, on the other hand, will provide a better approximation[3]by subtracting equation (5.7) from equation (5.5) and rearranging. Then,

$$U'(x) = \frac{U(x+h) - U(x-h)}{2h} + O(h^2)\,, \tag{5.9}$$

where $O(h^2)$ is represented the truncation error.

These methods can be expanded to higher-order derivatives. To cope with the partial differential equation for valuating the building option, we must approximate second-order derivatives which can be solved by adding equation (5.5) and (5.7), which becomes

$$U(x+h) + U(x-h) = 2U(x) + h^2 U''(x) + O(h^4)\,, \tag{5.10}$$

and rearranging yields

$$U''(x) = \frac{U(x+h) - 2U(x) + U(x-h)}{h^2} + O(h^2)\,, \tag{5.11}$$

where $O(h^4)$ and $O(h^2)$ are represented the truncation errors.

If we let U_m denote the value $U(mh)$ then this may be written as

$$U_{m+1} - U_m \approx h\frac{dU}{dx}(mh)\,. \tag{5.12}$$

[3] It should be noted that the forward and backward approximations may be useful to come up with efficient numerical schemes, depending on the type of boundary conditions.

The following table 5.1, we define a number of finite difference operators. These operators, which act on grid values $U_m = U(mh)$, form the main building blocks of Finite Difference Method (FDM). Moreover, this table also provides the first two terms in the corresponding Taylor series expansions.

Table 5.1[4]

Difference Operators

Operator	Symbol	Definition	Taylor series
Forward difference	Δ	$U_{m+1} - U_m$	$hU' + \frac{1}{2}h^2U'' + \ldots$
Backward difference	∇	$U_m - U_{m-1}$	$hU' - \frac{1}{2}h^2U'' + \ldots$
Half central difference	δ	$U_{m+\frac{1}{2}} - U_{m-\frac{1}{2}}$	$hU' - \frac{1}{24}h^2U'' + \ldots$
Full central difference	Δ_o	$\frac{1}{2}\left(U_{m+1} - U_{m-1}\right)$	$hU' + \frac{1}{6}h^3U''' + \ldots$
Second order central difference	δ^2	$U_{m+1} - 2U_m + U_{m-1}$	$h^2U'' - \frac{1}{12}h^4U'''' + \ldots$
Shift	E	U_{m+1}	$U + hU' + \ldots$
Average	μ	$\frac{1}{2}\left(U_{m+\frac{1}{2}} + U_{m-\frac{1}{2}}\right)$	$U + \frac{1}{8}h^2U'' + \ldots$

Noted:

1. We employ U' and U'' to denote $\frac{dU}{dx}$ and $\frac{d^2U}{dx^2}$, respectively.
2. Assume that functions are evaluated at $x = mh$

[4] We adapt this table from Higham, Desmond J. (2004).

Up to this point, we have already known the ways to approximate for the derivatives. In the next procedure, we aim to provide the principal steps to apply the Finite Difference Method (FDM) for solving the partial differential equation.

In general, the partial differential equation can be solved by these following steps.

5.2.1 Discretization

In computing an approximate solution to the PDE as shown in equation (5.4) , we usually handle with the unbound domain which is

$$\left\{ (x, y, t) \middle| x \geq 0, y \geq 0, t \in [0, T] \right\}. \tag{5.13}$$

Therefore, to apply FDM approximations for space variables, we must determine the upper bound of x $(X_{\max})$ and y $(Y_{\max})$ [5] in equation (5.13) . After we truncate this into a finite size computational domain, what we get is

$$(x, y, t) \in [0, X_{\max}] \times [0, Y_{\max}] \times [0, T] =: \Omega \times [0, T], \tag{5.14}$$

where $X_{\max}$ and $Y_{\max}$ are sufficient large.

For the computational domain in (5.14), the discretization is performed using a uniform space-time finite difference grid. Specifically, we divide the x-direction axis into $N_x + 1$ equally spaced points $\{i\Delta x\}_{i=0}^{N_x}$,the y-direction axis into $N_y + 1$ equally spaced points $\{j\Delta y\}_{j=0}^{N_y}$ and the time axis into $N_t + 1$ equally spaced

---

[5] It should be noted that $(X_{\max})$ and $(Y_{\max})$ are a realistic and practical approximation to infinity and is subjectively chosen depending on the maturity and type of the derivative contract. Wilmott states that $(X_{\max})$ and $(Y_{\max})$ do not have to be too large in practice; “Typically it should be three or four times the value of the exercise price or more generally, three to four times the value of the asset at which there is some important behavior.”

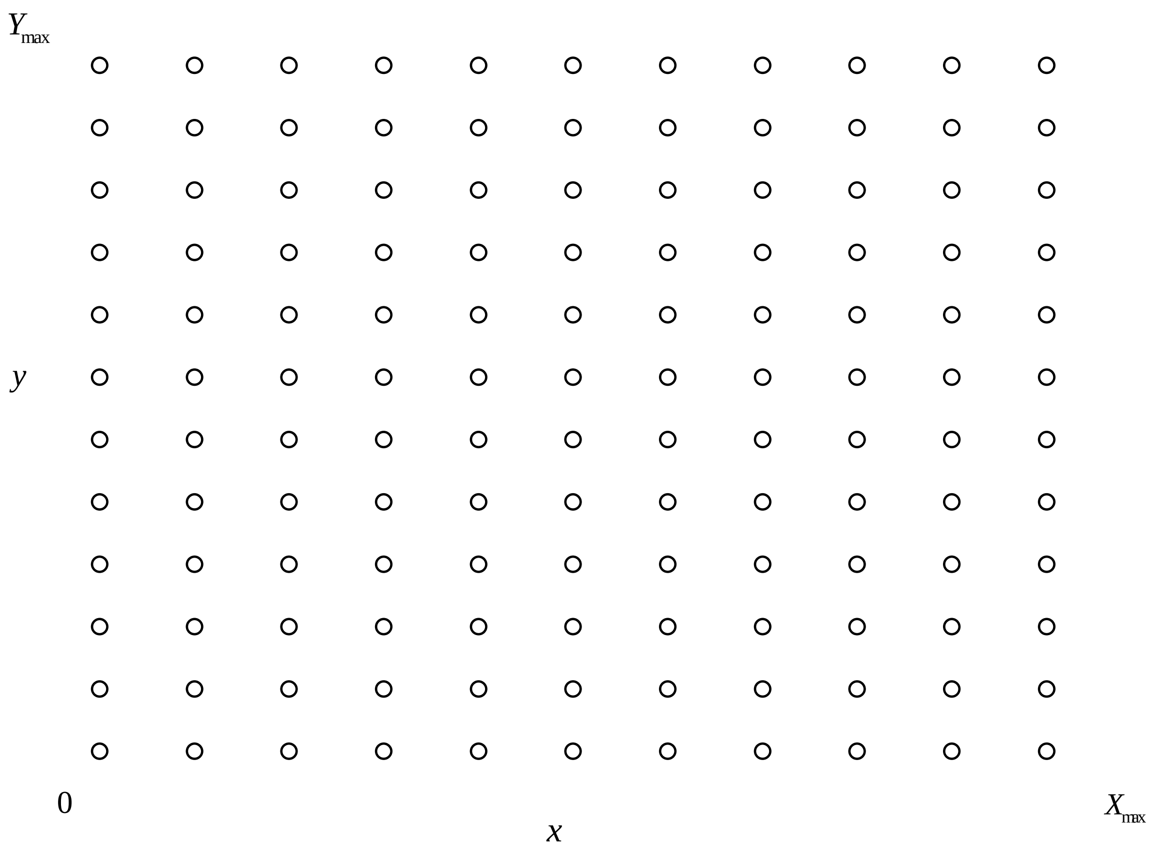
points $\{k\Delta t\}_{k=0}^{N_t}$. The grid steps to these directions are denoted by $\Delta x := \dfrac{X_{\max}}{N_x}$, $\Delta y := \dfrac{Y_{\max}}{N_y}$,and $\Delta t := \dfrac{T}{N_t}$.

Therefore, the grid point values of a finite difference approximation are denoted by

$$U_{i,j}^{(k)} \approx U\left(x_i, y_j, t_k\right) = U\left(i\Delta x, j\Delta y, k\Delta t\right). \tag{5.15}$$

Figure 5.1

Finite Difference Grid $\{i\Delta x, j\Delta k\}_{i=0,\, j=0}^{N_x, N_y}$



Noted: Points are spaced at a distance of Δx apart in the x-direction and Δy apart in the y-direction

Figure 5.1 illustrates the grid with the open circles indicate grid points where the solution is not yet known. Our task is to find numbers to put into the points marked with open circles. Although some of open circles can be determined the solution when we identify the boundary and initial conditions, the remaining points can be found out by using Finite Difference Method (FDM).

5.2.2 Boundary Condition

There is no exactly form for the boundary conditions. It depends on each type of problem. For real option which has the characteristics similar to American option; for each time before expiration, there is a critical value for the price of the underlying asset at which it is optimal to exercise the option. Depending on the option types (American put or American call option), it will also be optimal to exercise the optimal for prices above and below the critical price. Therefore, based on this right, we should cope with a free boundary, i.e., a boundary within the domain, which separates the exercise and no-exercise region.

5.2.3 Space Discretization

To apply Finite Difference Method (FDM) to equation (5.4), the key step is to replace differential operators with finite difference operators. In our PDE problem, domain involves three independent variables as described in (5.14). However, space discretization is firstly done only two directions, x and y.

The second-order and first-order spatial derivatives are approximated with standard second-order accurate central finite differences (see equation (5.9)). With the two space discretization, we must distinguish between difference operators in the x-and y-directions. We denote the difference operators for the first-order derivatives by

$$\Delta_x U_{i,j} = \frac{U_{i+1,j} - U_{i-1,j}}{2\Delta x} \quad \text{and} \quad \Delta_y U_{i,j} = \frac{U_{i,j+1} - U_{i,j-1}}{2\Delta y}. \tag{5.16}$$

and for the second-order derivatives, they are represented by

$$\delta_x^2 U_{i,j} = \frac{U_{i+1,j} - 2U_{i,j} + U_{i-1,j}}{\Delta x^2} \text{ and } \delta_y^2 U_{i,j} = \frac{U_{i,j+1} - 2U_{i,j} + U_{i,j-1}}{\Delta y^2}. \qquad (5.17)$$

Next, we consider the discretization of the second-order cross-derivative term. If we assume the coefficient b for the cross-derivative in (5.4) is non positive, we then obtain approximations from the Taylor's series approximation:

$$U(x_{i+1}, y_{i+1}) \approx U + \Delta x \frac{\partial U}{\partial x} + \Delta y \frac{\partial U}{\partial y} + \frac{1}{2}\left(\Delta x^2 \frac{\partial^2 U}{\partial x^2} + 2\Delta x \Delta y \frac{\partial U}{\partial x \partial y} + \Delta y^2 \frac{\partial^2 U}{\partial y^2} \right), \qquad (5.18)$$

$$U\left(x_{i-1}, y_{j-1}\right) \approx U - \Delta x \frac{\partial U}{\partial x} - \Delta y \frac{\partial U}{\partial y} + \frac{1}{2}\left(\Delta x^2 \frac{\partial^2 U}{\partial x^2} + 2\Delta x \Delta y \frac{\partial U}{\partial x \partial y} + \Delta y^2 \frac{\partial^2 U}{\partial y^2} \right), \qquad (5.19)$$

where the value for U and its derivative on the right side are evaluated at the grid point $\left(x_i, y_i\right)$. By summing the equations in (5.18) and (5.19), we obtain

$$2\Delta x \Delta y \frac{\partial U}{\partial x \partial y} \approx U\left(x_{i+1}, y_{i+1}\right) - 2U\left(x_i, y_i\right) + U\left(x_{i-1}, y_{j-1}\right) - \Delta x^2 \frac{\partial^2 U}{\partial x^2} - \Delta y^2 \frac{\partial^2 U}{\partial y^2}. \qquad (5.20)$$

Then, rearrange(5.20) gives

$$\frac{\partial U}{\partial x \partial y} \approx \frac{1}{2\Delta x \Delta y}\left[U\left(x_{i+1,} y_{j+1}\right) - 2U\left(x_i, y_j\right) + U\left(x_{i-1,} y_{j-1}\right)\right] - \frac{\Delta x}{2\Delta y}\frac{\partial^2 U}{\partial x^2} - \frac{\Delta y \partial^2 U}{2\Delta x \partial y^2}. \qquad (5.21)$$

Using equation (5.21) , the second-order derivative in equation (5.2) can be approximated as

$$
\begin{aligned}
a\frac{\partial^2 U}{\partial x^2} + b\frac{\partial^2 U}{\partial x \partial y} + c\frac{\partial^2 U}{\partial y^2} \approx \left[a - \frac{b\Delta x}{2\Delta y}\right]\frac{\partial^2 U}{\partial x^2} + \left[c - \frac{b\Delta y}{2\Delta x}\right]\frac{\partial^2 U}{\partial y^2} \\
+ \frac{b}{2\Delta x \Delta y}\left[U(x_{i+1,} y_{j+1}) - 2U(x_{i,} y_j) + U\left(x_{i-1,} y_{j-1}\right)\right]
\end{aligned}
. \qquad (5.22)
$$

Replacing differential operators with finite difference operators by using the central finite differences in (5.16), (5.17) and (5.22), we can approximate the partial differential equation (5.4) by the semi-discrete equation

$$
\begin{aligned}
\frac{\partial U}{\partial t} + \left[a - \frac{b\Delta x}{2\Delta y}\right]\delta_x^2 U_{i,j} + \left[c - \frac{b\Delta y}{2\Delta x}\right]\delta_y^2 U_{i,j} + d\delta_x U_{i,j} + e\delta_y U_{i,j} + fU_{i,j} \\
+ \frac{b}{2\Delta x \Delta y}\left[U_{i+1,j+1} - 2U_{i,j} + U_{i-1,j-1}\right] = 0.
\end{aligned}
\qquad (5.23)
$$

Then using the definitions (5.16) and (5.17) and rearranging terms, this equation has the form

$$
\begin{aligned}
\frac{\partial U}{\partial t} + \left(\frac{b}{2\Delta x \Delta y}\right)U_{i-1,j-1} + \left(\frac{1}{\Delta y^2}\left[c - \frac{b\Delta y}{2\Delta x}\right] - \frac{e}{2\Delta y}\right)U_{i,j-1} \\
+ \left(\frac{1}{\Delta x^2}\left[a - \frac{b\Delta x}{2\Delta y}\right] - \frac{d}{2\Delta x}\right)U_{i-1,j} \\
+ \left(-\frac{2}{\Delta x^2}\left[a - \frac{b\Delta x}{2\Delta y}\right] - \frac{2}{\Delta y^2}\left[c - \frac{b\Delta y}{2\Delta x}\right] - \frac{b}{\Delta x \Delta y} + f\right)U_{i,j} \\
+ \left(\frac{1}{\Delta x^2}\left[a - \frac{b\Delta x}{2\Delta y}\right] + \frac{d}{2\Delta x}\right)U_{i+1,j} \\
+ \left(\frac{1}{\Delta y^2}\left[c - \frac{b\Delta y}{2\Delta x}\right] + \frac{e}{2\Delta y}\right)U_{i,j+1} + \left(\frac{b}{2\Delta x \Delta y}\right)U_{i+1,j+1} = 0.
\end{aligned}
\qquad (5.24)
$$

Therefore, this space discretization is the semi-discrete equation which has the matrix representation

$$\frac{\partial U}{\partial t} i + \mathbf{M1U} + \mathbf{M2} = \mathbf{0}. \tag{5.25}$$

where $\mathbf{M1}$ is an $nm \times nm$ matrix , $\mathbf{U}$ is a vector of length nm, $\mathbf{M2}$ is an $nm \times 1$.

For example, if we set $n = 4$ and $m = 4$, the matrix $\mathbf{M1}$ will have $(4\times 4)\times(4\times 4)$ dimension and matrix $\mathbf{M2}$ will have 16×1 dimension which is represented in figure 5.2

5.2.4 Time Discretization

In equation (5.25), the first-order time derivative needs to be approximated. There are many ways to do this. Here, three second-order accurate methods, which are backward difference formula, Runge-Kutta scheme, and the Crank-Nicolson method, are clarified as well as the implicit Euler method.

The first-order accurate implicit Euler scheme is

$$\left(\frac{1}{\Delta t}\mathbf{I} + \mathbf{M1}\right)\mathbf{u}^{(k)} = \left(\frac{1}{\Delta t}\mathbf{I}\right)\mathbf{u}^{(\mathbf{k}-1)} - \mathbf{M2}, \quad \text{for} \quad k = 1, 2, \ldots, l. \tag{5.26}$$

The Runge-Kutta scheme consists the following steps:

$$\begin{aligned} \left(\mathbf{I} + \theta\Delta \mathrm{t}\mathbf{M1}\right)\bar{\mathbf{u}}^{(k)} &= \left(\mathbf{I} - \left(1-\theta\right)\Delta t\mathbf{M1}\right)\mathbf{u}^{(k-1)} - \mathbf{M2} \\ \left(\mathbf{I} + \theta\Delta t\mathbf{M1}\right)\mathbf{u}^{(k)} &= \left(\mathbf{I} - \frac{1}{2}\Delta t\mathbf{M1}\right)\mathbf{u}^{(k-1)} - \left(\frac{1}{2} - \theta\right)\Delta t\mathbf{M1}\bar{\mathbf{u}}^{(k)} - \mathbf{M2}, \end{aligned} \tag{5.27}$$

for $k = 1, 2, \ldots, l$.

Figure 5.2

Matrix M1 and M2 for $n=4, m=4$

$$
\begin{bmatrix}1\\1\\1\\1\\1\\1\\1\\1\\1\\1\\1\\1\\1\\1\\1\\1\end{bmatrix}
\left(\frac{\partial U}{\partial t}\right)+
\begin{bmatrix}
D & F & 0 & 0 & E & G & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
B & D & F & 0 & 0 & E & G & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & B & D & F & 0 & 0 & E & G & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & B+F & D & 0 & 0 & G & E & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
C & 0 & 0 & 0 & D & F & 0 & 0 & E & G & 0 & 0 & 0 & 0 & 0 & 0\\
A & C & 0 & 0 & B & D & F & 0 & 0 & E & G & 0 & 0 & 0 & 0 & 0\\
0 & A & C & 0 & 0 & B & D & F & 0 & 0 & E & G & 0 & 0 & 0 & 0\\
0 & 0 & A & C & 0 & 0 & B+F & D & 0 & 0 & G & E & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & C & 0 & 0 & 0 & D & F & 0 & 0 & E & G & 0 & 0\\
0 & 0 & 0 & 0 & A & C & 0 & 0 & B & D & F & 0 & 0 & E & G & 0\\
0 & 0 & 0 & 0 & 0 & A & C & 0 & 0 & B & D & F & 0 & 0 & E & G\\
0 & 0 & 0 & 0 & 0 & 0 & A & C & 0 & 0 & B+F & D & 0 & 0 & G & E\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & C+E & G & 0 & 0 & D & F & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & A & C+E & G & 0 & B & D & F & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & A & C+E & G & 0 & B & D & F\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & A & C+E & G & 0 & B+F & D
\end{bmatrix}
\begin{bmatrix}
U(dx,dy)\\ U(dx,2dy)\\ U(dx,3dy)\\ U(\Delta x,4\Delta y)\\ U(2\Delta x,\Delta y)\\ U(2\Delta x,2\Delta y)\\ U(2\Delta x,3\Delta y)\\ U(2\Delta x,4\Delta y)\\ U(3\Delta x,\Delta y)\\ U(3\Delta x,2\Delta y)\\ U(3\Delta x,3\Delta y)\\ U(3\Delta x,4\Delta y)\\ U(4\Delta x,\Delta y)\\ U(4\Delta x,2\Delta y)\\ U(4\Delta x,3\Delta y)\\ U(4\Delta x,4\Delta y)
\end{bmatrix}
+
\begin{bmatrix}
\varepsilon(dx,dy)\\ \varepsilon(dx,2dy)\\ \varepsilon(dx,3dy)\\ \varepsilon(dx,4dy)\\ \varepsilon(2dx,dy)\\ 0\\ 0\\ 0\\ \varepsilon(3dx,dy)\\ 0\\ 0\\ 0\\ \varepsilon(4dx,dy)\\ 0\\ 0\\ 0
\end{bmatrix}
=\mathbf{0}
$$

where

$\varepsilon(dx,dy) = A*U(0,0) + B*U(dx,0) + C*U(0,dy)$,

$\varepsilon(dx,2dy) = A*U(0,dy) + C*U(0,2dy)$,

$\varepsilon(dx,3dy) = A*U(0,2dy) + C*U(0,3dy)$,

$\varepsilon(dx,4dy) = A*U(0,3dy) + C*U(0,4dy)$,

$\varepsilon(2dx,dy) = A*U(dx,0) + B*U(2dx,0)$,

$\varepsilon(3dx,dy) = A*U(2dx,0) + B*U(3dx,0)$,

$\varepsilon(4dx,dy) = A*U(3dx,0) + B*U(4dx,0)$,

$$A = \frac{b}{2\Delta x\Delta y} \quad ,B = \left(\frac{1}{\Delta y^2}\left[c - \frac{b\Delta y}{2\Delta x}\right] - \frac{e}{2\Delta x}\right) \quad ,C = \left[\frac{1}{\Delta x^2}\left[a - \frac{b\Delta x}{2\Delta y}\right] - \frac{d}{2\Delta x}\right]$$

$$D = \left[-\frac{2}{\Delta x^2}\left[a - \frac{b\Delta x}{2\Delta y}\right] - \frac{2}{\Delta y^2}\left[c - \frac{b\Delta y}{2\Delta x}\right] - \frac{b}{\Delta x\Delta y} + f\right],$$

$$E = \left[\frac{1}{\Delta x^2}\left[a - \frac{b\Delta x}{2\Delta y}\right] + \frac{d}{2\Delta x}\right],$$

$$F = \left[\frac{1}{\Delta y^2}\left[c - \frac{b\Delta y}{2\Delta x}\right] + \frac{e}{2\Delta y}\right],\text{and}$$

$$G = \frac{b}{2\Delta x\Delta y}$$

The second-order backward difference formula (BDF2) is

$$\left(\frac{2}{3}\mathbf{M1}-\frac{1}{\Delta t}\mathbf{I}\right)\mathbf{u}^{(k)}=\frac{1}{3}\frac{1}{\Delta t}\mathbf{u}^{(k-2)}-\frac{4}{3}\frac{1}{\Delta t}\mathbf{u}^{(k-1)}-\mathbf{M2}, \tag{5.28}$$

for $k=2,...,l$.

Once the space and time discretizations are performed, the value of option at each period can be obtained by solving the sequence of linear complementarity problems:

$$\begin{aligned}&\mathbf{BU}^{(k+1)}\geq\mathbf{CU}^{(k)}\\&\mathbf{U}^{(k+1)}\geq\mathbf{g}\\&(\mathbf{BU}^{(k+1)}-\mathbf{CU}^{(k)})(\mathbf{U}^{(k+1)}-\mathbf{g})=0.\end{aligned} \tag{5.29}$$

In this study, we will solve the sequence of linear complementarity problem by employing the direct methods which are the Gaussian Elimination and the LU decomposition. We therefore present these methods in the following section.

5.3 Gaussian Elimination and LU Decomposition

Gaussian elimination is a common direct method for solving a linear system. Firstly, we will consider a simple case, and then we will study how to reduce the general problem to the simple one.

Let's consider the simple case is that of triangular matrix. **A** is called "lower triangular" if all nonzero elements lie on or below the diagonal which is

$$\mathbf{A}=\begin{bmatrix}\mathrm{a}_{11} & 0 & \cdots & 0\\ a_{21} & a_{22} & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ a_{n1} & a_{n2} & \cdots & a_{nn}\end{bmatrix}. \tag{5.30}$$

On the other hand, it is called “upper triangular” if all elements on or above the diagonal are all nonzero. Lastly if it is either upper or lower triangular, **A** is simply called a triangular matrix.

The linear systems in which **A** is triangular can be solved by back-substitution. Suppose that **A** is a lower triangular and nonsingular. Since all nondiagonal elements in the first row of **A** are zero, the first row of the system $\mathbf{AX} = \mathbf{b}$ can be reduced to $a_{11}x_1 = b_1$. Therefore, the solution is $x_1 = \frac{b_1}{a_{11}}$. After we get the solution of x_1, we can solve the solution of x_2 by putting the solution of x_1 into equation $a_{22}x_2 + a_{21}x_1 = b_2$. Proceeding down the matrix, we can obtain all component of x in sequence.

In general, the procedure of back-substitution for lower triangular matrix is as follows:

$$x_1 = \frac{b_1}{a_{11}} \tag{5.31}$$

$$x_k = \frac{b_k - \sum_{j=1}^{k-1} a_{kj}x_j}{a_{kk}}, \quad k = 2, 3, ..., n, \tag{5.32}$$

For **A** is upper triangular, we can similarly solve $\mathbf{AX} = \mathbf{b}$ beginning with $x_n = \frac{b_n}{a_{nn}}$.

In practice, **A** is usually not a triangular matrix. Hence, we must manipulate $\mathbf{AX} = \mathbf{b}$ by using elementary row operation in order to reduce it to an upper triangular system. This step is called “forward elimination”.

The applying forward elimination requires the bulk of the computational effort. This is particularly true for the large systems of equations.

This time consuming procedure can be avoided by using the LU decomposition method, which separate the time–consuming elimination of the matrix **A** from the manipulation of the right-hand side **b**. Thus, once **A** has been “decomposed,” multiple right-hand-side vectors can be evaluated in an efficient manner.

To solve $\mathbf{AX} = \mathbf{b}$ with general $\mathbf{A}$, we firstly factor $\mathbf{A}$ into the product of two triangular matrices, $\mathbf{A} = \mathbf{LU}$ where $\mathbf{L}$ is the lower triangular matrix and $\mathbf{U}$ is upper triangular. This is called the LU decomposition of **A**. We then replace the problem $\mathbf{AX} = \mathbf{b}$ with the equivalent problem $\mathbf{LUX} = \mathbf{b}$. Then, we can solve it by solving $\mathbf{LZ} = \mathbf{b}$. After we get $\mathbf{Z}$, we can get the value of $\mathbf{X}$ by solving $\mathbf{UX} = \mathbf{Z}$.

Up to this point, we have already gotten a good understanding about the Finite Difference Method (FDM). In the next chapter, we will employ this technique for solving the value of building option in order to determine the optimal time to develop land to be building.

CHAPTER 6

METHODOLOGY AND RESEARCH DESIGN

This chapter is composed of two main parts. In the first part, we present the econometric technique and sources of data which are applied to test rational bubble in the stock market, especially in the property stock. In the second part, we present the computational processes which are employed to do the policy simulations on building and resale options. In order to complete this part, we also explain how to construct most of parameters which are used to do the simulations in this study.

6.1 Testing the Existence of Asset Price Bubble in Stock Market

In order to test the rational bubble, we base on the test of rational bubble by Fukuta (1996) which is "given the real interest rate and the risk premium are stationary and that the sum of unconditional means of them is strictly larger than zero. If the first difference of a real stock prices movement is stationary, then the stock price behavior does not contain rational bubbles".

6.1.1 Methods to Test the Stationary

We apply the Augmented Dicky-Fuller (ADF) test, the Phillips-Perron (PP) test, and the Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) test to investigate whether there is a rational bubble in stock market in Thailand or not.

6.1.1.1 The Unit Root Test

From the unit root (stochastic) process

$$Y_t = \rho Y_{t-1} + u_t \,, \qquad (6.1)$$

where $-1 \leq \rho \leq 1$ and u_t is a white noise error term.

If $\rho = 1$, that is, in the case of the unit root[1], (6.1) becomes a random walk model without drift, which is a nonstationary stochastic process. Therefore, the general idea behind the unit root test is to find out that the estimated ρ is statistically equal to 1 or not, if it equals, then Y_t is nonstationary.

From (6.1), we manipulate by subtracting Y_{t-1} from both sides to get:

$$\begin{aligned} Y_t - Y_{t-1} &= \rho Y_{t-1} - Y_{t-1} + u_t \\ &= (\rho - 1) Y_{t-1} + u_t \end{aligned} \tag{6.2}$$

The equation (6.2) can be rewritten as

$$\Delta Y_t = \delta Y_{t-1} + u_t, \tag{6.3}$$

where $\delta = (\rho - 1)$ and Δ, as usual, is the first-difference operator. Therefore, we can estimate (6.3) and test the null hypothesis that $\delta = 0$. If $\delta = 0$,then $\rho = 1$, so that we have a unit root. This certainly means that the time series under consideration is nonstationary.

6.1.1.2 The Dickey-Fuller (DF) Test

The actual procedure of implementing the DF test involves several decisions. To allow for the various possibilities, the DF test is estimated in three different forms showed in table 6.1.

---

[1] When $\rho = 1$, we can write (6.1) as $Y_t - Y_{t-1} = u_t$. Now using the lag operator L so that $LY_t = Y_{t-1}$, $L^2 Y_t = Y_{t-2}$, and so on, we can write (6.1) as $(1 - L) Y_t = u_t$. The name unit root refers to the polynomial in the lag operator. If we set $(1 - L) = 0$, we obtain, $L = 1$ which is the name unit root.

Table 6.1

Three Different Forms of Random Walks

Types of random walk	Forms of equation
Y_t is a random walk	$\Delta Y_t = \delta Y_{t-1} + u_t$
Y_t is a random walk with drift	$\Delta Y_t = \beta_1 + \delta Y_{t-1} + u_t$
Y_t is a random walk with drift around a stochastic trend	$\Delta Y_t = \beta_1 + \beta_2 t + \delta Y_{t-1} + u_t$

where t is the time or trend variable. The null hypothesis is that $\delta = 0$ (the time series is nonstationary). The alternative hypothesis is that δ is less than zero (the time series is stationary)[2].

6.1.1.3 The Augmented Dickey-Fuller (ADF) Test

In the previous test, it was assumed that the error term u_t was uncorrelated. However, if this is not the case, Augmented Dickey-Fuller (ADF) test must be applied. This test is conducted by "augmenting" the preceding three equations by adding the lagged values of the dependent variable ΔY_t. For example, if we use the model random walk with drift around a stochastic trend. The ADF test consists of estimating the following regression:

$$\Delta Y_t = \beta_1 + \beta_2 t + \delta Y_{t-1} + \alpha_i \sum_{i=1}^{m} \Delta Y_{t-i} + \varepsilon_t \qquad (6.4)$$

[2] We rule out the possibility that $\delta > 0$,because in that case $\rho > 0$, in which case the underlying time series will be explosive. More technically, since (6.3) is a first-order difference equation, the so-called condition require that $|\rho| < 1$.

where $\varepsilon_t \sim i.i.d.\left(0, \sigma_\varepsilon^2\right)$ is a pure white noise error term and $\Delta Y_{t-1} = (Y_{t-1} - Y_{t-2})$ etc. In ADF we still test whether $\delta = 0$. Since the ADF test follows the same asymmetric distribution as the DF statistic, the same critical values can therefore be used.

According to equation(6.4), in order to apply the ADF, we must choose an appropriate lag length (m) for each variable. The criterion for choice of the appropriate lag length is to choose the lag-length (m), which minimizes Akaike Information Criterion (AIC):

$$AIC = \left(\frac{RSS}{T}\right) e^{(2k/T)}, \tag{6.5}$$

where RSS is the residual sum of squares, and T is the number of observations.

6.1.1.4 The Phillips-Perron (PP) Test

Moreover, due to the weakness of ADF test regarding the assumption of distribution of disturbance terms, Phillips-Perron unit root test, which is based on non-parametric approach, has been further implemented to cross-check whether ADF and PP of stationary test give the same consistent results. The Phillips-Perron test is carried out by estimating the following regression:

$$\Delta Y_t = \mu + \delta Y_{t-1} + \varepsilon_t . \tag{6.6}$$

The hypothesis testing is

$$H_0 : \delta = 0 \ (non-stationary),$$

$$H_a : \delta < 0 \ (stationary) . \tag{6.7}$$

6.1.1.5 The Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) Test

Based on unit root tests, these tests are set up using the unit root as the null hypothesis to be tested, Kwiatkowski et.al point out that these tests are not very powerful against the relevant alternative. Therefore, they propose new test (KPSS test) that set up the stationarity as the null hypothesis instead.

The KPSS test differs from the other unit root tests in that the series Y_t is assumed to be (trend-) stationary under the null hypothesis. The KPSS statistic is based on the residuals from the OLS regression of Y_t on the exogenous variable $\mathbf{X}_t$:

$$Y_t = \mathbf{X_t}'\boldsymbol{\delta} + u_t \,, \tag{6.8}$$

where $\mathbf{X}_t$ is the set of exogenous regressors and $\boldsymbol{\delta}$ is the set of coefficients.

The LM statistic is be defined as:

$$LM = \sum_t S(t)^2 / \left(T^2 f_o\right), \tag{6.9}$$

where f_o, is an estimator of the residual spectrum at frequency zero and where $S(t)$ is a cumulative residual function:

$$S(t) = \sum_{r=1}^{t} \hat{u}_r \,, \tag{6.10}$$

based on the residual $\hat{u}_t = Y_t - \mathbf{x}_t'\hat{\boldsymbol{\delta}}$.

It should be noted that the estimator of $\boldsymbol{\delta}$ used in this calculation differ from the estimator for $\boldsymbol{\delta}$ used by GLS detrending since it is based on a regression involving the origin data , and not on the quasi-differenced data.

6.1.2 Source of Data for Testing the Existence of Asset Price Bubble in Stock Market

In this part, we employ data for four variables, which are real SET index , real property stock index , the risk premium on SET index and property stock index, and the real rate of interest. All data are divided into three types of frequency data, which are yearly, quarterly, and monthly data, respectively.

For the real SET index and real property index, these variables are obtained from Stock Exchange of Thailand (SET). The data ranges for real SET index and property stock index are

1. The data range of yearly SET index is from 1975 to 2004.
2. The data range of quarterly SET index is from Q2:1975 to Q4:2004.
3. The data range of monthly SET index is from April: 1975 to December: 2004.
4. The data range of yearly property stock index is from 1988 to 2004.
5. The data range of quarterly property stock index is from Q2:1988 to Q4:2004.
6. The data range of monthly property stock index is from June: 1988 to December: 2004.

These variables are shown in real terms by deflating with consumer price index, which is obtained from International Financial Statistics (IFS)[3] (CPI, 2000=100).

For the risk premium, it is defined as the difference between return on risky asset and return on risk free assets. The return on SET index at period t is defined as the rate of change of SET index between period t and t-1 or

$$return_{SET}\ at\, t = \left[\left(\frac{SET\_INDEX_t - SET\_INDEX_{t-1}}{SET\_INDEX_{t-1}}\right)\times 100\right]. \qquad (6.11)$$

While the return on property stock index at period t is defined as the rate of change of property index between period t and t-1. Thus,

[3] The IFS is available on CD-ROM and the Internet.

$$return_{property}\ at\ t = \left[\left(\frac{PROPERTY\_INDEX_t - PROPERTY\_INDEX_{t-1}}{PROPERTY\_INDEX_{t-1}} \right) \times 100 \right]. \quad (6.12)$$

After we have already constructed the return on SET and property stock index, then, we can find the risk premium at any time t for SET and property stock index by the following equations:

$$risk\_premium_{SET}\ at\ t = return_{SET}\ at\ t - real\_rate\ of\ r\ at\ t,$$

$$risk\_premium_{property}\ at\ t = return_{property}\ at\ t - real\_rate\ of\ r\ at\ t. \quad (6.13)$$

where real rate of interest at time t is equal to deposit rate at time t – inflation at time t. These data are collected from International Financial Statistics (IFS).

6.2 The Analyzing of the Policy Simulations on the Building and Resale Options

One objective in this study is to analyze the effects of five policies on the optimal stopping time to develop land to be building and the resale option. These policies are

1. An increase in interest rate policy (r).
2. An increase in resale cost (c_2) policy.
3. An increase in government spending on the metropolitan transportation $(\bar{f})$ policy.
4. An increase in investors' confidence (ϕ) policy.
5. An increase in volatility of the noise in signals $(is = \frac{\sigma_f}{\sigma_s})$.

In order to study these effects, one must identify the optimal stopping time to develop land to be building. Therefore, in the following part, we present step by step the numerical and simulation techniques to study the policy simulations on building and resale options. We start with the financial technique employed to value the building option price and then follow by the explicit solution applied to value the resale option price. Finally we present the method to identify the optimal stopping time for exercising building option.

6.2.1 The Valuation of Building Option

In the previous chapter, we have already learned about the Finite Difference Method (FDM). We therefore employ it to value the price of building option in this chapter in order to identify the optimal stopping time to develop land to be building.

We begin with our main problem which is how to value building option price. From the model of property price with the heterogeneous beliefs, the land owner will develop land to be building when its value is equal to the immediate gain from developing land at that time. However, due to building option characteristics, which are similar to American option, the closed form solution rarely exists.

The price of building option therefore can be obtained by solving the partial differential equation with finite difference method. We begin with the partial differential equation for building option which is

$$L\left(P^{h,o}\left(\hat{f}_t^{o},g_t^{o}\right)\right)=\left[r-\left[\delta_f-\frac{\lambda\left(\overline{f}-\hat{f}_t^{o}\right)}{\hat{f}_t^{o}}\right]\right]\hat{f}_t^{o}\frac{\partial P^{h,o}\left(\hat{f}_t^{o},g_t^{o}\right)}{\partial \hat{f}_t^{o}}$$
$$+\left(r-\left[\delta_g-\rho\right]\right)g_t^{o}\frac{\partial P^{h,o}\left(\hat{f}_t^{o},g_t^{o}\right)}{\partial g_t^{o}}$$
$$+\frac{1}{2}\left(\left[\frac{\phi\sigma_s\sigma_f+\gamma}{\sigma_s}\right]^2+\left[\frac{\gamma}{\sigma_s}\right]^2+\left[\frac{\gamma}{\sigma_D}\right]^2\right)\frac{\partial^2 P^{h,o}\left(\hat{f}_t^{o},g_t^{o}\right)}{\left(\partial \hat{f}_t^{o}\right)^2}$$
$$+\rho_{\hat{f}g}\left(\sqrt{\left[\frac{\phi\sigma_s\sigma_f+\gamma}{\sigma_s}\right]^2+\left[\frac{\gamma}{\sigma_s}\right]^2+\left[\frac{\gamma}{\sigma_D}\right]^2}\right)\sigma_g\frac{\partial^2 P^{h,o}\left(\hat{f}_t^{o},g_t^{o}\right)}{\partial \hat{f}_t^{o}\partial g_t^{o}}$$
$$+\frac{1}{2}\sigma_g^2\frac{\partial^2 P^{h,o}\left(\hat{f}_t^{o},g_t^{o}\right)}{\left(\partial g_t^{o}\right)^2}-rP^{h,o}\left(\hat{f}_t^{o},g_t^{o}\right). \tag{6.14}$$

From equation(6.14), the price of building option can be obtained by solving a time dependent complementary problem which are

$$P^{h,o}\left(\hat{f}_t^{o},g_t^{o}\right)\geq\left[\left\{\frac{\overline{f}-R_a}{r}+\frac{\hat{f}_t^{o}-\overline{f}}{r+\lambda}-c_1\right\}+q^h\left(g_t^{o}\middle|\left(\hat{f}_t^{o},g_t^{o}\right)\in\mathbb{C}\right)\right], \tag{6.15}$$

$$\frac{\partial P^{h,o}\left(\hat{f}_t^{o},g_t^{o}\right)}{\partial t}+L\left(P^{h,o}\left(\hat{f}_t^{o},g_t^{o}\right)\right)\leq 0\,, \tag{6.16}$$

and

$$\left[\frac{\partial P^{h,o}\left(\hat{f}_t^{o},g_t^{o}\right)}{\partial t}+L\left(P^{h,o}\left(\hat{f}_t^{o},g_t^{o}\right)\right)\right]\times$$
$$\left[P^{h,o}\left(\hat{f}_t^{o},g_t^{o}\right)-\left[\left\{\frac{\overline{f}-R_a}{r}+\frac{\hat{f}_t^{o}-\overline{f}}{r+\lambda}-c_1\right\}+q^h\left(g_t^{o}\middle|\left(\hat{f}_t^{o},g_t^{o}\in\mathbb{C}\right)\right)\right]\right]=0. \tag{6.17}$$

In the next section, we then show how to solve such problem by using the Finite Difference Method (FDM).

6.2.1.1 The Finite Difference Method for Solving Building Option Price.

To find out solution to the building option pricing problem, we need to perform a numerical approximation of the two dimensional partial differential equation to equation (6.16) by using the Finite Difference Method (FDM).

The following 6 steps provide the details of how to apply Finite Difference Method to value the building option price.

Step 1: Changing unbound domain into a finite size

As stated in the previous chapter, we confront two variables with unbound domain. In order to employ finite difference approximations for space variables, one should firstly truncate these into a finite size computational domain which are

$$\left(\hat{f}^{o}, g^{o}, t\right) \in \left[0, \hat{f}^{o}_{\max}\right] \times \left[0, g^{o}_{\max}\right] \times \left[0, T\right], \tag{6.18}$$

where $\hat{f}^{o}_{\max}$ and $g^{o}_{\max}$ are sufficient large.

However, in our model the value of $g^{o}_{\max}$ can be a negative value for some periods. Thus, we should identify the domain of $g^{o}_{\max}$ which covers all possible values facing in our study.

To satisfy this, we then define our domain as

$$\left(\hat{f}^{o}, g^{o}, t\right) \in \left[0, \hat{f}^{o}_{\max}\right] \times \left[-g^{o}_{\max} + \Delta g^{o}, g^{o}_{\max}\right] \times \left[0, T\right]. \tag{6.19}$$

Step 2: Using a uniform space-time finite difference grid for the computational domain

After we have a finite size computational domain, the second task is to divide the domain using a uniform space-time finite difference grid.

We divide the $\hat{f}^0$-direction axis into $N_{\hat{f}^o}+1$ equally spaced points $\left\{i\Delta\hat{f}^o\right\}_{i=0}^{N_{\hat{f}^o}}$,the g^o-direction axis into $2N_{g^o}+1$[4] equally spaced points $\left\{j\Delta g^o\right\}_{j=0}^{2N_{g^o}}$ and the time axis into N_t+1 equally spaced points $\left\{k\Delta t\right\}_{k=0}^{N_t}$. The grid steps to these directions are denoted by

$$\Delta\hat{f}^o := \frac{\hat{f}^o_{\max}}{N_{\hat{f}^o}},\ \Delta g^o := \frac{g^o_{\max}}{N_{g^o}} \text{ ,and } \Delta t := \frac{T}{N_t}. \tag{6.20}$$

Therefore, the grid point values of a finite difference approximation are denoted by

$$U_{i,j}^{(k)} \approx U\left(\hat{f}_i^o, \hat{g}_j^o, t_k\right) = U\left(i\Delta\hat{f}^o, (-g_{\max} + j\Delta g^o), k\Delta t\right).^5 \tag{6.21}$$

Step 3: Space discretization

We apply the central finite difference schemes on space discretization and on a special approximation of the second-order cross-derivative term. From equation (6.16), we substitute equation (6.14) to obtain

---

[4]Because the domain of g^o can be a negative value, we therefore divide it into $2N_{g^o}+1$ equally spaced points $\left\{j\Delta g^o\right\}_{j=0}^{2N_{g^o}}$ in order to cover its domain.

[5] For example, $U_{1,1}^{(1)} \approx U\left(\hat{f}_1^o, \hat{g}_1^o, t_1\right) = U\left(\Delta\hat{f}^o, (-g_{\max} + \Delta g^o), \Delta t\right)$.

$$
\begin{aligned}
&\frac{\partial P^{h,o}\left(\hat{f}_t^o, g_t^o\right)}{\partial t}+\frac{1}{2}\left(\left[\frac{\phi\sigma_s\sigma_f+\gamma}{\sigma_s}\right]^2+\left[\frac{\gamma}{\sigma_s}\right]^2+\left[\frac{\gamma}{\sigma_D}\right]^2\right)\frac{\partial^2 P^{h,o}\left(\hat{f}_t^o, g_t^o\right)}{\left(\partial\hat{f}_t^o\right)^2}\\
&+\rho_{\hat{f}g}\left(\sqrt{\left[\frac{\phi\sigma_s\sigma_f+\gamma}{\sigma_s}\right]^2+\left[\frac{\gamma}{\sigma_s}\right]^2+\left[\frac{\gamma}{\sigma_D}\right]^2}\right)\sigma_g\frac{\partial^2 P^{h,o}\left(\hat{f}_t^o, g_t^o\right)}{\partial\hat{f}_t^o\partial g_t^o}\\
&+\frac{1}{2}\sigma_g^2\frac{\partial^2 P^{h,o}\left(\hat{f}_t^o, g_t^o\right)}{\left(\partial g_t^o\right)^2}\\
&+\left[r-\left[\delta_f-\frac{\lambda\left(\overline{f}-\hat{f}_t^o\right)}{\hat{f}_t^o}\right]\right]\hat{f}_t^o\frac{\partial P^{h,o}\left(\hat{f}_t^o, g_t^o\right)}{\partial\hat{f}_t^o}\\
&+\left(r-\left[\delta_g-\rho\right]\right)g_t^o\frac{\partial P^{h,o}\left(\hat{f}_t^o, g_t^o\right)}{\partial g_t^o}\\
&-rP^{h,o}\left(\hat{f}_t^o, g_t^o\right)=0.
\end{aligned}
\tag{6.22}
$$

The partial differential equation in equation (6.22) is now having the similar form of equation (5.4) in the chapter 5. As mentioned, after we approximate the partial differential equation by the semi-discrete equation, equation (5.4) has the form described in equation (5.24).

Therefore, to replace the coefficients of equation (5.24) by the coefficients of the building option equation in equation (6.22), what we need to do is to match the variables between equation (5.4) and (6.22). The results are reported in the table 6.2.

Table 6.2

The Comparison of the Parameters/Function between Equation (5.4) and (6.22)

Parameters/function in equation (5.4)	Parameters/function in equation (6.22)
$\frac{\partial^2 U}{\partial x^2}$	$\frac{\partial^2 P^{h,o}\left(\hat{f}_t^o, g_t^o\right)}{\left(\partial \hat{f}_t^o\right)^2}$
$\frac{\partial^2 U}{\partial x \partial y}$	$\frac{\partial^2 P^{h,o}\left(\hat{f}_t^o, g_t^o\right)}{\partial \hat{f}_t^o \partial g_t^o}$
$\frac{\partial^2 U}{\partial y^2}$	$\frac{\partial^2 P^{h,o}\left(\hat{f}_t^o, g_t^o\right)}{\left(\partial g_t^o\right)^2}$
$\frac{\partial U}{\partial x}$	$\frac{\partial P^{h,o}\left(\hat{f}_t^o, g_t^o\right)}{\partial \hat{f}_t^o}$
$\frac{\partial U}{\partial y}$	$\frac{\partial P^{h,o}\left(\hat{f}_t^o, g_t^o\right)}{\partial g_t^o}$
U	$P^{h,o}\left(\hat{f}_t^o, g_t^o\right)$
a	$\frac{1}{2}\left(\left[\frac{\phi\sigma_s\sigma_f+\gamma}{\sigma_s}\right]^2+\left[\frac{\gamma}{\sigma_s}\right]^2+\left[\frac{\gamma}{\sigma_D}\right]^2\right)$
b	$\rho_{\hat{f}g}\left(\sqrt{\left[\frac{\phi\sigma_s\sigma_f+\gamma}{\sigma_s}\right]^2+\left[\frac{\gamma}{\sigma_s}\right]^2+\left[\frac{\gamma}{\sigma_D}\right]^2}\right)\sigma_g$
c	$\frac{1}{2}\sigma_g^2$
d	$\left[r-\left[\delta_f-\frac{\lambda\left(\bar{f}-\hat{f}_t^o\right)}{\hat{f}_t^o}\right]\right]\hat{f}_t^o$
e	$\left(r-\left[\delta_g-\rho\right]\right)g_t^o$
f	$-r$

After we replace the coefficients, the space discretization scheme is changed to be

$$\frac{\partial P}{\partial t} + AP_{i-1,j-1} + BP_{i,j-1} + CP_{i-1,j} + DP_{i,j} + EP_{i+1,j} + FP_{i,j+1} + GP_{i+1,j+1} = 0, \qquad (6.23)$$

where

$$A = \frac{\rho\sqrt{\psi}\sigma_g}{2\Delta x \Delta y}$$

$$B = \left[\frac{1}{\Delta y^2}\left(\frac{1}{2}\sigma_g^2 - \frac{\rho\sqrt{\psi}\sigma_g \Delta y}{2\Delta x} \right) - \frac{\left(r - \left(\delta_g - \rho\right)\right) g_j^o}{2\Delta x} \right]$$

$$C = \left[\frac{1}{\Delta x^2}\left[\frac{\psi}{2} - \frac{\rho\sqrt{\psi}\sigma_g \Delta x}{2\Delta y} \right]\right] - \frac{\left[r - \left[\delta_f - \lambda \frac{\left(\bar{f} - \hat{f}_i^o\right)}{\hat{f}_i^o} \right]\right] \hat{f}_i^o}{2\Delta x}$$

$$D = \left[\frac{-2}{\Delta x^2}\left[\frac{\psi}{2} - \frac{\rho\sqrt{\psi}\sigma_g \Delta x}{2\Delta y} \right]\right] - \frac{2}{\Delta y^2}\left[\frac{\sigma_g^2}{2} - \frac{\rho\sqrt{\psi}\sigma_g \Delta y}{2\Delta x} \right] - \frac{\rho\sqrt{\psi}\sigma_g}{\Delta x \Delta y} - r$$

$$E = \left[\frac{1}{\Delta x^2}\left[\frac{\psi}{2} - \frac{\rho\sqrt{\psi}\sigma_g \Delta x}{2\Delta y} \right]\right] + \frac{\left[r - \left[\delta_f - \lambda \frac{\left(\bar{f} - \hat{f}_i^o\right)}{\hat{f}_i^o} \right]\right] \hat{f}_i^o}{2\Delta x}$$

$$F = \left[\frac{1}{\Delta y^2}\left[\frac{\sigma_g^2}{2} - \frac{\rho\sqrt{\psi}\sigma_g \Delta y}{2\Delta x} \right] + \frac{\left(r - \left(\delta_g - \rho\right)\right) g_j^o}{2\Delta y} \right]$$

$$G = \left[\frac{\rho\sqrt{\psi}\sigma_g}{2\Delta x \Delta y} \right]$$

for i=2,…, $N_x - 1$ and j=2,…, $N_y - 1$

where $\psi = \left[\frac{\phi\sigma_s\sigma_f + \gamma}{\sigma_s} \right]^2 + \left[\frac{\gamma}{\sigma_s} \right]^2 + \left[\frac{\gamma}{\sigma_D} \right]^2$

Step 4 : Boundary Condition

We start with the boundary condition in period T which is

$$P_{Amer}^{h,o}\left(i\Delta\hat{f}^{0}, j\Delta g^{o}, T\right) = \max\left\{\left[\left[\left\{\frac{\overline{f}-R_a}{r} + \frac{i\Delta\hat{f}^{0}-\overline{f}}{r+\lambda} - c_1\right\} + q^h\left((-g_{\max}^{o} + j\Delta g^{o})\middle|\left(\hat{f}_t^{o}, g_t^{o}\right)\in \mathbb{C}\, or\, \mathbb{S}\right)\right]\right], 0\right\}, \tag{6.24}$$

for $(\hat{f}_t^{o}, g_t^{o}) \in \left[0, \hat{f}_{\max}^{o}\right]\times\left[-g_{\max}^{o} + \Delta g^{o}, g_{\max}^{o}\right]$.

$$P(0, j\Delta g^{o}, t) = \max\left\{\left[\left[\left\{\frac{\overline{f}-R_a}{r} + \frac{-\overline{f}}{r+\lambda} - c_1\right\} + q^h\left((-g_{\max}^{o} + j\Delta g^{o})\middle|\left(\hat{f}_t^{o}, g_t^{o}\right)\in \mathbb{C}\, or\, \mathbb{S}\right)\right]\right], 0\right\}, \tag{6.25}$$

for $(g_t^{o}, t) \in \left[-g_{\max}^{o} + \Delta g^{o}, g_{\max}^{o}\right]\times\left[0, T\right]$.

$$P(0,0,t) = \max\left\{\left[\left[\left\{\frac{\overline{f}-R_a}{r} + \frac{-\overline{f}}{r+\lambda} - c_1\right\} + q^h\left(0\middle|\left(\hat{f}_t^{o}, g_t^{o}\right)\in \mathbb{C}\, or\, \mathbb{S}\right)\right]\right], 0\right\}, \tag{6.26}$$

for $t \in \left[0, T\right]$.

Let us consider the boundary at $\hat{f}^{o} = \hat{f}_{\max}^{o}$. The other boundary conditions on $g^{o} = g_{\max}^{o}$ can be done in the same way. At the grid point $(N_{\hat{f}^{o}}, j), j = 1,..., N_{g^{o}}$, the boundary condition is $\dfrac{\partial P(\hat{f}_{\max}^{o}, g_j, t)}{\partial\hat{f}^{0}} = 0$. We approximate this using the central finite difference operator and get

$$\Delta_{\hat{f}} P_{N_{\hat{f}^{o}}, j} = \frac{P_{N_{\hat{f}^{o}}+1, j} - P_{N_{\hat{f}^{o}}-1, j}}{2\Delta x} = 0. \tag{6.27}$$

From this, it follows that the fictitious grid point value $P_{N_{\hat{f}^o}+1,j}$ outside the computational domain has to be the same as the grid point value $P_{N_{\hat{f}^o}-1,j}$. We can also use this knowledge to eliminate all fictitious grid point values $P_{N_{\hat{f}^o}+1,j}$, appearing in the stencil (6.23).

The space discretization can also written in the semi-discrete equation which has the following matrix representation

$$\frac{\partial U}{\partial t} i + \mathbf{M1U} + \mathbf{M2} = \mathbf{0}. \tag{6.28}$$

Step5: Time discretization

The next step is to approximate the first-order time derivative. There are many methods to do this as we explain in the previous chapter.

In this study, we will use first-order accurate implicit Euler which has the following form

$$\left(\frac{1}{\Delta t}\mathbf{I} + \mathbf{M1}\right)\mathbf{u}^{(k)} = \left(\frac{1}{\Delta t}\mathbf{I}\right)\mathbf{u}^{(\mathbf{k}-1)} - \mathbf{M2} \quad \text{for} \quad k = 1, 2, \ldots, l. \tag{6.29}$$

Step 6: Solving equation (6.29) **by using LU decomposition**

We can obtain the solution of equation (6.29) by backward solving from period T,T-1,…,0.

For period T:

We assume firstly that the land owner will develop land to be building at period (T). The value of the building option at time T, therefore, equals to the immediate gain at time T which is calculated from the right hand side of equation (6.15) . Thus,

$$P^{h,o}\left(i\Delta\hat{f}^{0}, j\Delta g^{o}, T\right) = \max\left\{\left[\left\{\frac{\bar{f}-R_a}{r} + \frac{i\Delta\hat{f}^{0}-\bar{f}}{r+\lambda} - c_1\right\} + q^h\left((-g^{o}_{\max} + j\Delta g^{o})\middle|\left(\hat{f}^{o}_t, g^{o}_t\right) \in \mathbb{C} or \mathbb{S}\right)\right], 0\right\}$$

(6.30)

for $\forall i = 1, 2..., N_{\hat{f}^o}$ and $\forall j = 1, 2, ..., 2N_{g^o}$.

For period T-1:

We can obtain the value of building option $P^{h,o}_{PDE}(i\Delta\hat{f}^{o}, j\Delta g^{o})$ at time T-1 by solving equation (6.29) with LU decomposition. However, the land owner may exercise at this period (T-1) if the immediate gain is equal to the value of building option.

Hence, the value of building option, in the case where the land owner may exercise at time T-1, is equal to

$$P^{h,o}\left(i\Delta\hat{f}^{0}, j\Delta g^{o}, T-1\right) =$$

$$\max\left\{\max(\left[\left\{\frac{\bar{f}-R_a}{r} + \frac{i\Delta\hat{f}^{0}-\bar{f}}{r+\lambda} - c_1\right\} + q^h\left((-g^{o}_{\max} + j\Delta g^{o}\middle|\left(\hat{f}^{o}_t, g^{o}_t\right) \in \mathbb{C} or \mathbb{S}\right)\right], 0), P^{h,o}_{PDE}\left(i\Delta\hat{f}^{0}, j\Delta g^{o}, T-1\right)\right\}$$

for $\forall i = 1, 2..., N_{\hat{f}^o}$ and $\forall j = 1, 2, ..., 2N_{g^o}$.

(6.31)

For period T-2, T-3,…, 0 , we can identify the value of building option by using the same process as in period T-1.

However, in order to calculate the immediate gain from developing land to be building, one should know the value of resale option. Therefore, in the following section, we will present how to determine the value of resale option.

6.2.2 The Valuation of Resale Option

Since each agent in group o will develop land to be building at any time t if and only if the value of building option is equal to the immediate gain at that time, we therefore should know the value of immediate gain in order to compare its value with the value of building option at each period.

The value of immediate gain at each period is composed of two parts which are $\left\{\frac{\overline{f}-R_a}{r}+\frac{\hat{f}_t^o-\overline{f}}{r+\lambda}-c_1\right\}$ and $q^h\left(g_t^o \middle| \left(\hat{f}_t^o, g_t^o\right)\right)$.

The second part is called resale option value and we can obtain its value by applying the study of Scheinkman and Xiong (2003). The value of resale option in the case immediate gain can be obtained by using equation (4.39) which is

$$q^h\left(x \middle| \left(\hat{f}_t^o, x\right) \in \mathbb{S}\right) = \begin{cases} \dfrac{b}{h(-k^*)}h(x) & for\ \ (\hat{f}_t^o, x) \in \mathbb{S}, and\ \ x < k^* \\ \\ \dfrac{x}{r+\lambda}+\dfrac{b}{h(-k^*)}h(-x)-c_2 & for(\hat{f}_t^o, x) \in \mathbb{S}, and\ \ x > k^* \end{cases} \tag{6.32}$$

where $b \equiv q(-k^*) = \frac{1}{r+\lambda}\frac{h(-k^*)}{h'(k^*)+h'(-k^*)}$ which is referred as the bubble size and $h(x)$ is defined as

$$h(x) = \begin{cases} U\left(\dfrac{r}{2\rho}, \dfrac{1}{2}, \dfrac{\rho}{\sigma_g^2}\right) & if\quad x \le 0 \\ \\ \dfrac{2\pi}{\Gamma\left(\dfrac{1}{2}+\left(\dfrac{r}{2\rho}\right)\right)\Gamma\left(\dfrac{1}{2}\right)} M\left(\dfrac{r}{2\rho}, \dfrac{1}{2}, \dfrac{\rho}{\sigma_g^2}x^2\right) - U\left(\dfrac{r}{2\rho}, \dfrac{1}{2}, \dfrac{\rho}{\sigma_g^2}\right) & if\quad x > 0 \end{cases}$$

(6.33)

where

$M(\bullet,\bullet,\bullet)$ *and* $U(\bullet,\bullet,\bullet)$ are Kummer functions and define as

$$M(a,b,y) = 1 + \frac{ay}{b} + \frac{(a)_2}{b} + \ldots + \frac{(a)_n y^n}{(b)_n n!} + \ldots, \tag{6.34}$$

with $(a)_n$ is the Pochhammer symbol and define as

$$(a)_n = a(a+1)(a+2)\ldots(a+n-1) \text{ and } (a)_0 = 1. \tag{6.35}$$

For function $U(a,b,y)$, we define it as

$$U(a,b,y) = \frac{\pi}{\sin \pi b}\left[\frac{M(a,b,y)}{\Gamma(1+a-b)\Gamma(b)} - y^{1-b}\frac{M(1+a-b,2-b,y)}{\Gamma(a)\Gamma(2-b)}\right], \tag{6.36}$$

where $\Gamma(\bullet)$ is the gamma function.

The function $h(x)$ is positive and increasing in $(-\infty,0)$. Any solution $u(x)$ that is strictly positive and increasing in $(-\infty,0)$ must satisfy $u(x) = \beta_1 h(x)$ with $\beta_1 > 0$

From the lemma, for each $x \in \mathbb{R}$, $h(x) > 0, h'(x) > 0, h''(x) > 0, h'''(x) > 0$, $\lim_{x\to-\infty} h(x) = 0$, and $\lim_{x\to-\infty} h'(x) = 0$.

and k^* satisfies

$$\left[k^* - c_2(r+\lambda)\right]\left[h'(k^*) + h'(-k^*)\right] - h(k^*) + h(-k^*) = 0. \tag{6.37}$$

For each c_2, there exists a unique real solution pair $\left(k^*,b\right)$. Moreover, the function q constructed above is an equilibrium option value function. The optimal policy consists of exercising immediate if $g_t^o > k^*$; otherwise wait until the first time in which $g^o \geq k^*$.

Therefore, the process g_t^o will have the values in $\left(-\infty, k^*\right)$ where the value k^* acts as a barrier, and when g^o reaches k^*, a trade occurs, the owner's group switches, and the process is reoccurred at $-k^*$. The function $q\left(g^o\right)$ is the difference between the current owner's demand price and his fundamental valuation and can be legitimately called a bubble. When a trade occurs, this difference is

$$b \equiv q\left(-k^*\right) = \frac{1}{r+\lambda} \frac{h(-k^*)}{\left[h'\left(k^*\right)+h'\left(-k^*\right)\right]} . \tag{6.38}$$

When we study the policy simulations, these policies are not only effect the building option, but also effect the resale option. Therefore, to analyze these effects, we will concentrate on the following points:

6.2.2.1 The Trading Barrier $\left(k^*\right)$

As we have already known that the trading barrier is the minimum amount of difference in opinions that generates a trade. When the five policies in this study change, it will affect the value of trading barrier.

The value of trading barrier can be obtained by solving the following equation:

$$\left[k^* - c_2\left(r+\lambda\right)\right]\left[h'\left(k^*\right)+h'\left(-k^*\right)\right] - h\left(k^*\right) + h\left(-k^*\right) = 0 . \tag{6.39}$$

6.2.2.2 Duration between trades (τ)

From Scheinkman and Xiong (2003), they let $w(x,k,r) = E^{o}\left[e^{-r\tau(x,k)} \middle| x\right]$, with $\tau(x,k) = \inf\left\{s : g^{o}_{t+s} > k\right\}$ given $g^{o}_{t} = x \le k$. The term $w(x,k,r)$ is the discount factor applied to cash flows received the first time the difference in beliefs reaches the level of k given that the current difference in beliefs is x . Using proposition 2 in Scheinkman and Xiong, we have

$$w(x,k,r) = \frac{h(x)}{h(k)}. \tag{6.40}$$

Since w is the moment-generating function of τ ,

$$E\left[\tau\left(-k^{*},k^{*}\right)\right] = -\frac{\partial w\left(-k^{*},k^{*},r\right)}{\partial r}\Big|_{r=0}. \tag{6.41}$$

Equation (6.41) shows that when $c = 0$, the expected duration between trades is zero.

6.2.2.3 An Extra Volatility Component (η)

An extra source of price volatility comes from the option component. From Scheinkman and Xiong paper in proposition 3, the volatility from the option value component can be calculated from the following equation:

$$\eta(x) = \frac{\sqrt{2}\phi\sigma_{f}}{r+\lambda}\frac{h'(x)}{h'\left(k^{*}\right)+h'\left(-k^{*}\right)} \quad \forall x < k \tag{6.42}$$

6.2.2.4 The Bubble Size (b)

The function $q\left(g^{o}\right)$ is the difference between the current owner's demand price and his fundamental valuation and can be legitimately called a bubble. When the trade occurs, this difference is

$$b \equiv q\left(-k^{*}\right)=\frac{1}{r+\lambda} \frac{h\left(-k^{*}\right)}{\left(h'\left(k^{*}\right)+h'\left(-k^{*}\right)\right)}. \tag{6.43}$$

6.2.3 The Valuation of Optimal Stopping Time to Develop Land to be Building

When we have already known the value of building option from solving the partial differential equation with the Finite Difference Method (FDM), we can also know whether it is the optimal stopping time to develop land to be building or not in each period by comparing the value of building option with the value of immediate gain from developing land at that time. If the value of building option from solving partial differential equation is equal to the value of immediate gain, the owner will decide to develop land. On the other hand, if it is not, the owner will wait until the first period which is the value of building option is equal to the value of immediate gain.

However, in order to compare them, we should know the value of the conditional mean of the beliefs of agents in group A which is the land owners and the conditional mean of the beliefs of agents in group B. We can obtain these values by using the Monte Carlo simulation method.

From chapter 4, the conditional mean of the beliefs of agents in group A and B can be presented by the three-dimensional Brownian motions which are

$$d\hat{f}^{A} = -\lambda(\hat{f}^{A} - \overline{f})dt + \frac{\phi\sigma_s\sigma_f + \gamma}{\sigma_s^2}(ds^A - \hat{f}^{A}dt) + \frac{\gamma}{\sigma_s^2}(ds^B - \hat{f}^{A}dt)$$
$$+ \frac{\gamma}{\sigma_D^2}(dD - \hat{f}^{A}dt) \tag{6.44}$$

where

$$dW_t^{A,A} = \frac{1}{\sigma_s}(ds_t^A - \hat{f}_t^{A}dt), \tag{6.45}$$

$$dW_t^{A,B} = \frac{1}{\sigma_s}(ds_t^B - \hat{f}_t^{A}dt), \tag{6.46}$$

$$dW^{A,D} = \frac{1}{\sigma_D}\left(dD_t - \hat{f}_t^{A}dt\right). \tag{6.47}$$

$$d\hat{f}_t^{B} = -\lambda(\hat{f}_t^{B} - \overline{f})dt + \frac{\gamma}{\sigma_s^2}(ds_t^A - \hat{f}_t^{B}dt) + \frac{\phi\sigma_s\sigma_f + \gamma}{\sigma_s^2}(ds_t^B - \hat{f}_t^{B}dt) +$$
$$\frac{\gamma}{\sigma_D^2}(dD_t - \hat{f}_t^{B}dt) \tag{6.48}$$

where

$$dW_t^{B,A} = \frac{1}{\sigma_s}(ds_t^A - \hat{f}_t^{B}dt), \tag{6.49}$$

$$dW_t^{B,B} = \frac{1}{\sigma_s}(ds_t^B - \hat{f}_t^{B}dt), \tag{6.50}$$

$$dW^{B,D} = \frac{1}{\sigma_D}\left(dD_t - \hat{f}_t^{B}dt\right), \tag{6.51}$$

To simulate these paths which are the conditional mean of the beliefs of the agents in group A and B, we must firstly discretize time with the time step Δt.

For the term (6.45), (6.46), (6.47),(6.49), (6.50), and (6.51) , it is the standard Wiener process. Therefore, each term can be rewritten as $\sqrt{\Delta t} \times \varepsilon_t$ where $\varepsilon_t \sim N(0,1)$.

Next, we will present the processes to generate 15,000 paths of the conditional mean of the beliefs of the agents in group A and B and 15,000 paths of the difference in beliefs.

Step 1: At period 1, we set the initial value of the conditional mean of the beliefs of the agents in group A and B.

Step 2: At period2, we can obtain the value of the conditional mean of the beliefs of the agents in group A and B by simulating the equation (6.44) and (6.48) using Monte Carlo technique. After we get $d\hat{f}_1$ and $d\hat{f}_2$, we add these values to the initial values and get the conditional mean of the beliefs of the agents in period 2.

Step 3: For period 3, 4,…,T, we can find the conditional mean of the beliefs of two groups by repeating step (2). Finally, we will obtain 1 path for conditional mean of the beliefs of the agents in group A and 1 path for conditional mean of the beliefs of the agents in group B.

Step 4: We can find the path of difference in beliefs $\left(g^A\right)$ by subtracting the conditional mean of the beliefs of agents in group A from the conditional mean of the beliefs of agents in group B.

Step 5: We repeat steps (1), (2), (3), and (4) for 15,000 times so that we have 15,000 possible paths of the conditional mean of the beliefs of agents in group A and group B and 15,000 possible paths of the difference in beliefs.

Next, we identify the optimal stopping time to develop land to be building by following steps.

Step 1: We solve the partial differential equation in equation (6.22) in order to obtain the value of building option by using the Finite Difference Method (FDM) which is presented in section 6.2.1.1.

Step 2: We pick the first two paths of the conditional mean of the beliefs of agents in group A $\left(\hat{f}_A\right)$ (the land owner) and the difference in beliefs $\left(g^A\right)$ from 15,000 paths. These first two paths compose of the values of the conditional mean of the beliefs of agents in group A and the difference in beliefs for time $\tau = 1, 2, 3, \ldots, T$.

Step 3: We use values of the conditional mean of the beliefs of agents in group A and the difference in beliefs for time $\tau = 1$ to find the value of building option at time

$(\tau = 1)$ from the value of building option which has already solved from step1. In order to conclude whether it is the optimal stopping time to develop land to be building at period $(\tau = 1)$ or not, we should compare its building value with the value of immediate gain which can be obtained by substituting values of the conditional mean of the beliefs of agents in group A and the differences in beliefs at time $\tau = 1$ into the right hand side of equation (6.15). Then, we compare the value of building option and value of immediate gain. If value of building option is greater than the value of immediate gain, it means that it is not the optimal time to develop land to be building at time $\tau = 1$. Hence, we then repeat this process for next period. We do this process until we find out the optimal stopping time which is the period that value of building option is equal to value of immediate gain.

Step 4: We repeat steps (2) and (3) for paths 2, 3,…, 15,000 of the conditional mean of the beliefs of agents in group A $\left(\hat{f}_A\right)$ (the land owner) and the difference in beliefs $\left(g^A\right)$.

Step 5: Once we find all optimal stopping times for 15,000 paths of the conditional mean of the beliefs of agents in group A $\left(\hat{f}_A\right)$ and the difference in beliefs $\left(g^A\right)$, we then use these optimal stopping times to find the probability density function for the optimal stopping times in order to analyze the policy simulations.

6.2.4 Source of Data and Parameter Values for Analyzing of the Policy Simulations on the Building and Resale Options

As a final point, we are now identifying the values of all parameters used in the policy simulations for benchmark case. At first, we would like to construct values of all parameters based on Thailand' property market. However, we found out that these data are not available for Thailand. To continue our study, we therefore employ U.S. housing data from Davis Morris's paper[6]. Also, the Treasury bill rate and

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[6] All the housing data are come from paper "the rent-price ratio for the aggregate stock of owner-occupied housing. It is downloadable from http://morris.marginalq.com/2005-05-DLM_paper.pdf.

consumer price index are available in IFS data base. All parameter values are deflated by consumer price index.

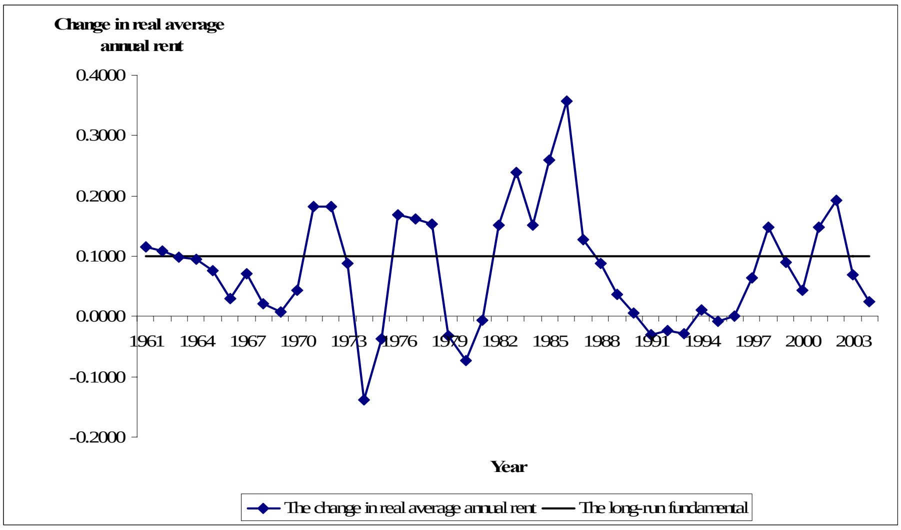
1. The volatility of the rental rate, (σ_D), can be calculated from the second moment of the real average annual rents. The real average annual rents are presented in the column 8 of table B1 in appendix B. Based on our calculation, we approximate it equals 1.052.
2. The mean reversion parameter of rental rate , (λ), is equal to 0 or 0.01.
3. The long-run mean of fundamental , $(\bar{f})$, is equal to 0.1. We approximate this value from the historical change in real average annual rent. From our model in chapter 3, the change in rental rate satisfies the following Brownian process:

$$dD_t = f_t dt + \sigma_D dZ_t^D \tag{6.52}$$

We therefore investigate this process by plotting the change in real average annual rent with respect to time. The result is presented in figure 6.1.

Figure 6.1

The Movement of the Change in Real Average Annual Rent and the Long-Run Fundamental from 1961-2004



From figure 6.1, we find out that even though the change in real average annual rent in each year quite fluctuates, its movements are around the value 0.1. Based on this fact ,we therefore assume the value 0.1 to represent the long-run fundamental.

4. The volatility of the fundamental, (σ_f), is derived from the second moment of the change in real average annual rent represented in column 10 of table B1 in appendix B and equals 0.096.
5. The volatility of signal, (σ_s), is assumed to be 0.05.
6. The overconfidence parameter, (ϕ), is equal to 0.216 and 0.9 for the low and high overconfidence cases, respectively.
7. The building cost, (c_1), is assumed to be 20.
8. The resale cost, (c_2), is equal to 1.289 by assuming it as 0.01% of real average house price.
9. The terminal date, (T), is equal to 40.
10. The maximum of the conditional mean of the beliefs of agents in group A , ($\hat{f}^{o}_{\max}$), is equal to 0.3.
11. The maximum of the difference in beliefs , ($g_{\max}$), is equal to 0.7.
12. Real interest rate for each time t is calculated from the U.S. Treasury bill rate minus inflation at that time. We approximate it as 0.013% by calculating the first moment of real interest rates over our sample size.

The following table summarizes and compares the parameters in which we apply for analyzing the policy implications.

Table 6.3

Parameter Value for Analyzing the Policy Simulations

Parameter	Definition	Case study	
		Low Overconfidence	High Overconfidence
(σ_D)	The volatility of the rental rate	1.052	1.052
(λ)	The mean reversion parameter of rental rate	0/0.01	0/0.01
$(\bar{f})$	The long-run mean of fundamental	0.1	0.1
(σ_f)	The volatility of the fundamental	0.096	0.096
(σ_s)	The volatility of signal	0.05	0.05
(ϕ)	The overconfidence parameter	0.216	0.9
c_1	The building cost	20	20
c_2	The resale cost	1.289	1.289
T	The terminal date	40	40
$\hat{f}^{o}_{\max}$	The maximum of the conditional mean of the beliefs of agents in group A	0.3	0.3
$g_{\max}$	The maximum of the differences in beliefs $g_{\max}$	0.7	0.7
r	The real interest rate	0.013	0.013

Noted:

1. We apply the parameter values in the case of high overconfidence level for simulating in the following cases: 1. An increase in interest rate policy and 2. An increase in resale cost policy.
2. We apply the parameter values in the case of low overconfidence level for simulating in the following cases: 1. An increase in government spending on transportation policy, 2. An increase in overconfidence level, and 3. An increase in information in signals.

CHAPTER 7

ECONOMETRIC AND SIMULATION RESULTS

This chapter presents the estimated and simulated results including their interpretations with regard to the methodology mentioned in the previous chapter. We divide this chapter into two sections. In the first section, in order to examine whether it had the rational bubble in the stock market or not, we apply the econometric tests as we state in the chapter 6. In the second section which discusses the policy simulations on the resale and building options consists of two parts. In the first part, we investigate the effects of five policies on the resale option by employing the explicit solution provided by Scheinkman and Xiong. In the second part, we present the simulation results of the five policies on the building option in order to analyze its effects on the optimal stopping time to develop land to be building. For comparable and observable purposes, we present these results by using the probability density function of the optimal stopping time to develop land to be building.

7.1 The Rational Bubble in the Stock Market

Without complete property price data in Thailand, we can not directly test the rational bubble in property market. However, many evidences[1] show that property price bubble is congenitally procyclical with equity price bubble. Given these deficiencies, it is constructive to supplement these data with information from stock market index. Therefore, we test the rational bubble by using SET index and stock market index for the property subsector. However, it should be noted that such data must be interpreted with care and can only be rough proxy for property price.

[1]For example, Aswin et.al (2003) find out that the peak in equity prices in Thailand tends to lead commercial and real estate prices by 1-2 years.

We examine the stationary of the first differences of the real SET index and real property stock index by employing the Augmented Dickey-Fuller (ADF) test, the Phillips-Perron (PP) test, and the Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) test. In order to check the robustness of the data series, we therefore test for two types of data series which are quarterly and monthly data.

It is useful to consider firstly the nature of the data series which are represented by figure 7.1. As shown in panel A ,B,C, and D, the real SET index, the real property stock index, the real interest rate, and the risk premium should be tested a stationary test by performing without time trend.

7.1.1 The Stationary Tests for Real SET Price

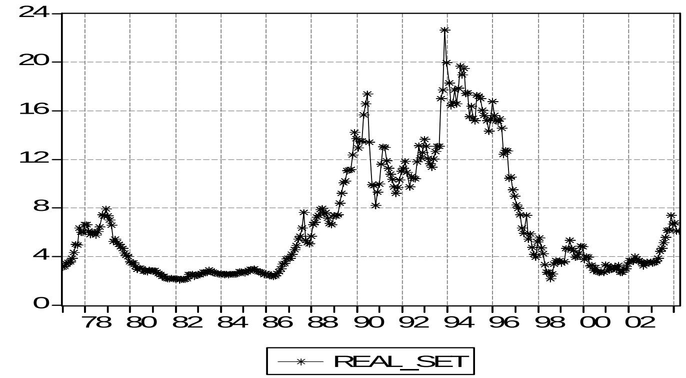
We begin with the stationary tests for real SET index. Before we analyze a sufficient condition for the absence of rational bubble, we should firstly test the stationary of the real interest rate and the risk premium. We divide the data range into four cases which are 1. Quarterly data from 1977:Q1- 2004:Q4, 2. Monthly data from 1977:M01-2004:M12, 3. Quarterly data from 1986:Q1-1996:Q4, and 4. Monthly data from 1986:M01-1996:M12.

Case 1: Quarterly Data from 1977:Q1- 2004:Q4

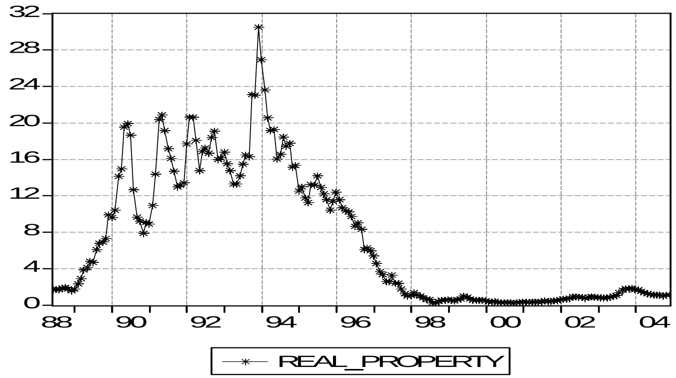
The results of the ADF, PP, and KPSS tests for the stationary of the real SET index are presented in the panel A of table 7.1. For the quarterly data from 1977:Q1-2004:Q4, we find out that the real interest rate is statistically stationary at 99% confidence interval when we use PP and KPSS tests. These results are shown in row 1 of panel A in table 7.1. However, the conclusion differs when we apply the ADF test. For the risk premium on SET index, it is statistically stationary at 99% confidence interval for all three tests as shown in row 5 of panel A in table 7.1. Armed with these tests, we can conclude that the real interest rate and the risk premium on SET index are stationary.

Figure 7.1

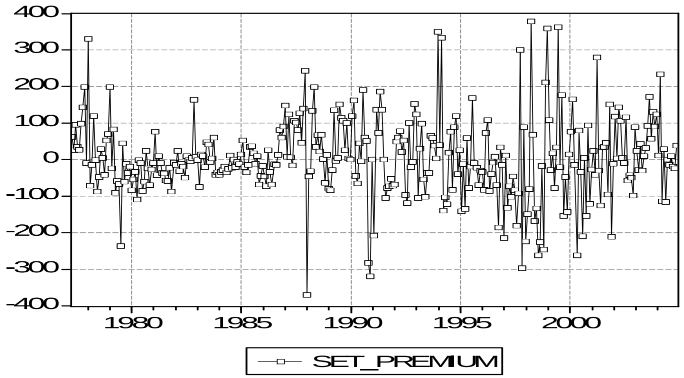
The Nature of the Data Series



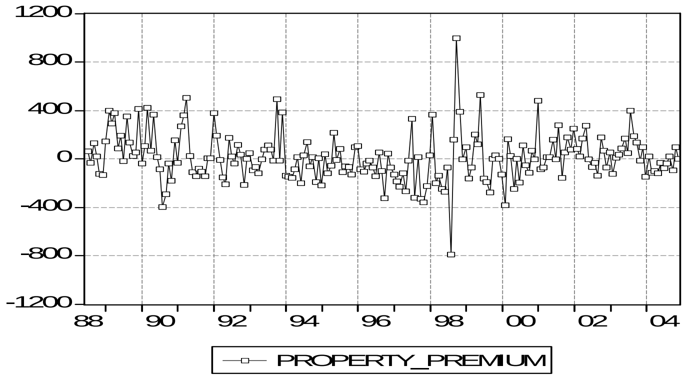
Panel A: Real SET Index



Panel B: Real Property Index



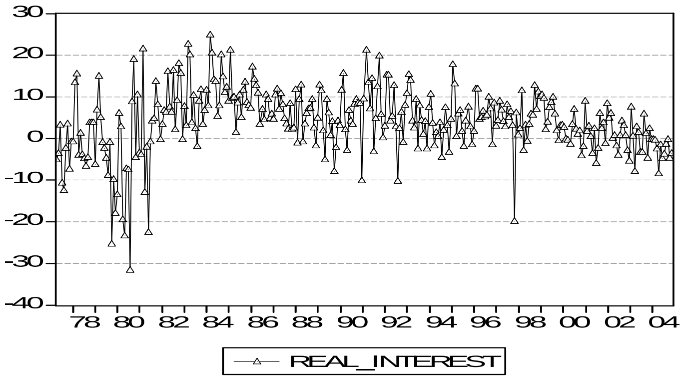
Panel C: Risk Premium on SET index



Panel D: Risk Premium on Property Index

Figure 7.1

The Nature of the Data Series (Continued)



Panel E: Real Interest Rate

Next, we test the real SET index which is the proxy of equity price. The results from ADF and PP tests in row 9 of panel A in table 7.1 show that the real SET index is not statistically stationary at 99%. However, when KPSS is applied, it shows that the real SET index is stationary.

We further employ ADF and PP tests again on the first difference of real SET index. The results in row 13 of panel A in table 7.1 show that the first difference of real SET index is statistically stationary at 99% confidence level.

According to these results, we therefore conclude that there was no rational bubble in stock market when we apply quarterly data from 1977:Q1-2004:Q4.

Case 2: Monthly Data from 1977:M01-2004:M12

In order to check the robustness of the data, we then retest the stationary of the real SET index again by applying monthly data from 1977:M01-2004:M12.

Firstly, we test the stationary of the real interest rate and the risk premium on SET index. The results are shown in row 2 and row 6 of panel A in table 7.1, respectively. For the real interest rate, it has the similar results as the previous case. PP and KPSS tests show that it is statistically stationary at 99% confidence interval. However, real interest rate can not reject the null hypothesis ($H_o:$ real interest rate is stationary) when we apply the ADF test. For risk premium on SET index, all tests show that it is statistically stationary at 99% confidence interval.

As a result, we summarize that the real rate of interest and risk premium on SET index are stationary.

When we employ these three tests on the real SET index where the results are presented in row 10 of panel A in table 7.1, we find out that two of three tests (ADF and PP tests) show that it is not statistically stationary at 99% confidence interval. We then test on the first difference of real SET index by using ADF and PP tests. The results presented in row 14 of panel A in table 7.1 show that the first difference of real SET index is stationary at 99% confidence interval.

As same as the previous conclusion, the rational bubble did not occur in stock market when we apply the monthly data from 1977:M01-2004:M12.

Due to our data range, one may argue that it may have the effects from two structural changes which are (1) drastically structural imparities of Thai stock market before and after 1986 and (2) the structural changes from the financial crisis in 1997.

Therefore, in order to avoid these effects, we then re-examine the stationary of real SET index by using the quarterly and monthly data within the range 1986-1996.

Case 3: Quarterly Data from 1986:Q1-1996:Q4

In this case, we test the stationary of real SET index by using the quarterly data from 1986:Q1 to 1996:Q4. The results are presented in panel A of table 7.1.

We initially test the stationary of real rate of interest and risk premium on SET index. The results presented in row 3 and 7 in panel A of table 7.1 confirm that real rate of interest and risk premium on SET index are statistically stationary at 99% confidence interval.

After we satisfy the condition of the stationary of real rate of interest and risk premium on SET index, we then test the stationary of real SET index. The results in row 11 of panel A in table 7.1 show that real SET index is not stationary at 99% confidence interval for ADF and PP tests and 95% confidence interval for KPSS test. Due to these results, we test the stationary of difference in SET index again. The results in row 15 of panel A in table 7.1 show that the difference in SET index is now statistically stationary for all three tests.

In sum, these results show that the sufficient condition for the absence of rational bubbles is satisfied by SET index data. Hence, we can conclude that Thailand’s stock prices did not contain manifest rational bubble during the period examined.

Case 4: Monthly Data from 1986:M01-1996:M12

Lastly, we test the stationary of real SET index by using monthly data from 1986:M01-1996:M12. The results in row 4 of panel A of table 7.1 show that the real interest rate is statistically stationary when we use PP and KPSS tests. However, it is not stationary when we apply the ADF test. For the risk premium in SET index, it is stationary when we use ADF and PP tests but not for KPSS test.

Obviously, even though the results are not consistent, we can also conclude that the real interest rate and risk premium on SET index are stationary by summarizing from two of three tests.

Next, we test the stationary of real SET index. The results are shown in row 12 of panel A in table 7.1. It is clearly see that the real SET index is not statistically stationary for all three tests at 99% confidence interval.

Hence, we then test the stationary of the first difference of real SET index. As shown in row 16 of panel A in table 7.1, all three tests show that the first difference of real SET index is statistically stationary at 99% confidence interval.

In a nutshell, we then conclude that there was no rational bubble in Thailand's stock prices during the period examined.

7.1.2 The Stationary Tests for Real Property Stock Price

According to the three boom and bust cycles in Thailand's property market, it is interesting to investigate whether it had the rational bubble in the property market or not. However, evidence on property prices is much less readily available. We thus supplement these data by using real property stock index to be a rough proxy for real estate prices.

We divide the data range into 4 cases which are (1) Quarterly data from 1988:Q2- 2004:Q4, (2) Monthly data from 1988:M06-2004:M12 (3) Quarterly data from 1988:Q2-1996:Q4, and (4) Monthly data from 1988:M06-1996:M12.

Case 1: Quarterly Data from 1988:Q2- 2004:Q4

Based on Quarterly data from 1988:Q2-2004:Q4, we firstly test the stationary for real interest rate and risk premium in property stock index. These results are provided in row 1 and 5 of panel B in table 7.1. For the real interest rate, only PP test shows that the real interest rate is statistically stationary at 99% confidence interval.

Table 7.1

Results of the Stationary Tests

Panel A: The Stationary Tests for the Real SET Index

Variable	Type of Data	Range	The Stationary Test		
			(1) ADF	(2) PP	(3) KPSS
Real interest rate	Quarterly	1977:Q1-2004:Q4	-2.1691	-5.1272*	0.2075
	Monthly	1977:M01-2004:M12	-2.25	-13.7431*	0.3152
	Quarterly	1986:Q1-1996Q4	-3.7034*	-5.7861*	0.2609
	Monthly	1986:M01-1996:M12	-2.2921	-8.8639*	0.3349
Risk premium on SET index	Quarterly	1977:Q1-2004:Q4	-10.8773*	-10.9069*	0.0746
	Monthly	1977:M01-2004:M12	-9.3526*	-16.5945*	0.1012
	Quarterly	1986:Q1-1996Q4	-7.827*	-7.3378*	0.4406
	Monthly	1986:M01-1996:M12	-9.7205*	-9.5863*	0.2422*
Real SET index	Quarterly	1977:Q1-2004:Q4	-2.0269	-1.9346	0.25
	Monthly	1977:M01-2004:M12	-1.9071	-1.7293	0.438
	Quarterly	1986:Q1-1996Q4	-1.7579	-2.0617	0.6681**
	Monthly	1986:M01-1996:M12	-1.7416	-1.886	1.1049*
D(real SET index)	Quarterly	1977:Q1-2004:Q4	-3.5909*	-12.8613*	0.1082
	Monthly	1977:M01-2004:M12	-4.7158*	-16.6763*	0.1115
	Quarterly	1986:Q1-1996Q4	-6.5613*	-8.6636*	0.1319
	Monthly	1986:M01-1996:M12	-6.4733*	-10.1982*	0.2801

Note:

1. ADF and PP test are based on the null hypothesis that the tested data series has a unit root. For, KPSS test is based on the null hypothesis that the tested data is stationary.
2. For quarterly data from 1977:Q1-2004Q4, the 99% and 95% of Mackinnon critical value are -3.4931 and -2.8889, respectively.
   For quarterly data from 1986:Q1-1996:Q4, the 99% and 95% of Mackinnon critical value are -3.5885 and -2.9297, respectively.
   For monthly data from 1977:01-2004:12, the 99%, and 95% of Mackinnon critical value are -3.4505 and -2.8703, respectively.
   For monthly data from 1986:01-1996:12, the 99%, and 95% of Mackinnon critical value are -3.4804 and -2.8834, respectively.
3. For KPSS test, the 99% and 95% of Kwiatkowski-Phillips-Schmidt-Shin value are 0.739 and 0.463, respectively.
4. The asterisk (*) and (**) represent significance at 1% and 5% level, respectively

Panel B: The Stationary Tests for the Real Property Stock Index

Variable	Type of Data	Range	The Stationary Test		
			(1) ADF	**(2) PP**	**(3) KPSS**
Real interest rate	Quarterly	1988:Q2-2004:Q4	-2.8208	-4.4905*	0.8016*
	Monthly	1988:M06-2004:M12	-1.532	-10.27859*	1.1826*
	Quarterly	1988:Q2-1996Q4	-5.4426*	-5.4426*	0.1391
	Monthly	1988:M06-1996:M12	-8.7895*	-7.8216*	0.0966
Risk premium on property stock index	Quarterly	1988:Q2-2004:Q4	-7.8564*	-7.8716*	0.2416
	Monthly	1988:M06-2004:M12	-10.908*	-10.8422*	0.4068
	Quarterly	1988:Q2-1996Q4	-3.826*	-4.6215*	0.6089**
	Monthly	1988:M06-1996:M12	-4.896*	-7.3481*	0.5901**
Real property stock index	Quarterly	1988:Q2-2004:Q4	-1.8564	-1.5269	0.5984**
	Monthly	1988:M06-2004:M12	-1.1908	-1.5929	0.9938*
	Quarterly	1988:Q2-1996Q4	-2.5065	-2.3255	0.2963
	Monthly	1988:M06-1996:M12	-2.8136	-2.29	0.4261
D(Real property index)	Quarterly	1988:Q2-2004:Q4	-8.229*	-9.499*	0.1971
	Monthly	1988:M06-2004:M12	-4.1239*	-11.0961*	0.1766
	Quarterly	1988:Q2-1996Q4	-5.8714*	-6.8746*	0.3403
	Monthly	1988:M06-1996:M12	-5.665*	-7.8996*	0.2182

Note:

1. ADF and PP test are based on the null hypothesis that the tested data series has a unit root. For, KPSS test is based on the null hypothesis that the tested data is stationary.
2. For quarterly data from 1988:Q2-2004Q4, the 99% and 95% of Mackinnon critical value are -3.5349 and -2.9069, respectively.
   For quarterly data from 1988:Q2-1996:Q4, the 99% and 95% of Mackinnon critical value are -3.6537 and -2.9571, respectively.
   For monthly data from 1988:06-2004:12, the 99%, and 95% of Mackinnon critical value are -3.4654 and -2.8768, respectively.
   For monthly data from 1988:06-1996:12, the 99%, and 95% of Mackinnon critical value are -3.5039 and -2.8936, respectively.
3. For KPSS test, the 99% and 95% of Kwiatkowski-Phillips-Schmidt-Shin value are 0.739 and 0.463, respectively.
4. The asterisk (*) and (**) represent significance at 1% and 5% level, respectively.
5. D (real SET index) is represented the first difference on real SET index.

For the risk premium in property stock index, all three tests show that the risk premium in property stock index is statistically stationary at 99% confidence interval.

Due to these results, we conclude that the real interest rate and risk premium in property stock index are stationary.

Next, we apply the stationary tests to real property stock index. The results are shown in row 9 of panel B in table 7.2. We find out that real property stock index is not stationary at 99% confidence interval for ADF and PP tests and 95% confidence interval for KPSS test. However, when we apply the stationary tests to the first difference in real property stock index, the results then show that it is stationary at 99% confidence interval for all three tests.

In recapitulation, we can conclude that there was no rational bubble in property stock prices for the period examined.

Case 2: Monthly data from 1988:M06-2004:M12

In this case, for the robustness purpose, we therefore employ monthly data from 1988:M06-2004:M12 to test the rational bubble in property stock index. As shown in row 2 of panel B in table 7.1, we find out that the real interest rate using PP test is significantly stationary at 99% confidence interval. For risk premium in property stock index, the results in row 6 of panel B in table 7.1 show that it is significantly stationary at 99% confidence interval when using ADF, PP, and KPSS tests, respectively.

Using these results, we therefore conclude that real interest rate and risk premium in property stock prices are stationary.

After we have already tested the stationary in real interest rate and risk premium in property stock index, next, we test the stationary in real property stock index. These results are shown in row 10 of panel B in table 7.1. We observe that the real property stock index is not statistically stationary with 99% confidence interval for ADF, PP, and KPSS tests.

Similarly, these tests are applied for the first difference in real property stock index. As shown in row 14 of panel B in table 7.1, the first difference in real property stock index is statistically stationary with 99% confidence interval.

Thus, we ascertain that real property stock index satisfies the sufficient condition for the absence of rational bubbles. In other word, there was no rational bubble in real property stock prices.

As same as real SET index, the data set may have the effects from two structural changes which are (1) drastically structural imparities of Thai stock market before and after 1986 and (2) the structural changes from the financial crisis in 1997.

We therefore re-examine whether the rational bubble occurred in property stock prices by using quarterly data from 1988:Q2-1996:Q4 and monthly data from 1988:M06-1996:M12.

Case 3: Quarterly data from 1988:Q2-1996:Q4

We begin with the tests for the stationary in real interest rate and risk premium in property stock index. The results in row 3 of panel B in table 7.1 show that the real interest rate is statistically stationary with 99% confidence interval for all three tests.

For risk premium in property stock index, the results are shown in row 7 of panel B in table 7.1. It is clearly see that the risk premium in property stock index is statistically stationary with 99% confidence interval when using ADF and PP tests; however, it is not stationary using KPSS test with 95% confidence interval.

From the above results, we conclude that the real interest rate and risk premium in property stock index are stationary.

Next, we test the stationary of real property stock index. The results in row 11 of panel B in table 7.1 show that it is not stationary with 99% confidence interval for ADF and PP tests.

Therefore, we use these tests again on the first difference of real property stock index. Now, it is statistically stationary with 99% confidence interval as shown in row 15 of panel B in table 7.1.

Armed with these results, there was no rational bubble in property stock index for this data range.

Case 4: Monthly data from 1988:M06-1996:M12.

Finally, we check the robustness of data by using monthly data from 1988:M06-1996:M12. We start with the tests for stationary in real interest rate and risk premium in property stock index. As shown in row 4 of panel B in table 7.1, the real interest rate is statistically stationary with 99% confidence interval when we test with ADF, PP, and KPSS tests, respectively. For the risk premium in property stock index, it is statistically stationary with 99% confidence interval when we test with ADF and PP tests. However, KPSS test shows that it is not stationary with 95% confidence interval. These results are shown in row 8 of panel B in table 7.1.

Based on these results, we conclude that the real interest rate and the risk premium in property stock index are stationary.

As same as the previous cases, we apply these three tests to test whether the real property stock index is stationary or not. The results are presented in row 12 of panel B in table 7.1. We detect that it is statistically not stationary with 99 % confidence interval when we apply the ADF and PP tests.

We apply these tests to the first difference in real property stock index. As illustrated in row 16 of panel B in table 7.1, it is statistically stationary for all three tests with 99% confidence interval.

According to these results, we conclude that the real property stock prices satisfy the sufficient condition for the absence of rational bubble. In short, rational bubble did not exist in the stock market. However, it should be noted that the tests for the unit roots may fail to detect the presence of the explosive rational bubbles that collapse periodically.

In an interesting paper, Evans (1991) highlighted the problem by demonstrating that standard unit root and the cointegration test for asset prices and underlying fundamentals can erroneously lead to acceptance of the no-bubble hypothesis when prices contain an explosive stochastic bubble which collapses from time to time. In his paper, he argues that the problem occurs from a maintained hypothesis of Dickey fuller and Bhargava tests which assume the process is linearly autoregressive. Under the null hypothesis, H_0 is assumed that there is a root of unity. Therefore, if Δd_t is a stable linear autoregressive process, then under the non bubble hypothesis the ΔP_t process will fall into the set of stable statistical alternatives to H_0. However, in cases that there exist a periodically collapsing bubble B_t then the ΔP_t process and the B_t process itself belong neither to null hypothesis nor to the explosive alternatives and actually fall outside the maintained hypothesis of linear autoregressive process.

Therefore, we should apply the other techniques to detect the rational bubble such as a stochastic unit root examined by McCabe and Tremayne (1995), Leybourne et al. (1996), and Granger and Swanson (1997) or a Markov-switching model.

Moreover, due to our data range, one may argue that it may have the effects from two structural changes which are 1. drastically structural imparities of Thai stock market before and after 1986 and 2. the structural changes from the financial crisis in 1997. The Augmented-Dickey Fuller test may has low power in the presence of a structural break. To remedy this problem, we may test the stationary of the stock index and property stock index by applying Zivot and Andrews (1992) test.

Zivot and Andrews (1992) test have, as its null hypothesis, that the dynamics of the respective series are characterized by a unit root. However, in that the Zivot and Andrews test makes allowance for the possible existence of a one-off structural change under the alternative hypothesis. This is an attractive feature of the test since they have demonstrated that the Augmented-Dickey Fuller test has low power in the presence of a structural break. It also allows a check to be made as to whether there has been a significant “regime shift” in the data generating process for the stock market index and property stock index.

7.2 Policy Simulations on the Resale and Building Options

The core purpose in this section is to analyze the policy simulations on the resale and building options in order to identify ways to deal with a possibility of future boom and bust in the property market and its effects on the economy.

We present our simulation results into 2 parts. In the first part, we study the effects of five policies on the resale option by passing through four variables: 1. the optimal trading barrier $\left(k^*\right)$, 2. the size of the bubble $\left(b\right)$, 3. the expected duration between trades $\left(\tau\right)$,and 4. the magnitude of the extra volatility component (η).

For the second part, we apply two financial techniques which are the Finite Difference Method (FDM) and Monte Carlo simulation to identify the optimal stopping time to develop land to be building when five policies are changed. Each simulation result in this part is presented in form of density function of the optimal stopping time.

7.2.1 Policy Simulations on the Resale Option

As illustrated in the chapter 4, the value of resale option is the implicit function in the value of building option. At any time t, the value of resale option $q\left(x\right)$ is at least as large as the gains realized from an immediate sale. The current owner of building chooses an optimal stopping time to exercise his resale option when the value of the option equals that of an immediate sale.

In this part, we aspire to analyze the effects of five policies on the resale option in order to study whether each policy stimulates or postpones the optimal stopping time to exercise the resale option. We present these results pass through four variables as mentioned.

7.2.1.1 The Effect of Real Interest Rate on Resale Option

We firstly investigate the effect of real rate of interest on resale option. In order to illustrate this effect, we employ the following parameter values: ϕ = 0.9, σ_f = 0.096, σ_s =0.05, σ_D = 1.052, c_2 = 1.289. Moreover, we also compare these effects in two cases: 1. when the speed of adjustment is equal to zero ($\lambda = 0$), and 2. when the speed of adjustment is not equal to zero ($\lambda \neq 0$).

Figure 7.2 shows the relations of the trading barrier (k^*), the size of bubble (b), the expected duration between trades (τ), and the extra volatility (η), with respect to the real rate of interest.

As illustrated in panel A of figure 7.2, an increase in real rate of interest boosts the trading barrier to go up. It means that agents in the property market tend to delay their trading when the real interest rate increases. The reason that agents tend to delay their trading comes from a decrease in immediate gain causing by the reducing in bubble size as illustrated in panel B of figure 7.2. Moreover, as we have already known that one of big difference between speculative in housing and stock is the source of fund for their investment. For speculative in property market, they typically finance loan from the commercial banks or financial institutions. Thus, an increase in real interest rate causes them to suffer from higher cost of investment. Hence, they will tend to delay their trading when the real interest rate increases.

Now, we consider the magnitude of trading barrier between two cases which are $\lambda = 0$ and $\lambda = 0.01$ with respect to the real rate of interest, we obviously find out that the magnitude of trading barrier in the former case is relatively smaller than the later case. The reason comes from the difference in speed of adjustment (λ). Based on the movement of conditional means of the beliefs of agents in group A and B which satisfy the mean reverting processes, it means that in the long run the conditional mean of the beliefs will adjust its value to the long-run fundamental value $(\bar{f})$. The time of adjustment depends on how large of speed of adjustment (λ). For the higher (λ), it can adjust to the long-run fundamental more quickly than the small one. However, the higher λ can also implied the higher cost of trading. Therefore, as depicted in panel A of figure 7.2, the trading barrier in case $\lambda = 0.01$ is relatively larger than in case $\lambda = 0$.

Another explanation is derived from the theoretical results in chapter 4. We have already known that for each trading cost $c \geq 0$, there exists a unique k^*. If $c = 0$, then $k^* = 0$. If $c > 0$, $k^* > c(r + \lambda)$.

Therefore, the values of k^* when $\lambda = 0$ and $\lambda = 0.01$ should have its values greater than 0.0168 and 0.0296, respectively. This explanation supports that trading barrier (k^*) when $\lambda = 0.01$ normally has the value bigger than trading barrier (k^*) when $\lambda = 0$.

For the size of bubble, an increase in real interest rate causes agents to bear the higher cost of investments. Therefore, agents tend to decrease their trading frequency. Since the volume of trading decreases, it then affects the size of bubble to decline as shown in panel B of figure 7.2. We can also see this effect by using the following equation:

$$b = \frac{1}{r+\lambda}\left[\frac{h\left(-k^*\right)}{h'\left(k^*\right)+h'\left(-k^*\right)}\right]. \tag{7.1}$$

Equation (7.1) shows the size of bubble in this model. If we calculate the partial derivative of equation (7.1) with respect to r, it will give us the negative value which means that the size of bubble decreases with respect to the real interest rate.

Also, if we recalculate the partial derivative of equation (7.1) with respect to λ, it will also give us the negative value which means that the size of bubble decreases with respect to the speed of adjustment. Hence, the size of bubble in case $\lambda = 0$ is relatively larger than in the case $\lambda = 0.01$.

In a nutshell, an increase in real rate of interest causes the agents to defer their trading as shown in panel C of table 7.2.

Let's us now show the effect of an increase in real interest rate on the extra volatility. Since we have already known that an increase in real interest rate generally reduces the size of bubble, in consequence, it causes the extra volatility component to decrease as shown in panel D of table 7.2. We can directly see this effect by using the following equation,

$$\eta = \frac{\sqrt{2}\phi\sigma_f}{r+\lambda}\left[\frac{h'\left(k^*\right)}{h'\left(k^*\right)+h'\left(-k^*\right)}\right]. \tag{7.2}$$

Equation(7.2) presents the value of extra volatility from the resale option value component. When we calculate the partial derivative of equation (7.2) with respect to real interest rate, it is apparent that the extra volatility decreases with respect to real interest rate.

From our numerical study, we can summarize that an increase in real interest rate can decrease the size of bubble in the property market by reducing the trading frequency among agents.

Moreover, it is interesting to investigate the effectiveness of this policy in order to decrease the size of bubble. Based on one big difference between houses and shares, most households and business depend on borrowing to purchase houses while using their saving to buy equity. In situation where there is the suspected property bubble, the low interest rate will be one factor that fuels the sharply raising property prices. The real economy will be more vulnerable when an increasing in the property prices may increase both the value of bank capital, to the extent that banks own property, and increase the value of property collateral, leading to a downward revision of the perceived risk of property lending. Consequently, an increase in property prices may increase the supply of credit to the property market, which further increases the price of real estate. As a result of such borrowing, property booms tend to be more dangerous than stock market bubbles, and are often followed by periods of prolonged economic weakness. There is a study by IMF found out that output losses after property price busts in developed countries have, on average, been twice as large as those after stock market crashes, and usually result in a recession.

According to the closed relationship between banking and property market, especially in the countries where banks play a dominant role in the real economy (bank-based system), an increasing in real rate of interest would probably be the effective policy implemented to handle with the property booms by passing the monetary policy transmission. This mechanism is as follows: an increase in the policy rate (for example: RP-14 day’s rate etc.) firstly causes short-term market interest rates to rise. When short term market interest rate increases, it then causes an increase in the lending rate which makes household and business to less demand on credit. This consequently decreases house buying and planed fixed investment which in turn contributes to a fall in property price and the size of bubble as we presented in panel B of figure 7.2.

Therefore, it is useful to ask what is the appropriate monetary policy response to asset price cycles, particularly to the property price boom. There are three different views which can be briefly discussed:

1. The first view suggests the monetary authority to do not thing in order to maintain economic and financial stability. One example is the U.S. monetary policy during 1999. The Federal Reserve Bank of U.S. (Fed) helped to inflate house prices by holding interest rates low for so long after equities crashed. This prevented a deep recession, but it may have merely delayed the needed economic adjustments.
2. The second view suggests the monetary authority to apply the slightly tightening monetary policy to temper the suspected asset price bubble such as an increase in an additional interest rate of perhaps 50 basis points to discourage a potentially excessive boom.
3. The last view suggests the monetary authority to use the aggressive monetary policy in order to eliminate a potential bubble. However, most economists dismiss the third view as impractical because a bubble is difficult to identify with certainty ex ante and monetary policy is a tool with long and variable lags.

Even though the views differ during the boom, economists tend to agree that monetary policy should react quickly and be more accommodative after bust.

7.2.1.2 The Effect of Resale Cost on Resale Option

We investigate the effect of resale cost by employing the following parameters values: ϕ = 0.9, σ_f = 0.096, σ_s =0.05, σ_D = 1.052, and $r = 0.013$. For λ, we divide it into two cases: 1. $\lambda = 0$ and 2. $\lambda = 0.01$. These results are illustrated in figure 7.3. In this figure, it shows the relations of the trading barrier (k^*), the size of bubble (b), the expected duration between trades (τ), and the extra volatility (η), with respect to resale cost (c_2). In panel A of the figure 7.3, it shows the optimal trading barrier with respect to resale cost where the optimal trading barrier represents the minimum level in the difference of beliefs that allows the owner of building to cover the trading cost with respect to resale cost. From panel A, we apparently see that when the trading cost is zero, the building owner sells the building immediately when it is profitable and these profits are infinitely small. However, when the resale cost increases, the value of the trading barrier also increases. The reason is because an increase in resale cost causes the immediate gain from trading to decline. Therefore, agents will tend to delay their trading because the value of resale option is larger than the value of immediate gain. Moreover, the speed of increasing is dramatic when the resale cost is near zero. The result comes from the $\frac{dk^*}{dc}$ which is infinite at the origin. When we compare the trading barrier in case $\lambda = 0$ and $\lambda = 0.01$, the magnitude of the trading barrier in the first case is relatively smaller than the second case when the resale cost increases.

As a result of an increase in resale cost, the trading frequency is greatly reduced as shown in Panel C. According to an increase in resale cost, it also reduces the size of bubble as shown in Panel B. Both trading frequency and size of bubble in case $\lambda = 0$ are relatively larger than in case $\lambda = 0.01$.

Although one could expect that the strong reduction in the trading frequency caused by increasing in resale cost should greatly reduce the extra volatility component, the result in Panel D is surprising and contrary to “conventional wisdom” about the effect of transaction cost. From Panel D, extra volatility slowly increases at the point where resale cost is nearly zero and then finally declines after the resale cost approaches higher level. We can explain this result by two opposing effects. Firstly, when the resale cost increases, the resale option value has to fall by the present value of transaction cost. It is called “present value effect”. Secondly, when the transaction cost increases, agents hold the resale option for longer period; therefore, each marginal agent holds fewer shares and requires a smaller risk premium. When agents require the lower risk premium, the resale option value becomes larger. We call “risk premium effect”.

Accordingly, the volatility from the resale option value component can either increase or decrease when we increase the resale cost. These two opposing effects reflect an increase in volatility of the option component for small trading cost. However, when the resale cost continuously increases, its effects then come to reduce the volatility of the option component with the small rate of decreasing.

In sum, an increase in resale cost causes the gain realized from an immediate sale to decrease. As a result of this effect, agents tend to delay their trading. Therefore, the bubble size and the extra volatility are relatively smaller.

7.2.1.3 The Effect of Overconfidence Level on Resale Option

We now examine the effect of overconfidence level. For this calibration, we apply the following parameter values from the previous case: σ_f = 0.096, σ_s =0.05, σ_D = 1.052, c_2 = 1.289, and $r = 0.013$ where λ are divided into two cases: (1) $\lambda = 0$ and (2) $\lambda = 0.01$. Figure 7.4 shows the relations of the trading barrier (k^*) , the size of bubble (b), the expected duration between trades (τ), and the extra volatility (η), with respect to overconfidence level (ϕ). As overconfidence level increases, the mean reversion parameter in the difference of beliefs decreases. On the contrary, the volatility parameter in the difference of beliefs increases.

Consequently, the resale option becomes more valuable to the asset owner. Therefore, the asset owner then decides to hold it for longer period and waits until the value of resale option is equal to the immediate gain from sale. As a result, trading barrier becomes higher as shown in panel A. For case $\lambda = 0$, the trading barrier is relatively smaller than case $\lambda = 0.01$. Since an increase in overconfidence level causes the volatility in difference of beliefs to increase, the size of bubble and the extra volatility therefore become larger. When we compare the magnitudes of these values in two cases which are 1. $\lambda = 0$ and 2. $\lambda = 0.01$, we find out that the former case has these values relatively larger than the later case.

For the expected duration between trades, it is determined by two offsetting effects. For the first effect, when overconfidence parameter increases, it causes the trading barrier to become larger; therefore, the building owner will generally hold the building longer than before. Thus, the expected duration between trades should be higher when the overconfidence increases. On the other hand, as a result of overconfidence increases, the volatility of the difference in beliefs increases, causing the duration between trades to be shorter. Nevertheless, Scheinkman and Wei Xiong (2003) show that when c is relatively small, the change in the trading barrier is second order. Therefore, the expected duration between trades actually declines when the overconfidence level increases. We present the effect of overconfidence on the expected duration between trades in the panel C of figure 7.4. This result coincides with the study of Scheinkman and Wei Xiong.

The effectiveness of the overconfidence level in increasing speculative trading has been hotly debated. Therefore, it is interesting to investigate what are the main factors that cause investors to be overconfident. There are many economists and psychologists that try to explain these factors. We start with the explanation by Robert. J. Shiller (2001) in his famous pocket book "Irrational Exuberance".

He concludes in his book that there are many theories explaining how investors seem to be overconfident. First, people tend to evaluate the probability that they are right on only the last step of their reasoning, forgetting how many other elements of their reasoning could be wrong. Second, they make probability judgments by looking for similarities to other known observations, and they forget that there are many other possible observations with which they could compare. Third, overconfidence may also have to do with hindsight bias[2] .

Another factor in overconfidence is magical thinking. Psychologists find that people have occasional feelings that certain actions will make them lucky even if they know logically that the actions cannot have an effect on their fortunes. For example, if they are asked how much money that they would demand to part with a lottery ticket they already hold, people will give a figure over four times greater if they chose the lottery number on the ticket by themselves. Fifth, people tend to make judgments in uncertain situations by looking for familiar patterns and assuming that future patterns

[2] Hindsight bias is a tendency to think that one would have known actual events were coming before they happened. It encourages a view of the world as more predictable than it really is (Shiller,2001).

will resemble past ones. This anomaly of human judgment is called the representativeness heuristic[3].

Barber and Odean (2001) also explain that the main factor causing investors to be overconfident is the abundant information. One may argue that more information should lead to better decision-making but they point out that this argument depends on the relevance of the information to the decision and on how well-equipped the decision marker is to use the information. They explain that additional information can lead to an "illusion of knowledge". It can occur when people are given more information on which to base a forecast or an assessment. Their confidence in the accuracy of their forecasts tends to increase much more quickly than the accuracy of those forecast and finally causes investors to become overconfident.

7.2.1.4 The Effect of Long-Run Fundamental on Resale Option

When the long-run fundamental increases, it is generally not affect the optimal stopping time to exercise resale option. As a result, an increase in long-run fundamental is not affect the trading barrier (k^*), the size of bubble (b), the expected duration between trades (τ), and the extra volatility (η) as illustrated in figure 7.5.

On the other hand, it causes the fundamental value of permanent building which is represented by $\left\{\frac{\overline{f}-R_a}{r}+\frac{\hat{f}_t^o-\overline{f}}{r+\lambda}-c_1\right\}$ to increase. Under such situation, the land owner has the tendency to exercise his building option prior to its exercise date as a result of higher fundamental value. We will discuss this effect in more details in the next part.

[3]See Robert J. Shiller (2005) and Nicholas Barberis, Andrei Shleifer, and Robert Vishny (1998).

7.2.1.5 The Effect of Information in Signals on Resale Option

For simplicity, we measure the information in each of two signals by $i_s = \frac{\sigma_f}{\sigma_s}$. When i_s increases, it means that there is more information for agents to disagree. However, because we measure the information in term $i_s = \frac{\sigma_f}{\sigma_s}$ therefore the information in signals can be increased by two reasons. First, holding the volatility of fundamentals σ_f constant, a decrease in σ_s is equivalent to an increase in the information. Second, holding the volatility of signals σ_s, an increase in σ_f is also equivalent to an increase in the information.

In order to study the effect of changes in the volatility of the noise in signals on resale option, we therefore present it into two cases which are: 1. a decrease in volatility of signals, and 2. an increase in volatility of fundamentals.

We firstly explain the numerical results of the first case which apply the following parameter values: σ_f = 0.096, σ_D = 1.052, $\phi = 0.9$, c_2 = 1.289 ,and $r = 0.013$. The speed of adjustment, λ, is also divided into two cases: 1. $\lambda = 0$ and 2. $\lambda = 0.01$.

The figure 7.6 depicts the relations of the trading barrier (k^*), the size of bubble (b), the expected duration between trades (τ), and the extra volatility (η), with respect to the changes in the volatility of the noise in signals (σ_s).

In the panel A of figure 7.6 , it shows the optimal trading barrier with respect to the changes in information in signals (i_s). When i_s increases, it means that there is more information in two signals which cause agents to more disagree[4]. Thus, the mean reversion parameter ρ of the difference in beliefs increases without the change

[4] It should be noted that the effect of i_s caused by a decrease in volatility of signals also depends on the overconfidence level. From our experiments, we find out that the size of bubble turns to decrease with respect to information in signals if we assume the overconfidence level relatively low. This result shows that an increase in information can also cause agents to less disagree when agents are not overconfident.

in volatility of the difference in beliefs $\left(\sigma_g\right)$. When the mean reversion parameter of the difference in beliefs increases, it then causes the optimal trading barrier to decrease. For the case $\lambda = 0$, the optimal trading barrier is relatively smaller than the case $\lambda = 0.01$. When the trading barrier decreases as i_s increases, it then brings about the duration between trades to drop as shown in panel C. In consequence, it causes agents to trade more and causes the size of bubble at the trading point to become larger because of an increase in trading frequency as depicted in panel B. As same as previous case, the magnitude of trading frequency, the size of bubble, and extra volatility in the case $\lambda = 0$ are relatively larger than the case $\lambda = 0.01$. Nonetheless, from panel D of figure 7.6, the extra volatility component (η) is almost independent of the information in each of two signals.

Next, we turn to explain the numerical results from the second case which apply the following parameter values: σ_s = 0.05, σ_D = 1.052, $\phi = 0.9$, c_2 = 1.289 ,and $r = 0.013$. We similarly divide λ into two cases: 1. $\lambda = 0$ and 2. $\lambda = 0.01$.

The figure 7.6 represents the relations of the trading barrier (k^*), the size of bubble (b), the expected duration between trades $\left(\tau\right)$, and the extra volatility $\left(\eta\right)$, with respect to the changes in the volatility of the fundamentals (σ_f).

When the volatility of fundamentals increases, the volatility of the difference of beliefs σ_g and the mean reversion parameter ρ also increase. An increase in the volatility parameter σ_g leads to an increase in value of resale option to the asset owner. On the other hand, an increase in the mean reversion parameter causes the resale option becomes less valuable to the asset owner. Therefore, an increase in volatility of fundamentals can either increase or decrease the optimal trading barrier. However, from our numerical study, we find out that an increase in the mean reversion parameter is second order. Thus the optimal trading barrier typically increases, as illustrated in panel A of figure 7.6. When we compare the magnitude of the optimal trading barrier between case $\lambda = 0$ and case $\lambda = 0.01$, we find out that the magnitude of the optimal trading barrier in the former case is relatively smaller than the later case.

For the size of bubble, when the volatility of fundamentals increases, it means that there is more information for agents to disagree. Therefore, agents tend to trade more aggressively. The result of higher volume in trade causes the size of bubble and extra volatility to increase as shown in panel B and D. These effects are relatively larger in case $\lambda = 0$ than in the case $\lambda = 0.01$.

For the duration between trades, it is determined by two offsetting effects as the volatility of fundamentals increases. On the one hand, the trading barrier becomes higher as explained, making the duration between trades longer. On the other hand, the volatility of the difference in beliefs σ_g increases, causing the duration to be shorter.

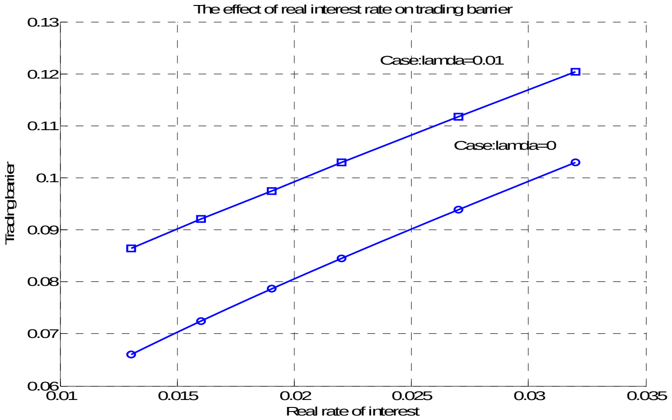
Nonetheless, our numerical result shows that the change in the trading barrier k^* is second order. Therefore the duration between trades typically decreases, as illustrated in panel C of figure 7.6. The duration between trades in case $\lambda = 0$ is relative smaller than in the case $\lambda = 0.01$.

From our numerical study, abundant information is an important factor that can speed up the asset price bubble. It generates disagreement among investors. Barber and Odean (2001) explain that when people who initially disagree on a topic are given arguments on either side of the issue, they become further polarized in their beliefs in circumstance such that they have variety of information. They are impressed by the arguments with which they already agree and they discount opposing views. Not only are people more impressed by arguments they favor, but they actively seek out confirming evidence. Moreover, more information can also lead to an illusion of knowledge as we have already explained.

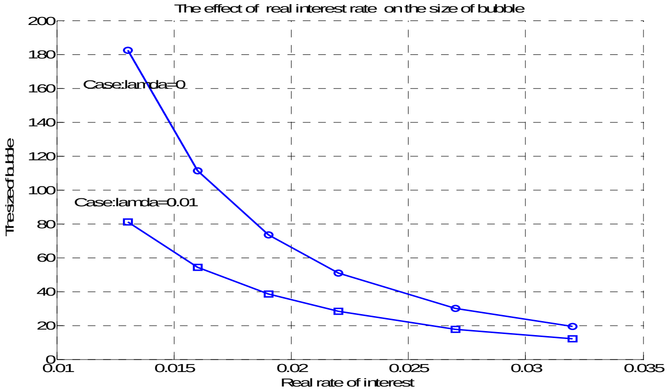
According to these results, investors are likely to become overconfident. They may believe that they have more ability to perform tasks such as investing in property market than they actually do.

Figure 7.2

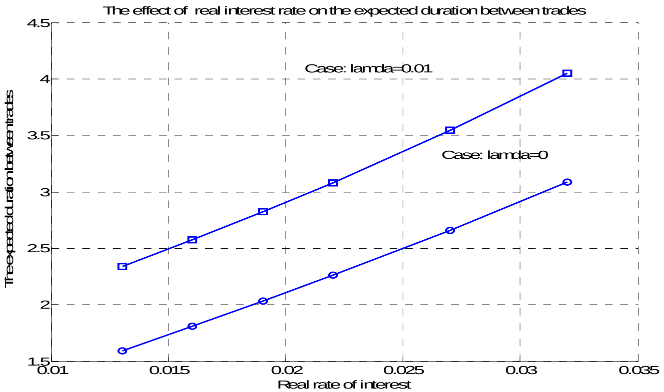
The Effect of Real Interest Rate on Trading Barrier, Bubble Size, Expected Duration between Trades, and Extra Volatility



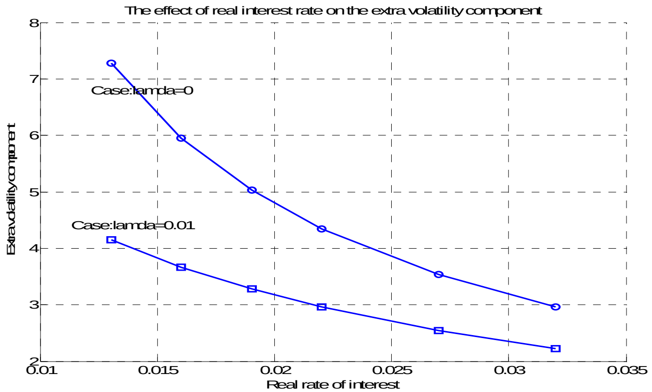
Panel A: Trading Barrier



Panel B: Size of Bubble



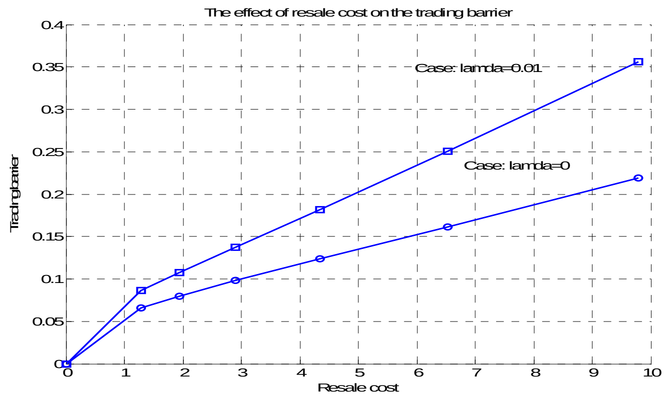
Panel C: Expected Duration between Trades



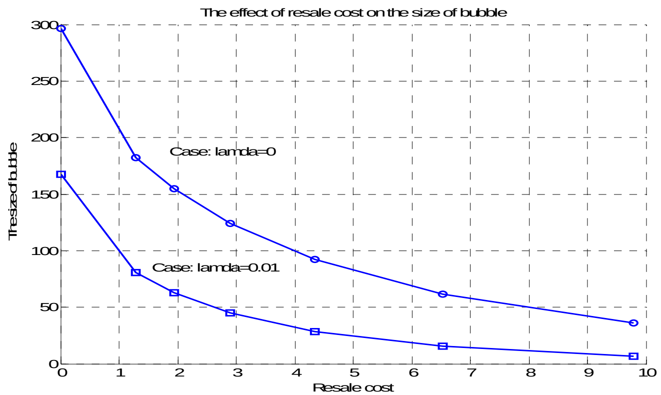
Panel D: Extra Volatility

Figure 7.3

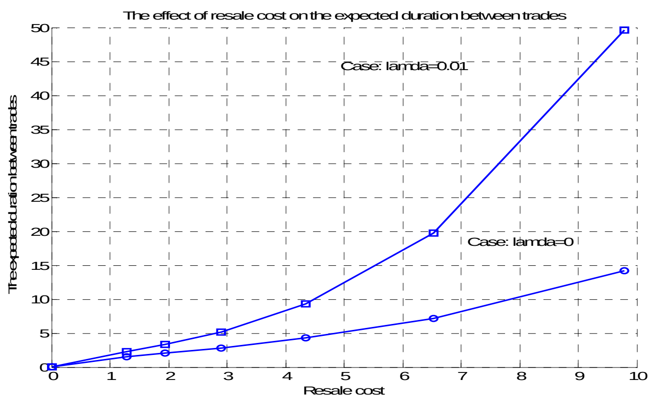
The Effect of Resale Cost on Trading Barrier, Bubble Size, Expected Duration between Trades, and Extra Volatility



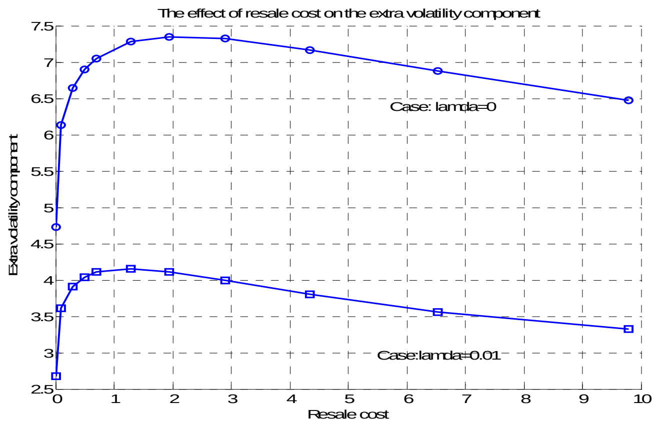
Panel A: Trading Barrier



Panel B: Size of Bubble



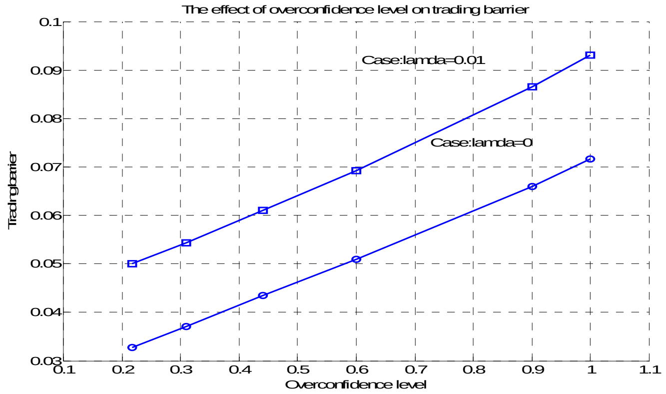
Panel C: Expected Duration between Trades



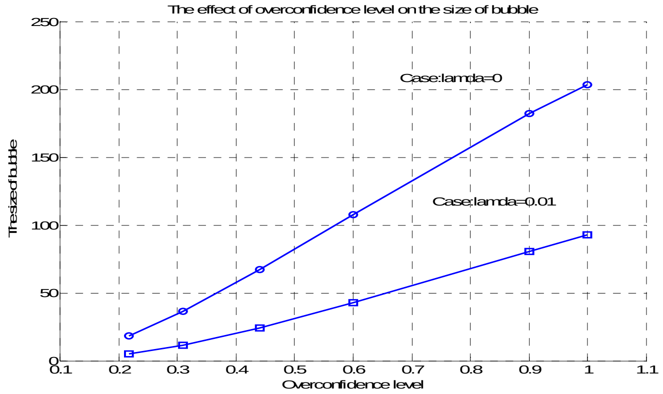
Panel E: Extra Volatility

Figure 7.4

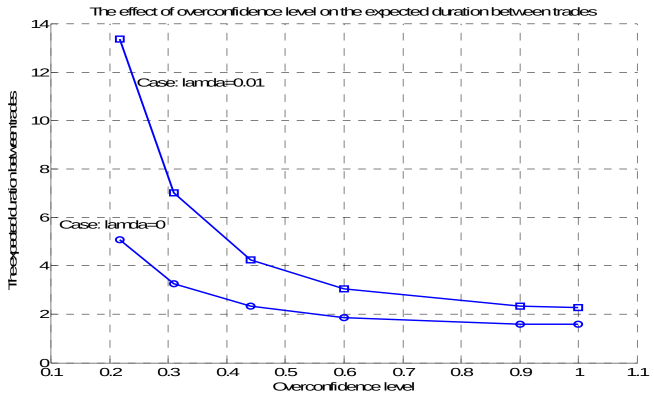
The Effect of Overconfidence Level on Trading Barrier, Bubble Size, Expected Duration between Trades, and Extra Volatility



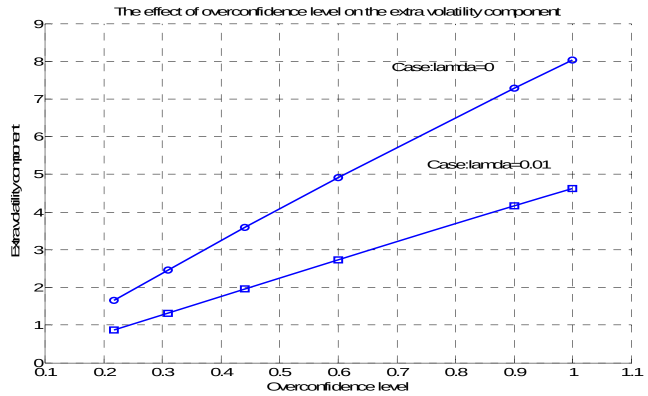
Panel A: Trading Barrier



Panel B: Size of Bubble



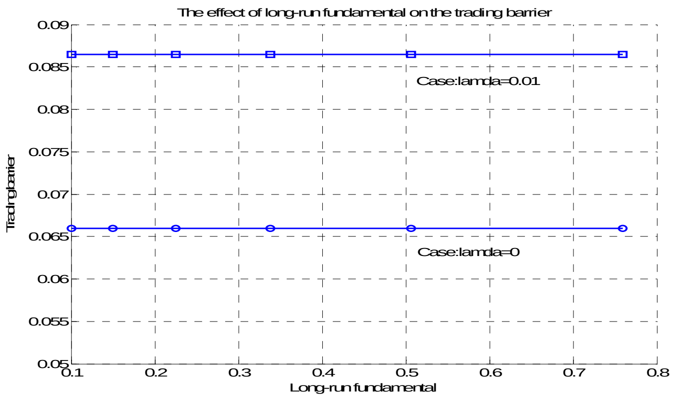
Panel C: Expected Duration between Trades



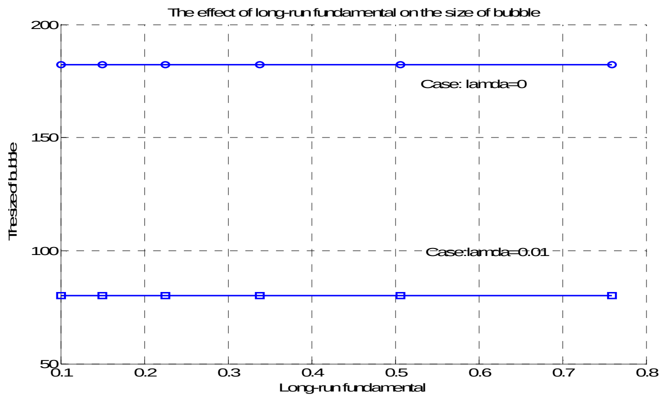
Panel D: Extra Volatility

Figure 7.5

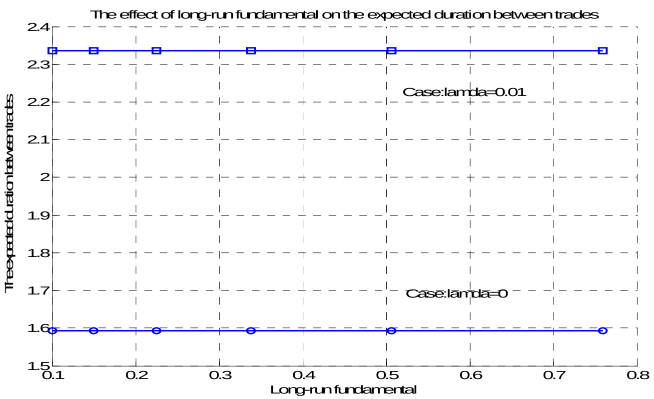
The Effect of Long-Run Fundamental on Trading Barrier, Bubble Size, Expected Duration between Trades, and Extra Volatility



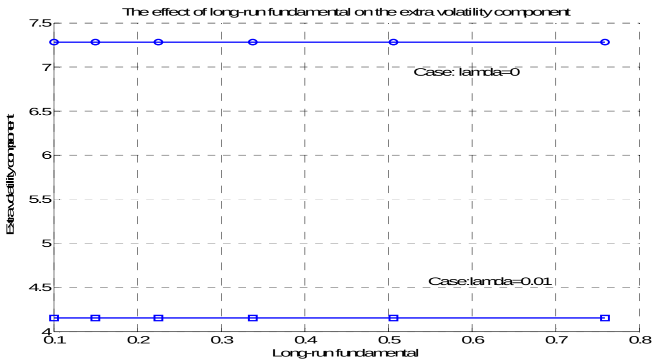
Panel A: Trading Barrier



Panel B: Size of Bubble



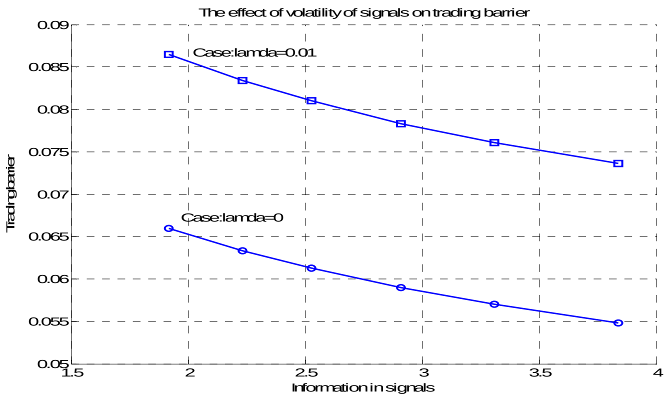
Panel C: Expected Duration between Trades



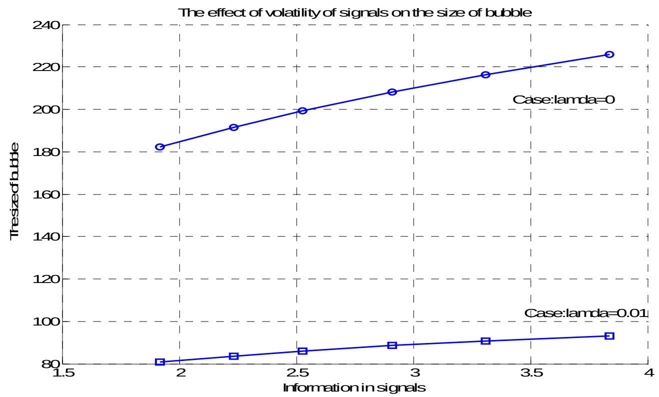
Panel D: Extra Volatility

Figure 7.6

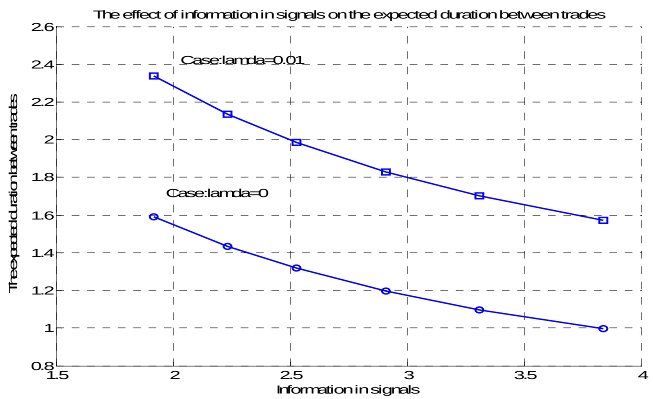
The Effect of Volatility of the Signals on Trading Barrier, Bubble Size, Expected Duration between Trades, and Extra Volatility



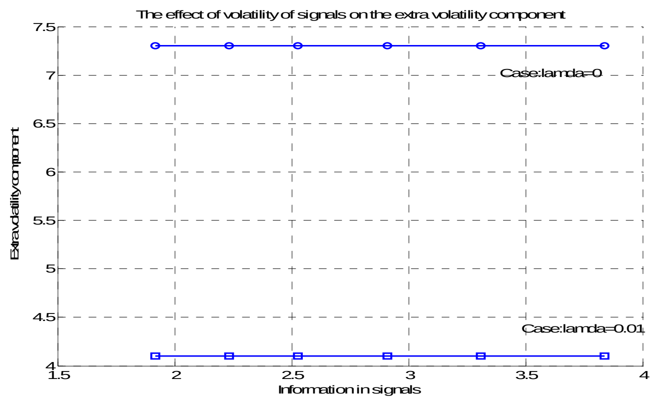
Panel A: Trading Barrier



Panel B: Size of Bubble



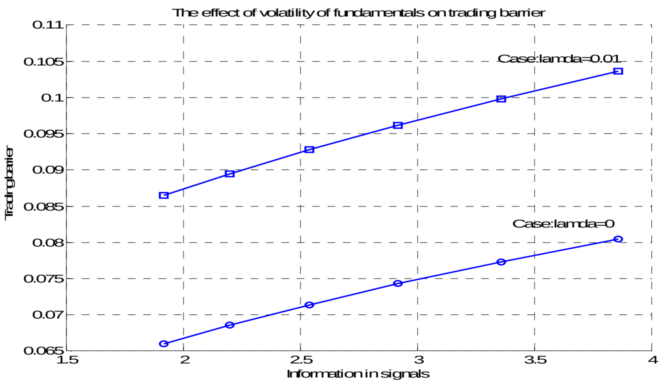
Panel C: Expected Duration between Trades



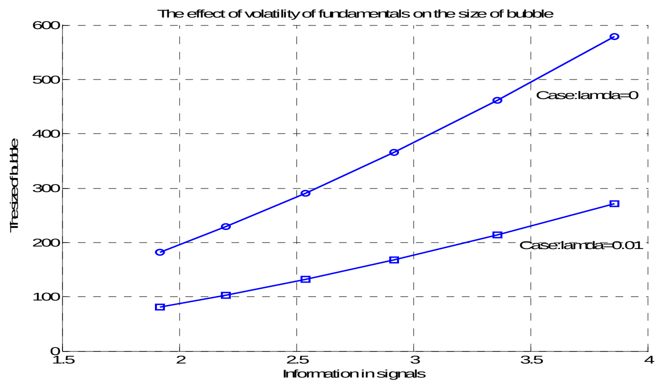
Panel D: Extra Volatility

Figure 7.7

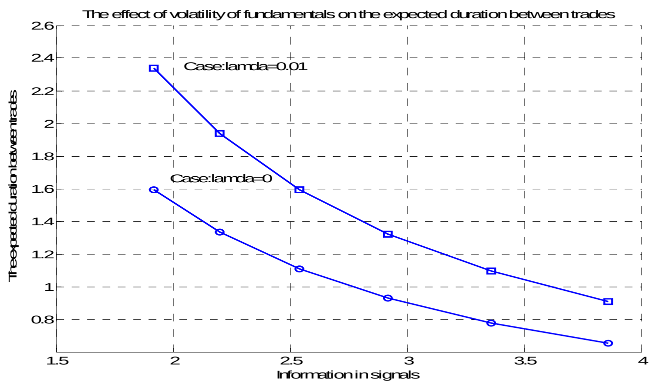
The Effect of Volatility of Fundamentals on Trading Barrier, Bubble Size, Expected Duration between Trades, and Extra Volatility



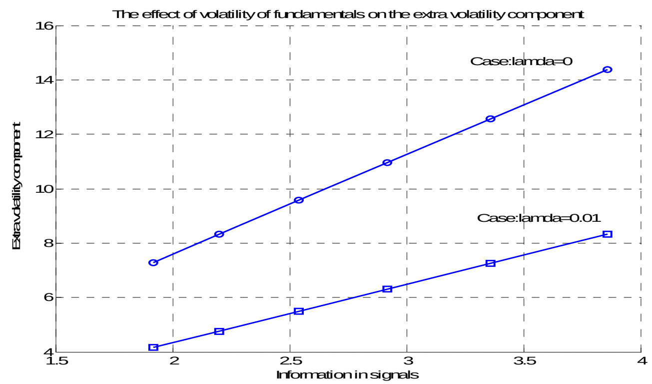
Panel A: Trading Barrier



Panel B: Size of Bubble



Panel C: Expected Duration between Trades



Panel D: Extra Volatility

7.2.2 Policy Simulations on the Building Option

In this section, we analyze the effects of five policies on the optimal stopping time to develop a piece of vacant land to be building. Based on our theoretical framework, it shows that overconfidence and the value of resale option among different agents can generate land prices bubble. Under such situation, the land owner has the tendency to exercise his building option prior to its optimal stopping date as a result of positive early exercise premium.

This situation can seriously generate dynamic inefficiency problem and causes the land price bubble in the economy. Therefore, it is interesting to study the effects of five policies on the building option in order to find the possible ways to deal with a possibility of future vulnerable housing bubble.

7.2.2.1 The Effect of Real Interest Rate on the Building Option

One important policy to slow down the land price bubble, in general, is an increase in interest rate policy. When the interest rate policy increases, most types of interest will also adjust its rates to higher levels. The higher cost of investment caused by an increase in interest rate then bears down on land owner. Because of the higher of cost of investment causing the lower of net present value of the immediate gain from developing land to be building, the land owner then generally tends to delay his building plan.

Nevertheless, an increase in interest rate may stimulate the land owner to develop land to be building faster than before if its effect can reduce the value of building option more than the present value of the immediate gain.

Based on these two unclear offsetting effects, it is therefore interesting to study these effects on the optimal stopping time to develop land to be building by using simulation technique. We employ the Finite Difference Method (FDM) and Monte Carlo simulation to identify the distribution of the optimal stopping time when interest rate is raised.

As pointed out earlier, due to the lack of housing data in Thailand, almost parameters applied in this section are based on U.S. housing data.

We simulate the distribution of the optimal stopping time when the interest rate is raised by using the following parameters:

σ_f = 0.096, σ_D = 1.052, $\sigma_s = 0.05$, $\phi = 0.9$, $\overline{f} = 0.1, r = 0.013$, $c_1 = 20$, c_2 = 1.289, and $\lambda = 0.01$[5]

Table 7.2 shows the probability that land owner decides to develop his land to be building at any time $\tau = t$ for $t = 2,3,...,\infty$. Each simulation is based on 15,000 experiments. We study the effect of an increase in interest rate on optimal stopping time by increasing real rate of interest from 0.013 to 0.016, 0.019, 0.022, 0.027, and 0.032, respectively.

The last row of table 7.2 represents the probability that land owner decides either to develop land after 40 years from now or to continuously utilize the benefit from the vacant land forever. An increase in this probability implies that the land owners tend to delay their developing land projects. Based on our simulation results, we find out that an increase in real interest rate significantly causes land owner to delay his project because this probability continuously increases when the real interest rate increases.

It is interesting to investigate the effect of raising rate of interest more closely. Therefore, we look at the probabilities to develop land to be building at each time τ =2, 3,…,40 when the real rate of interest vary from 0.013 until 0.032 . Surprisingly, we observe that when real rate of interest increases and is in the range 0.013-0.022, the probabilities to develop land to be building at time $\tau = 2$ and 3 slightly increase as shown in row 2 and 3 of table 7.2. The economic reason behind this situation is that, for a small rate of interest, even though the raising of real rate of interest brings about the land owner to bear the higher investment cost which reduced

[5] In this part, we only study the policy simulations by assuming that the mean reverting parameter is not equal to zero. This assumption bases on a cursory look at the historical data of the movement of the change in real average annual rent from 1961-2004 which suggests that these data are mean reverting, but that the rate of mean reversion is very slow. Therefore, we assume this parameter $\lambda = 0.01$.

the net present value of realized gain from develop land, the land owner may compensate this negative effect by immediately developing land and reinvesting as fast as possible in order to receive the higher reinvestment gain. Nonetheless, when real interest rate increases until reaching some value, the effect of cost of investment will be the dominant effect and causes land owner to delay his project. As shown in row 2, the probability to develop land at time $\tau = 2$ shapely decreases when we increase the real rate of interest from 0.1634 to 0.032.

For the probabilities to develop land to be building from period $\tau = 4,...,40$, these probabilities typically decrease when the real rate of interest increases.

In brief, our simulation shows that an increase in real rate of interest generally delays the optimal stopping time to develop land to be building as illustrated in figure 7.7. It means that an increase in real rate of interest can reduce the value of immediate gain much more the value of building option.

From figure 7.8, each panel represents the density function of the optimal stopping time from period $\tau = 2,3,...,40$ with respect to real rate of interest. We find out that when the real rate of interest increases, the height of density function will relatively decrease which means that an increase in real rate of interest normally causes land owner to delay his project.

However, it should be noted that although an increase in interest rate can cause the land owners to postpone their building projects, it can also cause agents in other sectors to delay their investments which can dynamically drop the growth of economy. Therefore, before we implement this policy in order to decrease the dynamic inefficiency in property market, we should firstly concern the trade off between the dynamic inefficiency in this market with the other sectors.

7.2.2.2 The Effect of Resale Cost on the Building Option

We search out the effect of resale cost on the building option by applying the following parameter values

σ_f = 0.096, σ_D = 1.052, $\sigma_s = 0.05$, $\phi = 0.9$, $\overline{f} = 0.1$, $r = 0.013$, $c_1 = 20$, c_2 = 1.289. Each simulation is based on 15,000 experiments.

From the previous section, an increase in resale cost generally reduces the size of bubble generated by trading frequency. When the size of bubble decreases, it then causes the value of resale option to decline. As illustrated in the theoretical model, the value of immediate gain from developing land to be building is composed of two components. One is the fundamental value and second is the value of resale option. Thus, the value of immediate gain also declines because of the reducing in the value of resale option. In consequence, it causes the land owner to delay his developing project due to the lower immediate gain.

We study this effect by simulating the probability density function of the optimal stopping time to develop land to be building when the resale cost increases.

Table 7.3 shows the probability density function of the optimal stopping time when the resale costs are 1.289, 1.934, 2.9, 4.35, 6.526, and 9.788, respectively.

In generally, we observe that the probabilities to develop land to be building for each period from $\tau = 2,...,5$ slightly decrease when the resale costs are 1.289, 1.934, 2.9, and 4.35, respectively as shown in row 2-4 of table 7.3 and significantly decrease when the resale costs are equal to 6.526 and 9.788, respectively. Nonetheless, the probabilities to develop land to be building from period $\tau = 6,...,40$ have no clear direction whether they decrease or increase when the resale cost goes up. This ambiguous direction may come from agents who tend to delay their land development projects with in 5 years from now but choose to develop them in the near future, says 6-40 years. Therefore, the probabilities to develop land to be building for period $\tau = 6,...,40$ increase in some periods in this range. For example, we find out that probability to develop land to be building for $21 \le \tau \le 40$ significantly increases when the resale cost goes up from 1.289 to 9.788 which means that agents postpone their projects to the next twenty years due to an increase in resale cost.

The effect of an increase in resale cost is also confirmed by an increase in the probability to develop land after 40 years from now and the probability to continuously utilize the benefit from the vacant land forever which is presented in the last row of table 7.3. This probability shows that an increase in resale cost typically reduces the probability to develop land to be building before the optimal period.

In short, based on this simulation, an increase in resale cost such as an increase in transfer fee etc. is the one alternative policy for the policymakers in order to handle with the property market booms.

In order to see this effect more clearly, we therefore present the probability density function of the optimal stopping time to develop land to be building when the resale costs are varied from 1.289, 1.934, 2.9, 4.35, 6.526, and 9.788, respectively in figure 7.9 from panel A-F . We find out that an increase in resale cost can slightly reduce the probability to develop land to be building within 40 years from now. This effect dramatically sees when the resale cost goes up from 1.289 to 6.526 and 9.788, respectively as shown in panel E and F.

However, it should be noted that even though this simulation find out that an increase in resale cost can typically reduce the probability to develop land to be building before the optimal time, this study is based on only one aspect. Before applying it, we must further study in other aspects such as its effect on social welfare etc. in order to implement it with the highest benefit for our economy.

7.2.2.3 The Effect of Overconfidence Level on the Building Option

Another factor that significantly determines the optimal stopping time to develop land to be building is come form the behavioral biases. We study the effect of the behavioral biases by passing through the overconfidence level which has the value in the range 0-1 in our model as presented in chapter 4.

Due to the fact that an increase in overconfidence level can either stimulate or postpone the optimal stopping time to develop land to be building as presented in chapter 4, therefore, it is interesting to study how these two offsetting effects impact to the optimal stopping time to develop land to be building.

In order to analyze this effect, we apply the following parameter values for this simulation:

$\sigma_f = 0.096$, $\sigma_D = 1.052$, $\sigma_s = 0.05$, $\overline{f} = 0.1$, $r = 0.013$, $c_1 = 20$, $c_2 = 1.289$.

As same as other cases, each simulation is based on 15,000 experiments.

Table 7.4 shows the probability density function to develop land to be building when the overconfidence parameters are 0.216, 0.309, 0.441, 0.63, 0.9, and1, respectively.

From our result, an increase in overconfidence level causes land owner to use his right to exercise the building option before the terminal date (T) more than before. These effects can be clearly found when the overconfidence level increases from 0.016 to 0.309 and 0.441, respectively (see panel A-C of figure7.10).

However, for the higher values of overconfidence level which are 0.63, 0.9, and 1, these probability density functions of optimal stopping time to develop land to be building are not much more clearly different (see panel D-F of figure 7.10).

Let's look at this effect in more details by using table 7.4. Table 7.4 represents the probability to develop land to be building with respect to time and overconfidence level. As shown in the last row, this probability typically decreases when the overconfidence level increases. It implies that the land owners tend to develop their projects within 40 years from now. Moreover, the probabilities to develop land to be building in 2 or 3 years from now also gradually increase as shown in row 2 and 3 of table 7.4.

In summary, an increase in overconfidence level is also the one important factor that causes land owner to speed up his investment in building because the higher gains realized from trading frequency.

7.2.2.4 The Effect of Long-Run Fundamental on the Building Option

Because the value of immediate gain from developing land to be building is composed of two components as we have already explained, therefore, the tendency that land owner will earlier develop land to be building may generate from either the higher value of resale option which represents the bubble component or the higher value of fundamental component.

In this part, we then study the effect of long-run fundamental on the building option. To complete the simulation, we again apply the following parameters:

σ_f = 0.096, σ_D =1.052, $\sigma_s = 0.05$, $\phi = 0.216$, $r = 0.013$, $c_1 = 20$, c_2 = 1.289. Each simulation is relied on 15,000 experiments.

However, in order to see this effect with the small effect of bubble component, we therefore change the over confidence parameter from 0.9 to 0.216.

Table 7.5 presents the probability density function of the optimal stopping time to develop land to be building when the values of long-run fundamental are 0.1, 0.15, 0.225, 0.338, 0.506, and 0.759, respectively.

The result shows that an increase in fundamental component is one factor that stimulates the land owner to develop land to be building more quickly than before (see panel A-E of figure 7.11). From the last row of table 7.5, the probability to develop land after 40 years from now and the probability to continuously utilize the benefit from the vacant land forever are sharply drop which confirm that an increase in long-run fundamental generally causes the land owner to investment in their projects more quickly than before.

From this simulation, it is not surprisingly that why the central cities such as Bangkok, Chiang Mai , Khon Kaen etc. have the higher growth of development in property market more than other areas in Thailand. One reason is because these areas have high infrastructural developments which increase the long-run fundamental in buildings.

7.2.2.5 The Effect of Information in Signals on the Building Option

Information in signals is also one factor which can speed up the optimal stopping time to develop land to be building by passing through an increase in bubble value. We have already known that when there is more information in two signals which causes agents to more disagree, it then causes them to trade more aggressively and perform worse in their trading. In consequence of this, the aggressive trading therefore generates the larger value of bubble which causes the value of resale option to increase. Since the value of resale option increases, it dynamically causes the value of building option to increase. At the same time, an increase in the value of resale option also increases the value of immediate gain from developing land to be building. According to these effects, an increase in signal therefore can either increase or decrease the optimal stopping time to develop land to be building.

Due to the unclear result of the effect of the information in signals, we therefore study this effect by simulating the probability density function of the optimal stopping time to develop land to be building when the information in signals increases.

However, because we measure the information in term $i_s = \dfrac{\sigma_f}{\sigma_s}$ therefore the information in signals can be increased by two reasons. First, holding the volatility of fundamentals σ_f constant, a decrease in σ_s is equivalent to an increase in the information. And second, holding the volatility of signals σ_s, an increase in σ_f is also equivalent to an increase in the information.

Hence, in our simulation, we study the effect of an increase in information in two cases which are 1. a decrease in volatility of signals, and 2. an increase in volatility of fundamentals.

Let's begin with the first case. In order to do the simulation in this case, we base on these parameter values: σ_f = 0.096, σ_D =1.052, $\phi = 0.216$, $\overline{f} = 0.1$, $r = 0.013$, $c_1 = 20$, c_2 = 1.289. Each simulation is relied on 15,000 experiments. For the volatility of signals, we vary its value to be 0.05, 0.043, 0.038, 0.033, 0.029, and 0.025 respectively.

We present the simulation results from a decrease in volatility of signals in table 7.6. Table 7.6 presents the probability density function of the optimal stopping time to develop land to be building when the information in signals are 1.92, 2.21, 2.54, 2.92, 3.36, and 3.86 respectively.

We find out that an increase in information in signals caused by a decrease in volatility of signals has small effect on the optimal stopping time to develop land to be building as depicted in figure 7.6 from panel A-F. By comparing the values of probability to develop land to be building in each column in table 7.6, its values insignificantly differ from other ones. Moreover, if we look at the last row of this table which represents the probability to develop land to be building after 40 years from now and the probability to continuously utilize the benefit from land forever, we find out that this probability slightly increases when there is more information for agents.

One explanation is because of the value of overconfidence level. As we have already noted that an increase in information caused by a decrease in volatility of signals can either increase or decrease the size of bubble. In case that overconfidence level is relatively low, an increase in information can reduce disagreement among agents. Therefore, the size of bubble decreases when information increases as shown in this simulation. A decrease in the size of bubble then causes the value of immediate gain to decrease. Finally, agents tend to delay their projects even though there is more information.

We further study the effect of an increase in information caused by an increase the volatility of fundamentals. Our simulation results are based on the following parameter values:

$$\sigma_s = 0.05,\ \sigma_D = 1.052,\ \phi = 0.216,\ \bar{f} = 0.1,\ r = 0.013,\ c_1 = 20,\ c_2 = 1.289.$$

For the volatility of fundamentals, we vary its value to be 0.096, 0.11, 0.127, 0.146, 0.168, and 0.193, respectively.

The results are shown in table 7.7 which presents the probability density function of the optimal stopping time to develop land to be building when the information in signals are 1.92, 2.21, 2.54, 2.92, 3.36, and 3.86, respectively.

As explained, when the volatility of fundamentals increases, holding the volatility of signals constant, it causes the information in signals to increase. An increase in information can cause agents to trade more and consequently incur higher size of bubble. When the size of bubble increases, it increases the value of immediate gain from developing land to be building and also increases the value of building option. These effects can either stimulate or postpone the optimal stopping time to develop land to be building.

However, after we simulated, we find out that an increase in volatility of fundamentals causes the land owner to develop land to be building faster than before. It represents by a decrease in the probability to develop land to be building after 40 years from now and the probability to continuously utilize the benefit from the vacant land forever combining with an increase in the probability to develop land to be building in the next 2-3 years from now.

We also represent the density of the optimal stopping time when the volatility of fundamentals increases in figure 7.13. It is very clear that when the volatility of fundamentals increases, the density function of the optimal stopping time tends to increase in the left tail which means that each land owner decides to develop land to be building earlier than before.

In a nutshell, an increase in volatility of fundamentals is the main factor which causes agents to have more information to disagree. In consequence, the size of bubble increases because of the aggressive trading among agents. An increase in size of bubble can also cause the value of immediate gain to increase. Finally, the land owner tends to develop land to be building prior to the exercise date because of a positive value of early exercise premium.

Ultimately, it should be noted that although we increase the value of information in signals in two cases: 1. a decrease in volatility of signals and 2. an increase in volatility of fundamentals by the same proportion, the results from these two cases quite differ. It means that actually, the results come from the effects of volatility of signals and volatility of fundamentals individually.

Table 7.2
The Effect of Real Interest rate on Density function of the Optimal Stopping Time by Using Monte Carlo Simulation

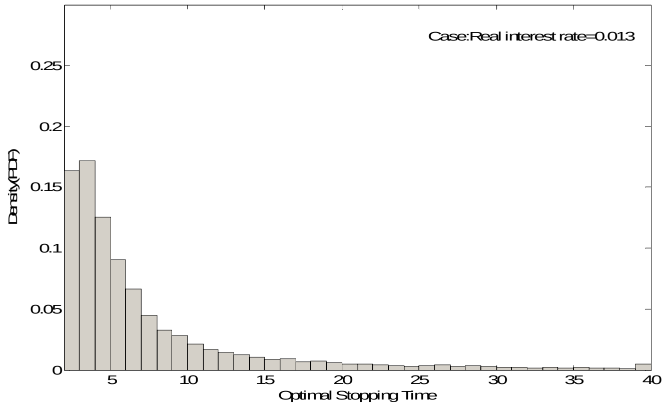
Probability at time τ	$r=0.013$	$r=0.016$	$r=0.019$	$r=0.022$	$r=0.027$	$r=0.032$
$P(\tau=2)$	0.1634	0.1717	0.1825	0.1939	0.1267	0.0001
$P(\tau=3)$	0.1719	0.1768	0.1797	0.1829	0.1419	0.0018
$P(\tau=4)$	0.1253	0.1245	0.1247	0.1225	0.0989	0.0053
$P(\tau=5)$	0.0909	0.0888	0.0871	0.0852	0.0767	0.0073
$P(\tau=6)$	0.0662	0.0643	0.0611	0.0593	0.0545	0.0097
$P(\tau=7)$	0.0447	0.0429	0.0416	0.0388	0.0395	0.0109
$P(\tau=8)$	0.033	0.0319	0.0317	0.0303	0.0298	0.0099
$P(\tau=9)$	0.0282	0.0272	0.0259	0.0244	0.0246	0.0107
$P(\tau=10)$	0.0212	0.0201	0.0191	0.0174	0.0205	0.0117
$P(11\le\tau\le20)$	0.0906	0.0937	0.0881	0.0829	0.1059	0.0919
$P(21\le\tau\le40)$	0.0616	0.0534	0.0512	0.0507	0.0768	0.1128
$P(\{\tau>40\}\cup\{\tau=\infty\})$	0.103	0.1047	0.1073	0.1117	0.2042	0.7279

Note:

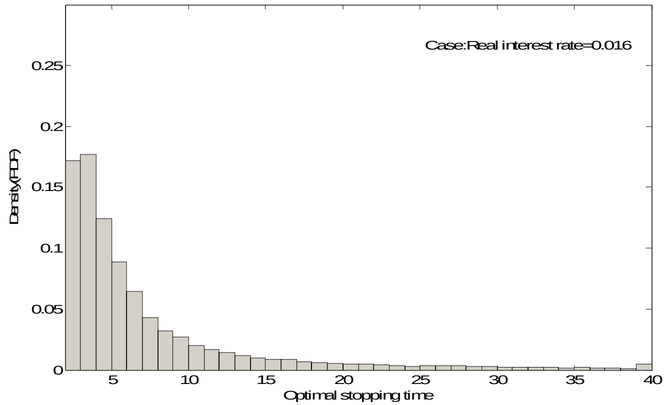
1. Each experiment is based on Monte Carlo simulation 15,000 times and uses the initial value of $\hat{f}_A = 0.03$ and $\hat{f}_B = 0.06$.
2. $P(\tau=i)$ for $i=2,3,4,\ldots,\infty$ presents the probability to develop land to be building at time i where the initial date is represented by $\tau=1$. For example, $P(\tau=9)=0.05$ means that the probability to develop land to be building at time $\tau=9$ equals 0.05.

Figure 7.8

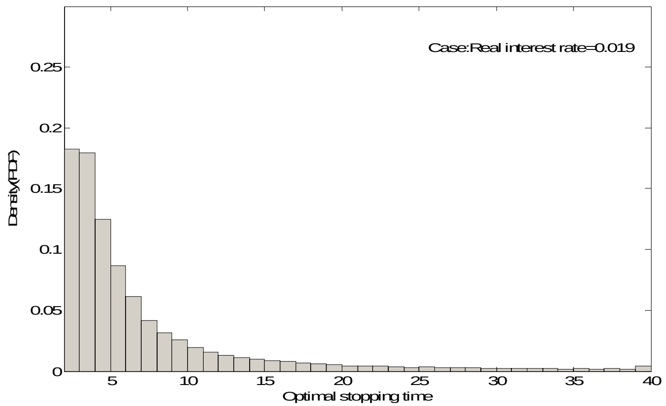
The Effect of Real Interest rate on Density function of the Optimal Stopping Time by Using Monte Carlo Simulation



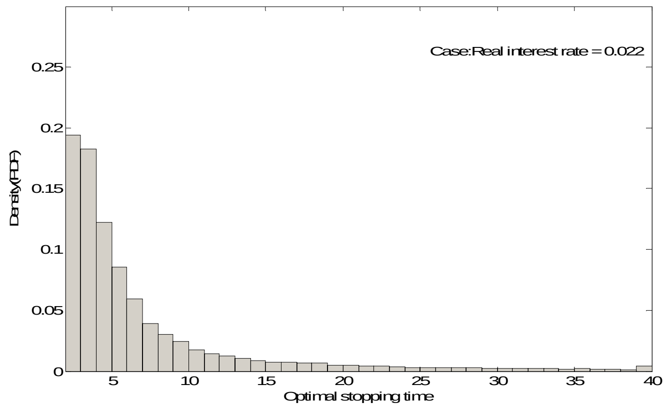
Panel A: Real interest rate = 0.013



Panel B: Real interest rate = 0.016



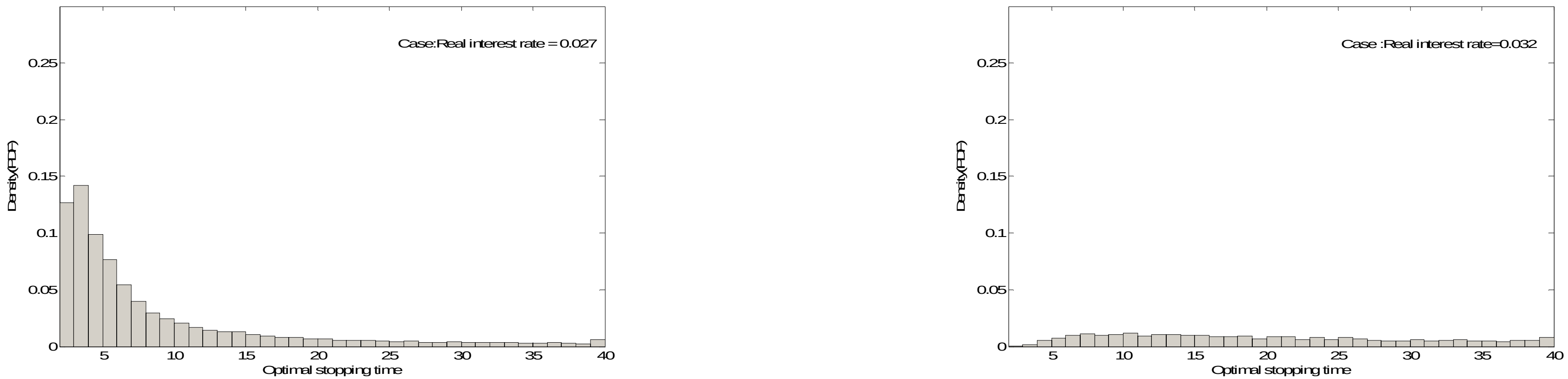
Panel C: Real interest rate = 0.019



Panel D: Real interest rate = 0.022

Figure 7.8 (Continued)

The Effect of Real Interest rate on Density function of the Optimal Stopping Time by Using Monte Carlo Simulation



Panel E: Real interest rate = 0.027 **Panel F: Real interest rate = 0.032**

Table 7.3
The Effect of Resale Cost on Density function of the Optimal Stopping Time by Using Monte Carlo Simulation

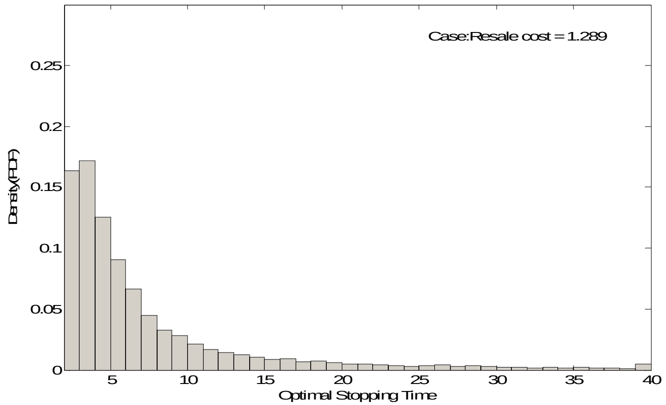
Probability at time τ	$C2=1.289$	$C2=1.934$	$C2=2.9$	$C2=4.35$	$C2=6.526$	$C2=9.788$
$P(\tau=2)$	0.1634	0.158	0.1553	0.1372	0.0901	0.0189
$P(\tau=3)$	0.1719	0.1714	0.1691	0.1622	0.1325	0.0513
$P(\tau=4)$	0.1253	0.1236	0.1235	0.1205	0.1085	0.0556
$P(\tau=5)$	0.0909	0.0918	0.0909	0.0909	0.0907	0.0551
$P(\tau=6)$	0.0662	0.0669	0.0667	0.0663	0.0651	0.0462
$P(\tau=7)$	0.0447	0.0449	0.046	0.048	0.0518	0.0405
$P(\tau=8)$	0.033	0.0333	0.033	0.0351	0.0388	0.0348
$P(\tau=9)$	0.0282	0.0291	0.0294	0.0304	0.0324	0.0312
$P(\tau=10)$	0.0212	0.0215	0.0216	0.0226	0.0246	0.0287
$P(11\le\tau\le20)$	0.0906	0.0993	0.101	0.1081	0.1311	0.1697
$P(21\le\tau\le40)$	0.0616	0.0551	0.0569	0.0606	0.0803	0.1351
$P(\{\tau>40\}\cup\{\tau=\infty\})$	0.103	0.1051	0.1066	0.1181	0.1541	0.3329

Note:

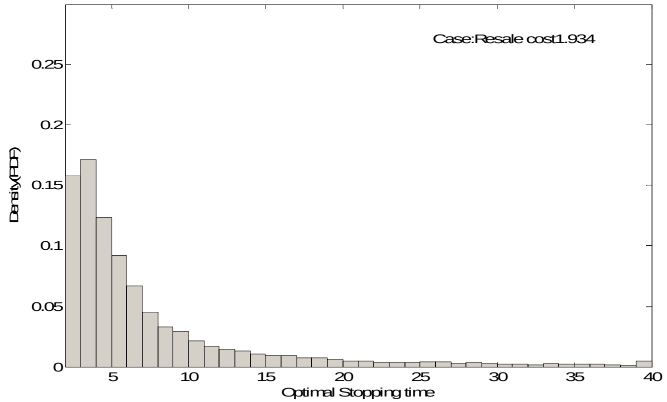
1. Each experiment is based on Monte Carlo simulation 15,000 times and uses the initial value of $\hat{f}_A = 0.03$ and $\hat{f}_B = 0.06$.
2. $P(\tau=i)$ for $i=2,3,4,...,\infty$ presents the probability to develop land to be building at time i where the initial date is represented by $\tau=1$. For example, $P(\tau=9)=0.05$ means that the probability to develop land to be building at time $\tau=9$ equals 0.05.

Figure 7.9

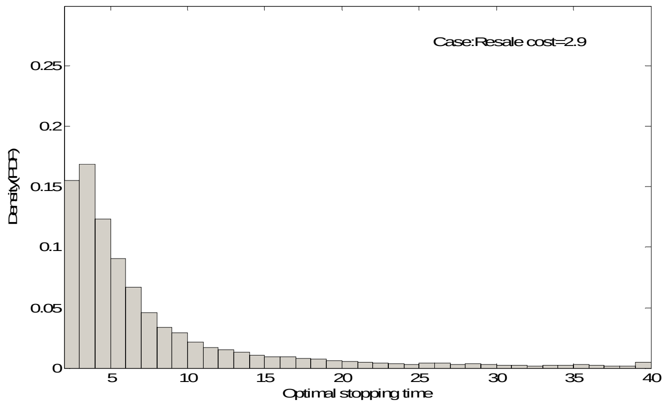
The Effect of Resale Cost on Density function of the Optimal Stopping Time by Using Monte Carlo Simulation



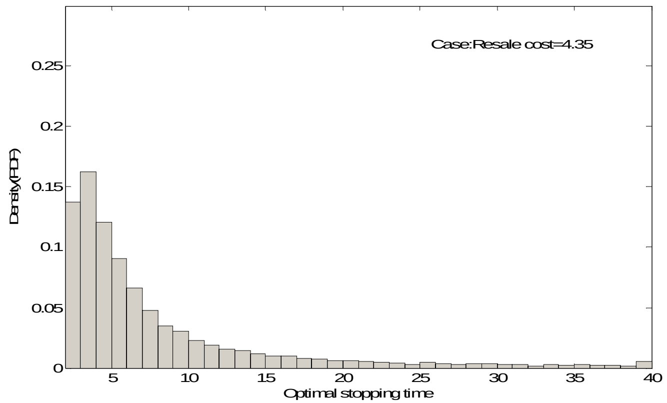
Panel A: Resale cost = 1.289



Panel B: Resale cost = 1.934



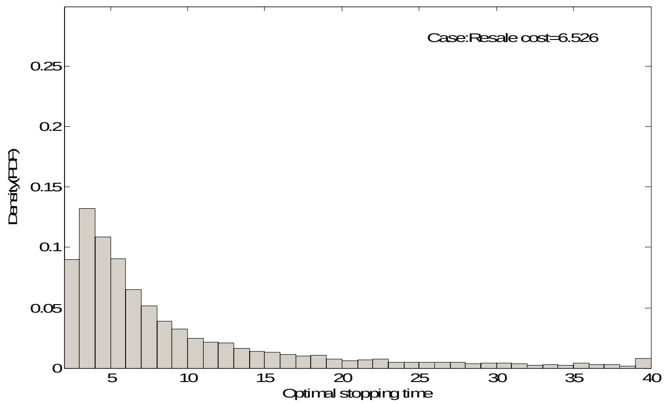
Panel C: Resale cost = 2.9



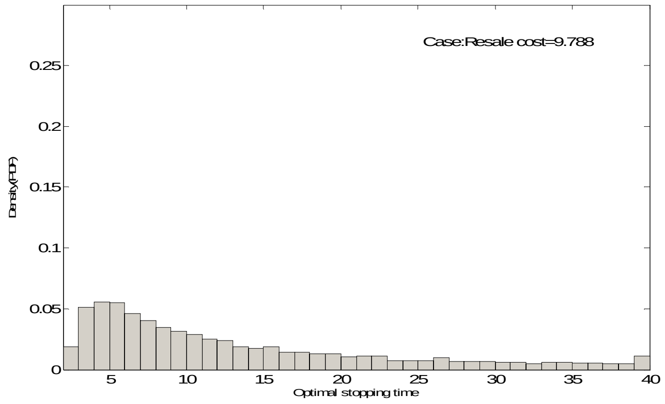
Panel D: Resale cost = 4.35

Figure 7.9 (Continued)

The Effect of Resale Cost on Density function of the Optimal Stopping Time by Using Monte Carlo Simulation



Panel E: Resale cost = 6.526



Panel F: Resale cost = 9.788

Table 7.4
The Effect of Overconfidence Level on Density function of the Optimal Stopping Time by Using Monte Carlo Simulation

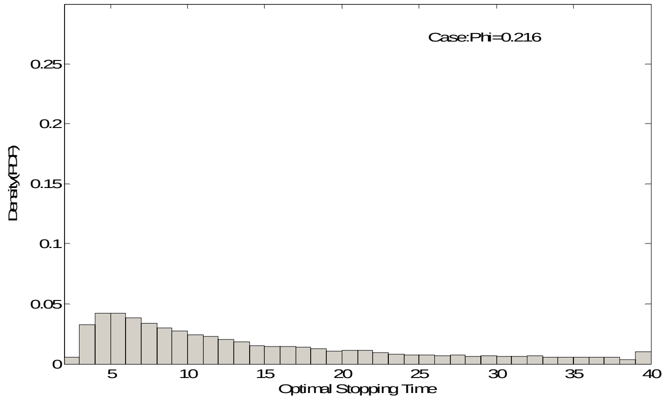
Probability at time τ	$\phi = 0.216$	$\phi = 0.309$	$\phi = 0.441$	$\phi = 0.63$	$\phi = 0.9$	$\phi = 1$
$P(\tau = 2)$	0.0052	0.0617	0.2312	0.2079	0.1634	0.1448
$P(\tau = 3)$	0.0324	0.1118	0.189	0.1889	0.1719	0.1611
$P(\tau = 4)$	0.0423	0.0891	0.1275	0.1304	0.1253	0.1241
$P(\tau = 5)$	0.0421	0.071	0.0815	0.0848	0.0909	0.0906
$P(\tau = 6)$	0.0383	0.0541	0.0505	0.0556	0.0662	0.0676
$P(\tau = 7)$	0.0341	0.0415	0.037	0.0399	0.0447	0.0458
$P(\tau = 8)$	0.0301	0.0328	0.0251	0.0302	0.033	0.0366
$P(\tau = 9)$	0.0277	0.0283	0.0195	0.0203	0.0282	0.0306
$P(\tau = 10)$	0.0243	0.0242	0.0173	0.0179	0.0212	0.0239
$P(11 \le \tau \le 20)$	0.1532	0.1343	0.0739	0.0786	0.0906	0.1091
$P(21 \le \tau \le 40)$	0.1303	0.0955	0.0458	0.049	0.0616	0.0609
$P(\{\tau > 40\} \cup \{\tau = \infty\})$	0.44	0.2557	0.1017	0.0965	0.103	0.1049

Note:

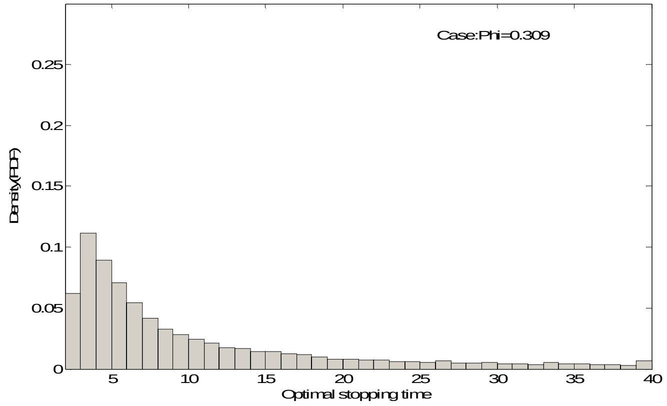
1. Each experiment is based on Monte Carlo simulation 15,000 times and uses the initial value of $\hat{f}_A = 0.03$ and $\hat{f}_B = 0.06$.
2. $P(\tau = i)$ for $i = 2, 3, 4, \ldots, \infty$ presents the probability to develop land to be building at time i where the initial date is represented by $\tau = 1$. For example, $P(\tau = 9) = 0.05$ means that the probability to develop land to be building at time $\tau = 9$ equals 0.05.

Figure 7.10

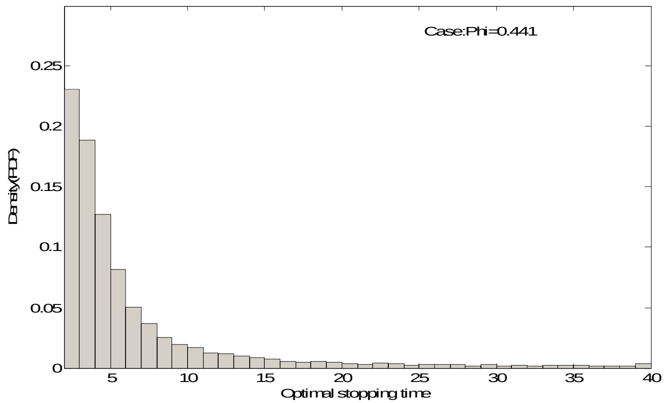
The Effect of Overconfidence Parameter on Density function of the Optimal Stopping Time by Using Monte Carlo Simulation



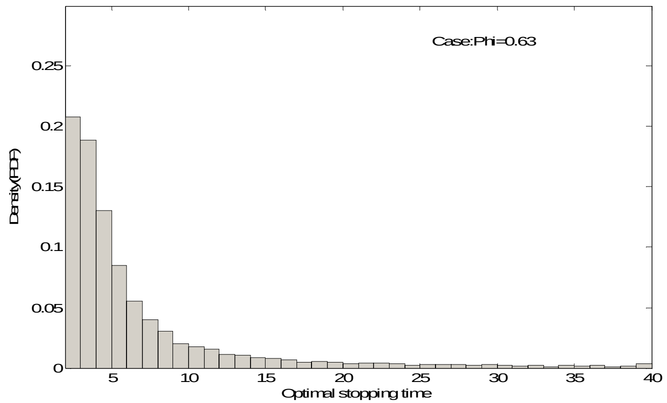
Panel A: Phi = 0.216



Panel B: Phi = 0.309



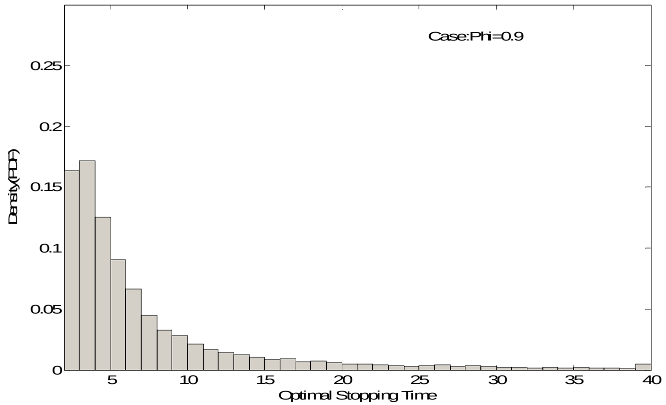
Panel C: Phi = 0.441



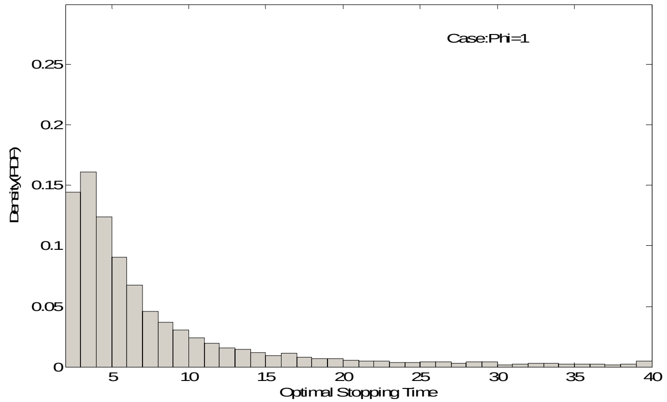
Panel D: Phi = 0.6

Figure 7.10 (Continued)

The Effect of Overconfidence Level on Density function of the Optimal Stopping Time by Using Monte Carlo Simulation



Panel E: Phi = 0.9



Panel F: Phi = 1

Table 7.5

The Effect of Long-Run Fundamental on Density function of the Optimal Stopping Time by Using Monte Carlo Simulation

Probability at time τ	$\bar{f} = 0.1$	$\bar{f} = 0.15$	$\bar{f} = 0.225$	$\bar{f} = 0.338$	$\bar{f} = 0.506$	$\bar{f} = 0.759$*
$P(\tau = 2)$	0.0052	0.013	0.0294	0.1013	0.2775	N.A.
$P(\tau = 3)$	0.0324	0.05	0.0813	0.1307	0.1717	N.A.
$P(\tau = 4)$	0.0423	0.0583	0.0773	0.1003	0.1144	N.A.
$P(\tau = 5)$	0.0421	0.0533	0.0657	0.0763	0.0777	N.A.
$P(\tau = 6)$	0.0383	0.0468	0.0537	0.0567	0.0511	N.A.
$P(\tau = 7)$	0.0341	0.0375	0.0425	0.0463	0.038	N.A.
$P(\tau = 8)$	0.0301	0.0325	0.0347	0.0341	0.0304	N.A.
$P(\tau = 9)$	0.0277	0.032	0.0317	0.0316	0.0233	N.A.
$P(\tau = 10)$	0.0243	0.0244	0.0276	0.0251	0.0173	N.A.
$P(11 \le \tau \le 20)$	0.1532	0.1591	0.1531	0.1366	0.0815	N.A.
$P(21 \le \tau \le 40)$	0.1303	0.1286	0.123	0.0911	0.0466	N.A.
$P(\{\tau > 40\} \cup \{\tau = \infty\})$	0.44	0.3645	0.28	0.1699	0.0705	N.A.

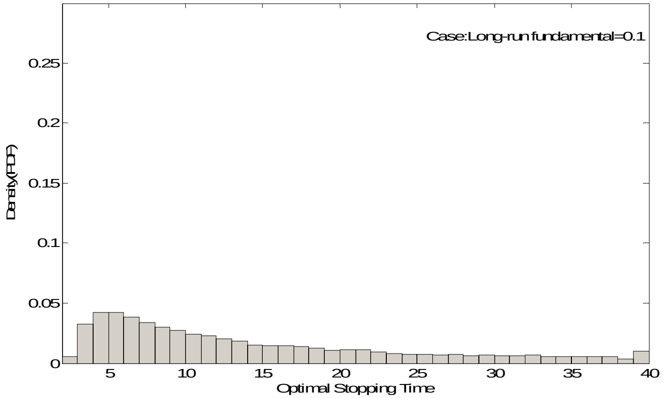
Note: 1. When the long-run fundamental is equal to 0.759, land owner will decide to develop land to be building at the beginning date which is $\tau = 1$.

2. Each experiment is based on Monte Carlo simulation 15,000 times and uses the initial value of $\hat{f}_A = 0.03$ and $\hat{f}_B = 0.06$.

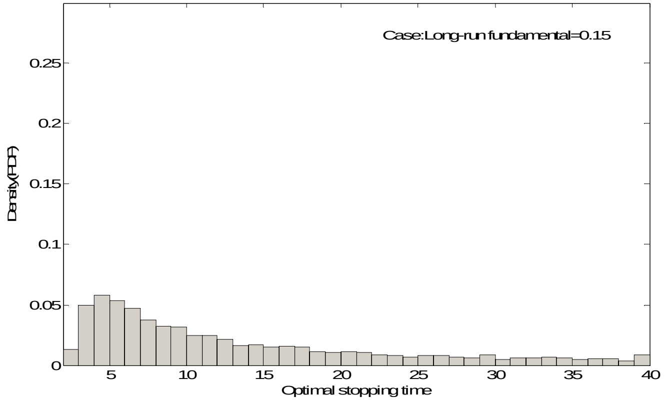
3. $P(\tau = i)$ for $i = 2, 3, 4, ..., \infty$ presents the probability to develop land to be building at time i where the initial date is represented by $\tau = 1$. For example, $P(\tau = 9) = 0.05$ means that the probability to develop land to be building at time $\tau = 9$ equals 0.05.

Figure 7.11

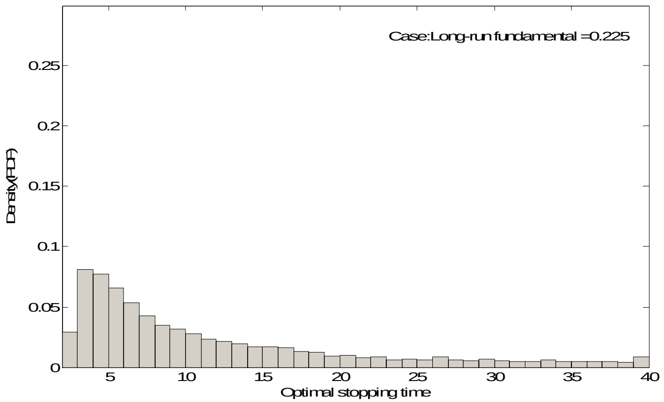
The Effect of Long-Run Fundamental on Density function of the Optimal Stopping Time by Using Monte Carlo Simulation



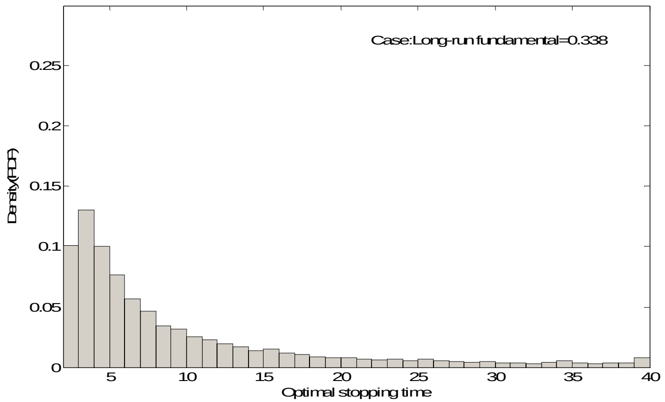
Panel A: Long-run fundamental = 0.1



Panel B: Long-run fundamental = 0.15



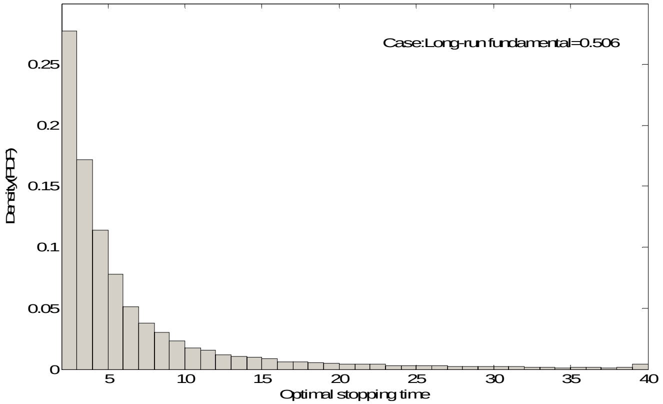
Panel C: Long-run fundamental = 0.225



Panel D: Long-run fundamental = 0.338

Figure 7.11 (Continued)

The Effect of Long-Run Fundamental on Density function of the Optimal Stopping Time by Using Monte Carlo Simulation



Panel E: Long-run fundamental = 0.506

Table 7.6
The Effect of Volatility of Signals on Density function of the Optimal Stopping Time by Using Monte Carlo Simulation

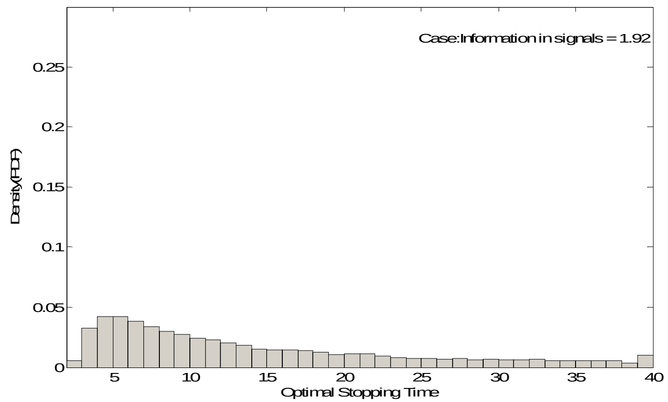
Probability at time τ	$i_s = 1.92$	$i_s = 2.21$	$i_s = 2.54$	$i_s = 2.92$	$i_s = 3.36$	$i_s = 3.86$
$P(\tau = 2)$	0.0052	0.0045	0.0043	0.004	0.0033	0.0031
$P(\tau = 3)$	0.0324	0.0298	0.0278	0.0258	0.0234	0.0203
$P(\tau = 4)$	0.0423	0.0395	0.037	0.0348	0.0321	0.0288
$P(\tau = 5)$	0.0421	0.041	0.0392	0.0369	0.0336	0.0307
$P(\tau = 6)$	0.0383	0.0381	0.0361	0.0346	0.0323	0.0293
$P(\tau = 7)$	0.0341	0.0334	0.032	0.0305	0.0289	0.0257
$P(\tau = 8)$	0.0301	0.0294	0.0281	0.0276	0.0263	0.0239
$P(\tau = 9)$	0.0277	0.0267	0.0244	0.0245	0.0238	0.0224
$P(\tau = 10)$	0.0243	0.0237	0.0227	0.0217	0.0211	0.0206
$P(11 \le \tau \le 20)$	0.1532	0.1535	0.1519	0.148	0.1426	0.1353
$P(21 \le \tau \le 40)$	0.1303	0.1319	0.1308	0.1285	0.1275	0.126
$P(\{\tau > 40\} \cup \{\tau = \infty\})$	0.44	0.4485	0.4657	0.4831	0.5051	0.5339

Note:

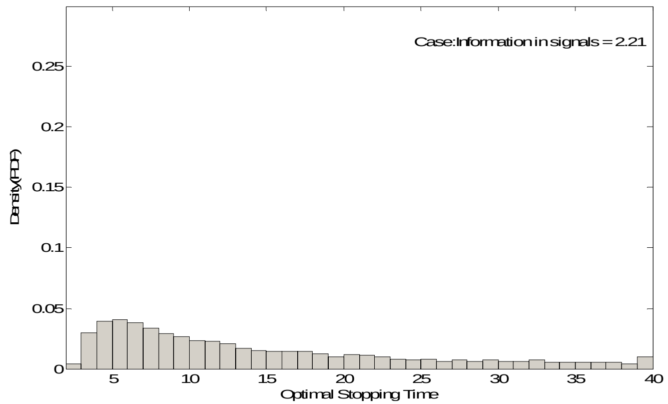
1. Each experiment is based on Monte Carlo simulation 15,000 times and uses the initial value of $\hat{f}_A = 0.03$ and $\hat{f}_B = 0.06$.
2. $P(\tau = i)$ for $i = 2,3,4,...,\infty$ presents the probability to develop land to be building at time i where the initial date is represented by $\tau = 1$. For example, $P(\tau = 9) = 0.05$ means that the probability to develop land to be building at time $\tau = 9$ equals 0.05.

Figure 7.12

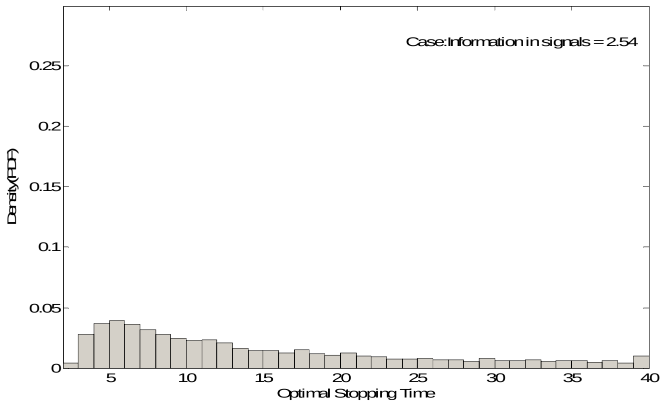
The Effect of Volatility of Signals on Density function of the Optimal Stopping Time by Using Monte Carlo Simulation



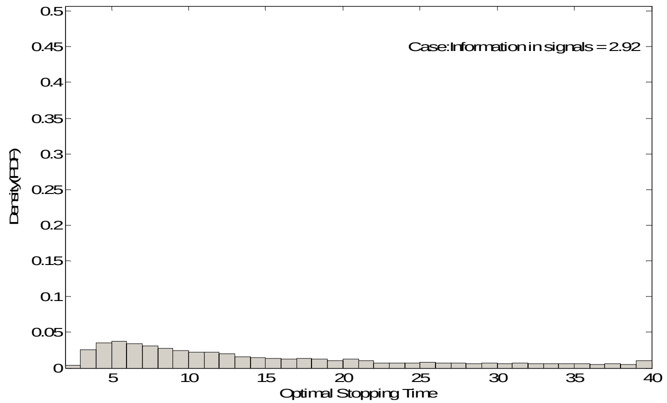
Panel A: Information in signals = 1.92



Panel B: Information in signals = 2.21



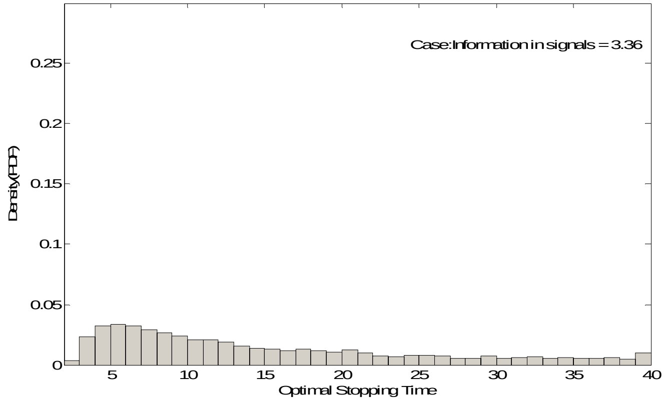
Panel C: Information in signals = 2.54



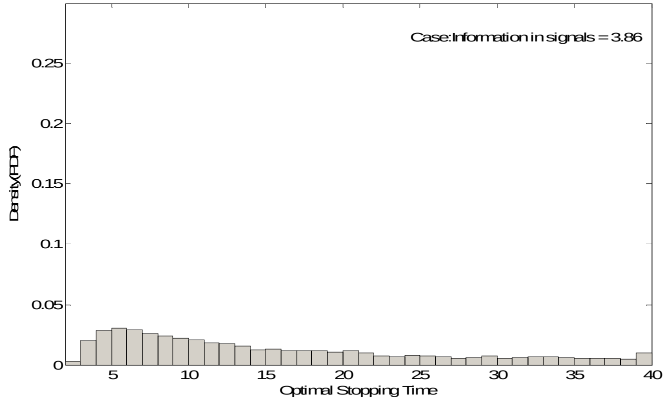
Panel D: Information in signals = 2.92

Figure 7.12 (continued)

The Effect of Volatility of Signals on Density function of the Optimal Stopping Time by Using Monte Carlo Simulation



Panel E: Information in signal = 3.36



Panel F: Information in signal = 3.86

Table 7.7

The Effect of Volatility of Fundamentals on Density function of the Optimal Stopping Time by Using Monte Carlo Simulation

Probability at time τ	$i_s = 1.92$	$i_s = 2.21$	$i_s = 2.54$	$i_s = 2.92$	$i_s = 3.36$	$i_s = 3.86$
$P(\tau = 2)$	0.0052	0.0276	0.0954	0.223	0.3895	N.A.
$P(\tau = 3)$	0.0324	0.0745	0.1246	0.1606	0.1725	N.A.
$P(\tau = 4)$	0.0423	0.068	0.0863	0.0871	0.0797	N.A.
$P(\tau = 5)$	0.0421	0.0563	0.0639	0.0599	0.0513	N.A.
$P(\tau = 6)$	0.0383	0.0468	0.0499	0.0417	0.0294	N.A.
$P(\tau = 7)$	0.0341	0.0381	0.0386	0.0324	0.0232	N.A.
$P(\tau = 8)$	0.0301	0.033	0.0277	0.0252	0.0163	N.A.
$P(\tau = 9)$	0.0277	0.0283	0.0261	0.0189	0.0133	N.A.
$P(\tau = 10)$	0.0243	0.0243	0.0211	0.018	0.0123	N.A.
$P(11 \le \tau \le 20)$	0.1532	0.1472	0.1233	0.0918	0.0608	N.A.
$P(21 \le \tau \le 40)$	0.1303	0.1154	0.0898	0.0656	0.0426	N.A.
$P(\{\tau > 40\} \cup \{\tau = \infty\})$	0.44	0.3405	0.2533	0.1758	0.1091	N.A.

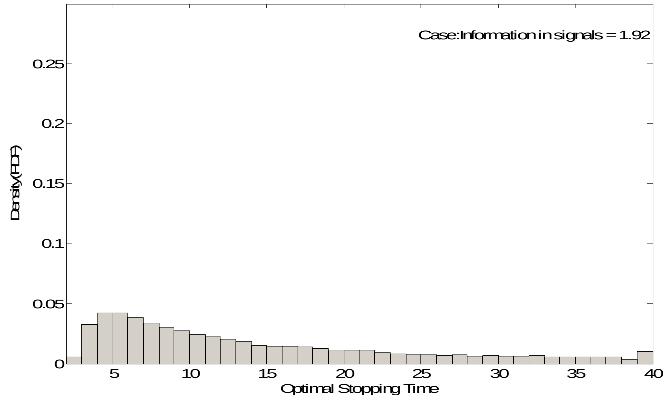
Note: 1. When the volatility of fundamentals is equal to 0.759, land owner will decide to develop land to be building at the beginning date which is $\tau = 1$.

2. Each experiment is based on Monte Carlo simulation 15,000 times and uses the initial value of $\hat{f}_A = 0.03$ and $\hat{f}_B = 0.06$.

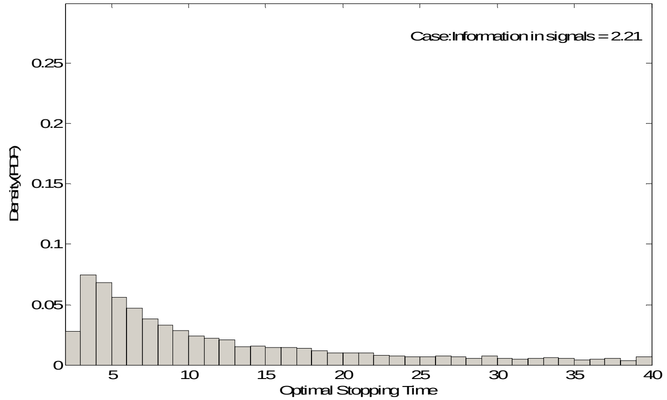
3. $P(\tau = i)$ for $i = 2, 3, 4, \ldots, \infty$ presents the probability to develop land to be building at time i where the initial date is represented by $\tau = 1$. For example, $P(\tau = 9) = 0.05$ means that the probability to develop land to be building at time $\tau = 9$ equals 0.05.

Figure 7.13

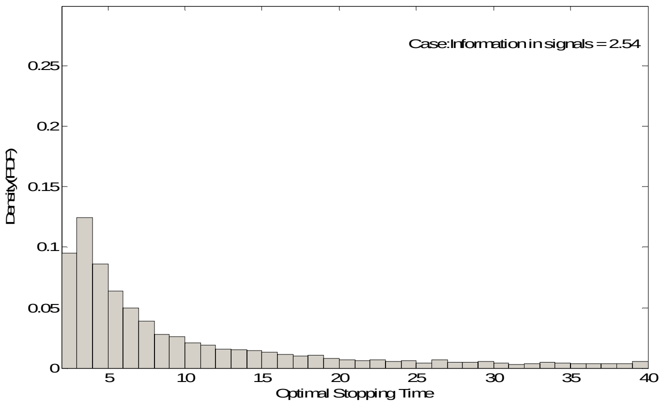
The Effect of Volatility of Fundamentals on Density function of the Optimal Stopping Time by Using Monte Carlo Simulation



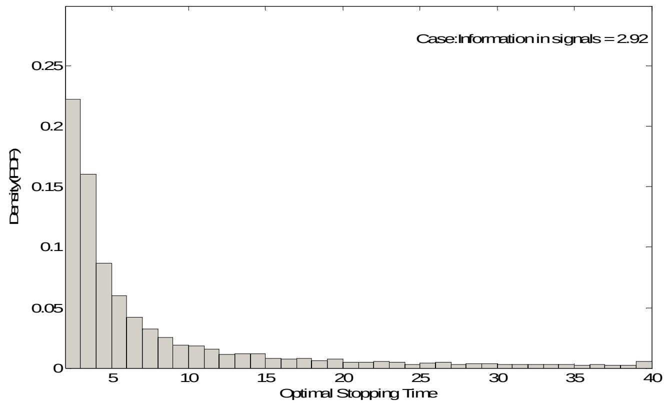
Panel A: Information in signals = 1.92



Panel B: Information in signals = 2.21



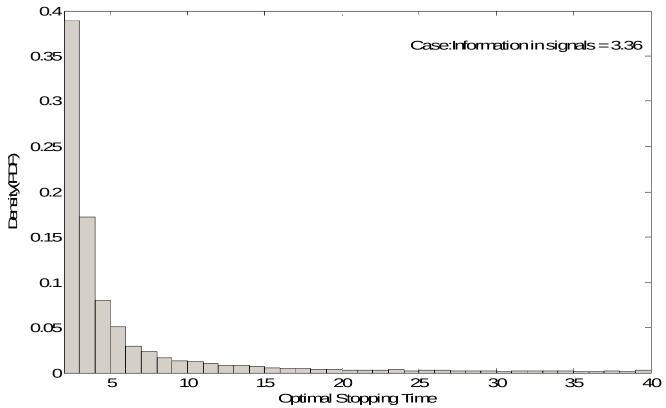
Panel C: Information in signals = 2.54



Panel D: Information in signals = 2.92

Figure 7.13 (continued)

The Effect of Volatility of Fundamentals on Density function of the Optimal Stopping Time by Using Monte Carlo Simulation



Panel E: Information in signal = 3.36

CHAPTER 8

CONCLUSION

8.1 Summary

In this study, we analyze how five policies which are (1) an increase in interest rate policy (2) an increase in resale cost (3) an increase in overconfidence level (4) an increase in long-run fundamental, and (5) an increase in information in signals affect the decision of land owner to develop land to be building by employing Preechametta (2005) model. Based on this model, the main reason causing the bubble in property market comes from the tendency of agents to overestimate the precision of their knowledge, provides a convenient way to generate heterogeneous beliefs. Moreover, this model shows that abundant information is an important factor that can speed up the property price bubble by generating disagreement among investors. Barber and Odean (2001) explain that when people who initially disagree on a topic are given arguments on either side of the issue, they become further polarized in their beliefs in circumstance such that they have variety of information.

In the equilibrium, each land owner will decide to develop a piece of vacant land to be building if and only if his building option has the value equals the immediate gain from developing land to be building. This building option has the characteristics similar to American option. That is, the land owner can exercise it for any time t from the initial date. From the land owner’s view point, he tends to develop land to be building prior to the exercise date if he receives the higher gain from early exercise premium.

The positive early exercise premium typically generates from the value of bubble which is in the value of resale option. In this model, it shows that the larger size of bubble will lead the land owner to have the early exercise which brings about the dynamic inefficient in land utility.

In light of these concerns, in this study, we aim to analyze the main factors that determine the decision of land owner about the optimal stopping time to develop land to be building in order to find out ways to handle with all possible bubbles in property market in the future. Nevertheless, based on the characteristics of building option, there is no explicit solution for it. We therefore employ the Finite Difference Method (FDM) and Monte Carlo techniques to identify its optimal stopping time.

Our core policy simulation results indicate that an increase in interest rate policy can generally decrease the size of bubble and, in turn, a delay in land development. This is so because during the period of rising interest rate, the reduction in the gain from investing the new development project immediately is much more significant than the reduction in building option value of the new development project. Besides, an increase in resale cost such as transfer fee can reduce the aggressive trading in property market as well.

This study also finds out that an increase in long-run fundamental caused by large investment in infrastructure and an increase in overconfidence level play a significant role in stimulating land owner to develop land to be building.

However, for the policy simulation on the effect of an increase in information in signals, this study shows that level of overconfidence is of crucial importance in determining this effect. In the case of low overconfidence level in this study, an increase in information brought about by a decrease in volatility of signals has insignificantly effect on the optimal time to develop land to be building. On the contrary, an increase in information caused by an increase in volatility of fundamentals drives significantly the land owner to develop land to be building prior the exercise date because of a positive value of early exercise premium.

The above distinct results generated from the different effects of a decrease in volatility of signals and an increase in volatility of fundamentals can be obtained because, in contrast to the case of a decrease in volatility of signals, when the volatility of fundamentals increases, it also increases the volatility of the difference in beliefs which causes the size of bubble to increase.

This study also tests the rational bubble in the stock market and sub-stock market (property stock market). In a nutshell, the results that come out from these tests demonstrate that both Thailand stock and property stock prices satisfy a sufficient condition for the absence of rational bubbles. In other words, rational bubble did not exist in stock and property stock prices for the period examined.

8.2 Limitations of the Study and Suggestions for Further Study

This chapter will not completely finish without illustrating the limitations of this study and some suggestions for the future studies.

1. One of limitations of this study comes from the difficulties and inadequacy of Thailand' property market database. Therefore, it can only do the policy simulations by applying the U.S. property data. By this problem, we strongly suggest all relevant organizations to begin systematically collecting and constructing all necessary standard property databases. In the near future, with better data, the government and policy makers can enable to more effective control the expansion of the property sector which may contain the asset price bubble. And also, for researchers, they can employ these data to study their work related property market in order to find the ways to deal with all possible bubbles in property market.
2. Based on Preechametta (2005), it is interesting for researchers to develop the model which has other choices for land owner to develop his vacant land.
3. There are some variables in this model that need to be identified such as overconfidence level, the speed of adjustment of difference in beliefs (see Alpert and Raiffa (1982)).
4. Ultimately, for the rational bubble testing, even though we find out from our results that there is no bubble in stock market in our data range, some evidences still point out that there are the rational bubbles in both stock and property market in the past. Therefore, it is interesting to employ other methods to detect them such as a stochastic unit root examined by McCabe and Tremayne (1995), Leybourne et al. (1996), and Granger and Swanson (1997) or a Markov-switching model, and Zivot and Andrews (1992) test.

APPENDICES

APPENDIX A

Proof of equation 4.6[1]

The stationary variance of γ decrease with ϕ

From $$\gamma \equiv \frac{\sqrt{\left[\lambda + \left(\phi\sigma_f / \sigma_s\right)\right]^2 + \left(1-\phi^2\right)\left[\left(2\frac{\sigma_f^2}{\sigma_s^2}\right) + \left(\frac{\sigma_f^2}{\sigma_D^2}\right)\right]} - \left[\lambda + \left(\phi\frac{\sigma_f}{\sigma_s}\right)\right]}{\left(\frac{1}{\sigma_D^2}\right) + \left(\frac{2}{\sigma_s^2}\right)} \tag{A.1}$$

Let $\upsilon(\phi) = \lambda + \phi\left(\frac{\sigma_f}{\sigma_s}\right)$ and $i(\phi) = \left(1-\phi^2\right)\left[\left(\frac{2\sigma_f^2}{\sigma_s^2}\right) + \left(\frac{\sigma_f^2}{\sigma_D^2}\right)\right]$

$$\frac{dr}{d\phi} = \frac{\frac{1}{2}\frac{2\nu\left(\frac{d\nu}{d\phi}\right) + \frac{di}{d\phi}}{\sqrt{\nu^2 + i}} - \frac{d\nu}{d\phi}}{\left(\frac{1}{\sigma_D^2}\right) + \left(\frac{2}{\sigma_s^2}\right)} \leq 0$$

Proof of equation 4.16

The process for g^A can be derived by using equation 4.7 and 4.11

$$d\hat{f}^A = -\lambda(\hat{f}^A - \bar{f})dt + \frac{\phi\sigma_s\sigma_f + \gamma}{\sigma_s^2}(ds^A - \hat{f}^A dt) + \frac{\gamma}{\sigma_s^2}(ds^B - \hat{f}^A dt)$$
$$+ \frac{\gamma}{\sigma_D^2}(dD - \hat{f}^A dt) \tag{A.2}$$

[1] These proofs are summarized from Scheinkman and Xiong (2003).

, and

$$d\hat{f}_t^{B} = -\lambda(\hat{f}_t^{B} - \overline{f})dt + \frac{\gamma}{\sigma_s^2}(ds_t^A - \hat{f}_t^{B}dt) + \frac{\phi\sigma_s\sigma_f + \gamma}{\sigma_s^2}(ds_t^B - \hat{f}_t^{B}dt) + \frac{\gamma}{\sigma_D^2}(dD_t - \hat{f}_t^{B}dt) \tag{A.3}$$

Then using equation (A.3) minus equation (A.2) , we get

$$dg^A = d\hat{f}^B - d\hat{f}^A = -\left[\lambda + \frac{2\gamma + \phi\sigma_s\sigma_f}{\sigma_s^2} + \frac{\gamma}{\sigma_D^2}\right]g^A dt + \frac{\phi\sigma_f}{\sigma_s}\left(ds^B - ds^A\right). \tag{A.4}$$

Using the formula for γ , we can write dg^A as

$$dg^A = -\rho g^A dt + \frac{\phi\sigma_f}{\sigma_s}\left(\sigma_s dW_B^A - \sigma_s dW_A^A\right), \tag{A.5}$$

where

$$\rho = \sqrt{\left(\lambda + \phi\frac{\sigma_f}{\sigma_s}\right)^2 + \left(1-\phi^2\right)\sigma_f^2\left(\frac{2}{\sigma_s^2} + \frac{1}{\sigma_D^2}\right)}. \tag{A.6}$$

The result follows by writing

$$W_g^A = \frac{1}{\sqrt{2}}\left(W_B^A - W_A^A\right). \tag{A.7}$$

Proof of equation 4.36

The value of option $q(x)$ should be at least as large as the gains realized from an immediate sale. Using Ito's lemma and the evolution equation for g_t^o, Scheinkman and Wei Xiong (2003) get the conditions as

$$q(x) \geq \frac{x}{r+\lambda} + q(-x) - c \tag{A.8}$$

, and

$$\frac{1}{2}\sigma_g^2 q'' - \rho x q' - rq \leq 0 \,. \tag{A.9}$$

To construct the function q, the continuation region will be an interval $\left(-\infty, k^*\right)$,with $k^* > 0$. k^* is the minimum amount of difference in opinions that generate the trade. q must satisfy the second order ordinary differential equation, even though only in the continuation region.

$$\frac{1}{2}\sigma_g^2 u'' - \rho x u' - ru = 0 \,. \tag{A.10}$$

Let $v(y)$ be a solution to the differential equation

$$y v''(y) + (\frac{1}{2} - y) v'(y) - \frac{r}{2\rho} v(y) = 0 \,. \tag{A.11}$$

It can verify that

$$u(x) = v\left(\left(\frac{\rho}{\sigma_g^2}\right) x^2\right), \tag{A.12}$$

is the solution of equation (A.10).

The general solution of equation (A.12) is (See Abramowitz and Stegun 1964, ch. 13)

$$v(y) = \alpha M\left(\frac{r}{2\rho}, \frac{1}{2}, y\right) + \beta U\left(\frac{r}{2\rho}, \frac{1}{2}, y\right), \tag{A.13}$$

where $M\left(\bullet,\bullet,\bullet\right) and\, U\left(\bullet,\bullet,\bullet\right)$ are Kummer functions define as

$$M(a,b,y) = 1 + a\frac{y}{b} + \frac{(a)_2 y^2}{\left(b\right)_2 2!} + ... + \frac{\left(a\right)_n y^n}{(b)_n n!} + ..., , \tag{A.14}$$

With $(a)_n = a(a+1)(a+2)...(a+n-1)$ and $(a_0) = 1$ and

$$U(a,b,y) = \frac{\pi}{\sin \pi b}\left[\frac{M(a,b,y)}{\Gamma(1+a-b)\Gamma\left(b\right)} - y^{1-b}\frac{M\left(1+a-b, 2-b, y\right)}{\Gamma(a)\Gamma(2-b)}\right], \tag{A.15}$$

where

$M_y(a,b,y) > 0$ for all y>0

$M(a,b,y) \to \infty, and\, U\left(a,b,y\right) \to 0\, as\; y \to \infty.$

Given a solution u to equation (A.10), we can construct two solutions to equation (A.12) by using the value of the function for $x < 0\, and\;\, for\, x > 0$. We shall denote that the corresponding linear combinations of M and U by

$$\alpha_1 M + \beta_1 U \text{ and } \alpha_2 M + \beta_2 U .$$

To guarantee that $u(x)$ is positive and increasing for $x < 0$, α_1 must be zero. Therefore,

$$u(x) = \beta_1 U(\frac{r}{2\rho}, \frac{1}{2}, \frac{\rho}{\sigma_g^2}x^2) \quad \text{if } x \le 0 . \tag{A.16}$$

For other region, the solution can be express as

$$u(x)=\alpha_2 M\left(\frac{r}{2\rho},\frac{1}{2},\frac{\rho}{\sigma_g^2}x^2\right)+\beta_2 U\left(\frac{r}{2\rho},\frac{1}{2},\frac{\rho}{\sigma_g^2}x^2\right) \quad \text{if } x>0 \qquad \text{(A.17)}$$

The solution must be continuously differentiable at x = 0. From the definition of the two Kummer function, we can show that

$$x\to 0^-, u(x)\to \frac{\beta_1\pi}{\Gamma(\frac{1}{2}+\left(\frac{r}{2\rho}\right)\Gamma\left(\frac{1}{2}\right)} \qquad u'(x)\to \frac{\beta_1\pi\sqrt{\rho}}{\sigma_g\Gamma\left(\frac{r}{2\rho}\right)\Gamma\left(\frac{3}{2}\right)}$$

$$x\to 0^+, u(x)\to \alpha_2+\frac{\beta_2\pi}{\Gamma\left(\frac{1}{2}+\left(\frac{r}{2\rho}\right)\Gamma\left(\frac{1}{2}\right)\right)} \qquad u'(x)\to -\frac{\beta_2\pi\sqrt{\rho}}{\sigma_g\Gamma\left(\frac{r}{2\rho}\right)\Gamma\left(\frac{3}{2}\right)}$$

(A.18)

Using (A.18) we will find that

$\beta_2=-\beta_1$, and

$$\alpha_2=\frac{2\beta_1\pi}{\Gamma\left(\frac{1}{2}+\left(\frac{r}{2\rho}\right)\right)\Gamma\left(\frac{1}{2}\right)}.$$

The function value at x = 0 equals

$$u(x)=\frac{\beta_1\pi}{\Gamma\left(\frac{1}{2}+\frac{r}{2\rho}\right)\Gamma\left(\frac{1}{2}\right)}.$$

where

$$x \to 0^-, u(x) \to \frac{\beta_1 \pi}{\Gamma(\frac{1}{2} + \left(\frac{r}{2\rho}\right)\Gamma\left(\frac{1}{2}\right)}$$
$$x \to 0^+, u(x) \to \frac{2\beta_1 \pi}{\Gamma\left(\frac{1}{2} + \left(\frac{r}{2\rho}\right)\right)\Gamma\left(\frac{1}{2}\right)} - \frac{\beta_1 \pi}{\Gamma\left(\frac{1}{2} + \left(\frac{r}{2\rho}\right)\Gamma\left(\frac{1}{2}\right)\right)} \quad . \qquad \text{(A.19)}$$

Equation (A.19) can be rewritten to the following from

$$x \to 0^-, u(x) \to \beta_1 h(x) \qquad where \quad h(x) = \frac{\pi}{\Gamma(\frac{1}{2} + \left(\frac{r}{2\rho}\right)\Gamma\left(\frac{1}{2}\right)} = U(\frac{r}{2\rho}, \frac{1}{2}, \frac{\rho}{\sigma_g^2} x^2)$$
$$x \to 0^+, u(x) \to \frac{2\beta_1 \pi}{\Gamma\left(\frac{1}{2} + \left(\frac{r}{2\rho}\right)\right)\Gamma\left(\frac{1}{2}\right)} M\left(\frac{r}{2\rho}, \frac{1}{2}, \frac{\rho}{\sigma_g^2} x^2\right) - \frac{\beta_1 \pi}{\Gamma\left(\frac{1}{2} + \left(\frac{r}{2\rho}\right)\Gamma\left(\frac{1}{2}\right)\right)} \qquad \text{(A.20)}$$

Thus,

$$h(x) = \begin{cases} U(\frac{r}{2\rho}, \frac{1}{2}, \frac{\rho}{\sigma_g^2} x^2) & if \ x \le 0 \\ \frac{2\pi}{\Gamma\left(\frac{1}{2} + \left(\frac{r}{2\rho}\right)\right)\Gamma\left(\frac{1}{2}\right)} M\left(\frac{r}{2\rho}, \frac{1}{2}, \frac{\rho}{\sigma_g^2} x^2\right) - U\left(\frac{r}{2\rho}, \frac{1}{2}, \frac{\rho}{\sigma_g^2} x^2\right) & if \ x > 0 \end{cases}$$

(A.21)

APPENDIX B

PARAMETER VALUES

In order to identify parameter values, we employ the United State housing data from Davis Morris's paper[1], the Treasury bill rates and consumer price index are available in IFS data base. These data are presented in table B1.

Table B1

The United State Data

YEAR (1)	AVERAGE ANNUAL RENT (Ten U.S.$) (2)	AVERAGE HOUSE PRICE (Ten U.S. $) (3)	TREASURY BILL RATE (4)	CONSUMER PRICE INDEX (5)	INFLATION (6)	REAL TREASURY BILL RATE (7)=(4)-(6)	REAL AVERAGE ANNUAL RENT (8)=(2)/(5)	REAL AVERAGE HOUSE PRICE (9)=(3)/(5)	CHANGE IN REAL AVERAGE ANNUAL RENT (10)
1960	81.98	1462.83	2.94	17.19	1.54	0.51	4.7691	85.0979	-
1961	84.84	1512.51	2.38	17.37	1.05	1.33	4.8840	87.0761	0.1149
1962	87.72	1558.89	2.78	17.57	1.15	1.63	4.9926	88.7246	0.1086
1963	90.52	1602.81	3.16	17.78	1.20	1.96	5.0911	90.1469	0.0985
1964	93.40	1684.52	3.55	18.01	1.29	2.26	5.1860	93.5326	0.0949
1965	96.36	1762.30	3.95	18.31	1.67	2.28	5.2629	96.2482	0.0769

[1] All the housing data are available in paper "the rent-price ratio for the aggregate stock of owner-occupied housing". It is downloadable from http://morris.marginalq.com/2005-05-DLM_paper.pdf.

YEAR (1)	AVERAGE ANNUAL RENT (Ten U.S.$) (2)	AVERAGE HOUSE PRICE (Ten U.S. $) (3)	TREASURY BILL RATE (4)	CONSUMER PRICE INDEX (5)	INFLATION (6)	REAL TREASURY BILL RATE (7)=(4)-(6)	REAL AVERAGE ANNUAL RENT (8)=(2)/(5)	REAL AVERAGE HOUSE PRICE (9)=(3)/(5)	CHANGE IN REAL AVERAGE ANNUAL RENT (10)
1966	99.83	1878.39	4.88	18.86	3.00	1.88	5.2934	99.5967	0.0305
1967	103.96	2023.47	4.33	19.38	2.76	1.57	5.3643	104.4102	0.0709
1968	108.78	2193.59	5.35	20.2	4.23	1.12	5.3853	108.5937	0.0210
1969	114.79	2223.69	6.69	21.29	5.40	1.29	5.3919	104.4477	0.0066
1970	122.55	2119.97	6.44	22.55	5.92	0.52	5.4347	94.0118	0.0427
1971	132.04	2176.01	4.34	23.51	4.26	0.08	5.6163	92.5566	0.1817
1972	140.83	2355.54	4.07	24.29	3.32	0.75	5.7979	96.9758	0.1816
1973	151.85	2627.88	7.03	25.8	6.22	0.81	5.8857	101.8559	0.0878
1974	164.62	2917.62	7.87	28.64	11.01	-3.14	5.7478	101.8721	-0.1379
1975	178.49	3189.43	5.82	31.26	9.15	-3.33	5.7100	102.0292	-0.0378
1976	194.27	3517.47	4.99	33.05	5.73	-0.74	5.8782	106.4287	0.1682
1977	212.59	4003.19	5.27	35.2	6.51	-1.24	6.0396	113.7270	0.1614
1978	234.66	4673.61	7.22	37.89	7.64	-0.42	6.1931	123.3469	0.1535
1979	259.73	5375.48	10.04	42.16	11.27	-1.23	6.1606	127.5020	-0.0325
1980	291.26	5970.81	11.62	47.85	13.50	-1.88	6.0870	124.7817	-0.0736
1981	321.01	6439.58	14.08	52.79	10.32	3.76	6.0808	121.9849	-0.0062
1982	349.27	6768.71	10.73	56.04	6.16	4.57	6.2326	120.7836	0.1518
1983	374.35	7091.55	8.62	57.84	3.21	5.41	6.4722	122.6063	0.2396
1984	399.70	7518.88	9.39	60.34	4.32	5.07	6.6241	124.6085	0.1519
1985	430.18	8062.84	7.49	62.49	3.56	3.93	6.8839	129.0260	0.2598
1986	460.93	8774.21	5.97	63.65	1.86	4.11	7.2417	137.8509	0.3577
1987	486.56	9565.62	5.83	66.03	3.74	2.09	7.3687	144.8678	0.1270
1988	512.11	10374.25	6.67	68.68	4.01	2.66	7.4564	151.0520	0.0877
1989	539.39	11206.62	8.12	71.99	4.82	3.30	7.4926	155.6691	0.0362
1990	568.99	11755.08	7.51	75.88	5.40	2.11	7.4986	154.9167	0.0059

YEAR (1)	AVERAGE ANNUAL RENT (Ten U.S.$) (2)	AVERAGE HOUSE PRICE (Ten U.S. $) (3)	TREASURY BILL RATE (4)	CONSUMER PRICE INDEX (5)	INFLATION (6)	REAL TREASURY BILL RATE (7)=(4)-(6)	REAL AVERAGE ANNUAL RENT (8)=(2)/(5)	REAL AVERAGE HOUSE PRICE (9)=(3)/(5)	CHANGE IN REAL AVERAGE ANNUAL RENT (10)
1991	590.68	11996.97	5.41	79.09	4.23	1.18	7.4685	151.6876	-0.0301
1992	606.62	12250.85	3.46	81.48	3.02	0.44	7.4450	150.3540	-0.0234
1993	622.13	12508.96	3.02	83.89	2.96	0.06	7.4160	149.1114	-0.0291
1994	639.26	12790.11	4.27	86.08	2.61	1.66	7.4263	148.5840	0.0104
1995	656.48	13185.69	5.51	88.49	2.80	2.71	7.4187	149.0077	-0.0077
1996	675.76	13669.13	5.02	91.09	2.94	2.08	7.4186	150.0618	-0.0001
1997	697.53	14203.69	5.07	93.22	2.34	2.73	7.4826	152.3675	0.0641
1998	722.24	14882.98	4.82	94.66	1.54	3.28	7.6298	157.2257	0.1472
1999	746.65	15670.97	4.66	96.73	2.19	2.47	7.7189	162.0073	0.0891
2000	776.23	16725.98	5.84	100	3.38	2.46	7.7623	167.2598	0.0434
2001	813.44	17954.08	3.45	102.83	2.83	0.62	7.9105	174.5996	0.1483
2002	846.40	19138.33	1.61	104.46	1.59	0.02	8.1026	183.2120	0.1920
2003	872.95	20486.61	1.01	106.83	2.27	-1.26	8.1714	191.7683	0.0688
2004	899.10	22525.03	1.38	109.69	2.68	-1.30	8.1967	205.3517	0.0254
MEAN	392.290	8052.816	5.64	-	4.28	1.34	6.513	128.865	0.078
S.D	267.399	6044.943	2.71	-	2.94	2.02	1.052	30.848	0.096

Noted: The change in real average annual rent at time t = real average annual rent at time t – real average annual rent at time t-1.

APPENDIX C

MATLAB ALGORITHMS

The following appendix generally provides MATLAB algorithms applied to solve the explicit solution for resale option and numerical study on policy simulations. Moreover, we also provide the MATLAB algorithms used to simulate the optimal building option price which has the characteristics similar to American option.

In order to consistent with chapter 6, this appendix C will organize into 2 parts. The first part is the MATLAB algorithms which are written for the analytical method (closed-form solution) to find the optimal resale option and to study the policy simulation when some parameters are vary in the model.

The second part is the Monte Carlo simulation and the Finite Difference Method (FDM) codes which are written to approximate the optimal stopping time for building option.

C.1. Algorithm for the optimal resale option

Due to chapter 6, we can employ the explicit solution (closed-form solution) to find the optimal resale option. In order to accomplish this task, we firstly write the codes for function $h(x)$.

Here are the codes for function $h(x)$:

The Codes for Function $h(x)$

%%

Step1: Write an M-file for function an.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function an=pochhammer1(n,r_rate,rho)
%a(n)=a(a+1)(a+2)(a+3)...(a+n-1)
a=r_rate/(2*rho);
if n==1
    an=a;
```

```
else
   an=pochhammer1(n-1,r_rate,rho)*(a+n-1);
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Step2: Write an M-file for function bn.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function bn=pochhammer2(n)
%b(n)=b(b+1)(b+2)(b+3)...(b+n-1)
b=0.5;
if n==1
  bn=b;
else
  bn=pochhammer2(n-1)*(b+n-1);
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Step3: Write an M-file for function cn.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function cn=pochhammer3(n,r_rate,rho)
%c(n)=c(c+1)(c+2)(c+3)...(c+n-1)
a=r_rate/(2*rho);
b=0.5;
c=1+a-b;
if n==1
   cn=c;
else
   cn=pochhammer3(n-1,r_rate,rho)*(c+n-1);
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Step4: Write an M-file for function dn.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function dn=pochhammer4(n)
%d(n)=d(d+1)(d+2)(d+3)...(d+n-1)
b=0.5;
d=2-b;
if n==1
   dn=d;
else
   dn=pochhammer4(n-1)*(d+n-1);
```

```
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Step5: Write an M-file for function series1.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function series1=kummer1(n,r_rate,rho,x,phi,sigma_g)
%Series1 is the kummer function M(a,b,y)
if n==0
   series1=1;
else
   y=(rho*(x^2))/(sigma_g^2);
   series1=kummer1(n-1,r_rate,rho,x,phi,sigma_g)+...
   (pochhammer1(n,r_rate,rho)*(y^n))/(pochhammer2(n)*factorial(n));
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Step6: Write an M-file for function series2.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function series2=kummer2(n,r,rho,x,phi,sigma_g)
%Series2 is the kummer function M(a,b,y)
if n==0
   series2=1;
else
   y=(rho*(x^2))/(sigma_g^2);
   series2=kummer2(n-1,r,rho,x,phi,sigma_g)+...
   (pochhammer3(n,r,rho)*(y^n))/(pochhammer4(n)*factorial(n));
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Step7: Write an M-file for function U.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function U=kummer3(n,r_rate,rho,x,phi,sigma_g)
% U is the kummer function U(a,b,y)
a=r_rate/(2*rho);
b=0.5;
c=1+a-b;
d=2-b;
y1=(rho*(x^2));
y2=(sigma_g^2);
y=y1/y2;
```

```
U1=(gamma(1+a-b)*gamma(b));
U2=y^(1-b);
U3=(gamma(a)*gamma(d));
U4=kummer1(n,r_rate,rho,x,phi,sigma_g);
U5=kummer2(n,r_rate,rho,x,phi,sigma_g);
U6=U4/U1;
U7=(U2*U5)/U3;
U=pi*(U6-U7);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Step8: Write an M-file for function hvalue.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function hvalue=h(n,r_rate,rho,x,phi,sigma_g)
% hvalue is defined as the value of explicit function h.
if x<=0
   hvalue=kummer3(n,r_rate,rho,x,phi,sigma_g);
else
   a=r_rate/(2*rho);
   part1h=2*(pi);
   part2h=gamma(0.5+(a))*gamma(0.5);
   hvalue=((part1h/part2h)*kummer1(n,r_rate,rho,x,phi,sigma_g))...
   -kummer3(n,r_rate,rho,x,phi,sigma_g);
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

After we get the codes for function $h(x)$, we can also construct the function $g(x)$ as illustrated in equation (6.32). Since q is continuous and continuously differentiable at k^*, taking the value of $q(k^*)$ and the value of first derivative of function $q(x)$ at k^*, we will get equation (6.37) which we can write MATLAB codes to find the optimal k^*.

Here are the codes for the optimal k^*:

The Codes for the Optimal k^*

%%

Step1: Write an M-file for function hbar_plus.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function hbar_plus=hbarplus(n,r_rate,rho,x,phi,sigma_g)
% hbar_plus is defined as the value of first derivative of explicit function h when x is a positive
value.
 a=r_rate/(2*rho);
 b=0.5;
 c=1+a-b;
 d=2-b;
if n==0
    hbar_plus=(pi/(gamma(a)*gamma(2-b)))*(sqrt(rho)/sigma_g);
else
    hbar_plus=hbarplus(n-1,r_rate,rho,x,phi,sigma_g)+...
    (((2*pi)/(gamma(0.5+a)*gamma(0.5)))*(pochhammer1(n,r_rate,rho)/...
     pochhammer2(n))*((rho/sigma_g^2)^n)*(2*n)*(x^(2*n-1))*(1/factorial(n)))...
     -((pi/(gamma(1+a-b)*gamma(b)))*(pochhammer1(n,r_rate,rho)/pochhammer2(n))*...
    ((rho/sigma_g^2)^n)*(2*n)*(x^(2*n-1))*(1/factorial(n)))+...
    (pi/(gamma(a)*gamma(2-b)))*(sqrt(rho)/sigma_g)*...
    (pochhammer3(n,r_rate,rho)/pochhammer4(n))*((rho/(sigma_g^2))^n)*...
     ((2*n)+1)*(x^(2*n))*(1/factorial(n));
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Step2: Write an M-file for function hbar_minus.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function hbar_minus=hbarminus(n,r_rate,rho,x,phi,sigma_g)
% hbar_minus is defined as the value of first derivative of explicit function h when x is a
negative value.
if n==0
    a=r_rate/(2*rho);
    b=0.5;
    c=1+a-b;
    d=2-b;
```

```
    hbar_minus=(pi/(gamma(a)*gamma(2-b)))*(sqrt(rho)/sigma_g);
else
    a=r_rate/(2*rho);
    b=0.5;
    c=1+a-b;
    d=2-b;
    hbar_minus=hbarminus(n-1,r_rate,rho,x,phi,sigma_g)+...
    ((pi/(gamma(1+a-b)*gamma(b)))*(pochhammer1(n,r_rate,rho)/pochhammer2(n))*...
   ((rho/sigma_g^2)^n)*(2*n)*((-x)^(2*n-1))*(1/factorial(n)))+...
    (pi/(gamma(a)*gamma(2-b)))*(sqrt(rho)/sigma_g)*...
    (pochhammer3(n,r_rate,rho)/pochhammer4(n))*((rho/(sigma_g^2))^n)*...
    ((2*n)+1)*((-x)^(2*n))*(1/factorial(n));
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Step3: Write an M-file for function k_star.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function k_star=kstar(n,r_rate,rho,phi,sigma_g,c_2,lamda)
% Find the optimal trading barrier where k_star is represented the optimal trading barrier.
a=r_rate/(2*rho)
b=0.5;
c=1+a-b;
d=2-b;
syms x % to solve the equation
hbar=hbarminus(n,r_rate,rho,x,phi,sigma_g)...
+hbarplus(n,r_rate,rho,x,phi,sigma_g);
h=(-((2*pi)/(gamma(0.5+a)*gamma(0.5))))...
*kummer1(n,r_rate,rho,x,phi,sigma_g)...
+2*kummer3(n,r_rate,rho,x,phi,sigma_g);
k=solve(((x-c_2*(r_rate+lamda))*hbar)+h);
ksolution=double(k);
i=find(abs(imag(ksolution))<eps); %index of real k
k_star=double(k(i));
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Since the model provides the relation among resale option price, resale option price volatility, and share turnover and we have already known that these are endogenous variables. Therefore, their relationship will typically depend on which exogenous variable is shifted.

To analyze the effect of the shift in exogenous, we thus write MATLAB codes for the effect of an increase in real interest rate, the effect of an increase in resale cost, the effect of an increase in overconfidence level, and the effect of an increase in information in signals, respectively on (a) Trading barrier, (b) Bubble, (c) Duration between trades, and (d) Extra volatility component. These codes are illustrated below.

The Codes for the Numerical Analysis

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Step1: Write the codes for the effect of real interest rate on (a) Trading barrier, (b) Bubble, (c) Duration between trades, and (d) Extra volatility component.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Parameters setting
sigma_f=  ;% The volatility of fundamentals.
sigma_d= ;% The volatility of dividend (in this study, it is the volatility of rents).
sigma_s= ;% The volatility of signals.
f_bar= ;% The long-run fundamental.
r_rate=[ ] ;% Vector of real interest rates.
phi= ; % Overconfidence level.
lamda= ; % Mean reverting parameter.
n=; % Number of term/s in pochhammer function.
c_1= ; % The building cost.
c_2= ; % The resale cost.
%Setting rho
rho1=(lamda+(phi*(sigma_f/sigma_s)))^2;
rho2=(1-(phi^2));
rho3=(sigma_f^2)*((2/(sigma_s^2))+(1/(sigma_d^2)));
rho=sqrt(rho1+(rho2*rho3));
%Setting gamma
gamma1=(lamda+(phi*(sigma_f/sigma_s)))^2;
gamma2=(1-(phi^2));
gamma3=(2*((sigma_f^2)/(sigma_s^2)))+((sigma_f^2)/(sigma_d^2));
gamma4=sqrt(gamma1+(gamma2*gamma3));
gamma5=lamda+(phi*(sigma_f/sigma_s));
gamma6=(1/(sigma_d^2))+(2/(sigma_s^2));
```

```
gamma=(gamma4-gamma5)/gamma6;
%Setting sigma_g
sigma_g=(sqrt(2))*phi*sigma_f;
%The effect of an increase in real interest rate level on trading barrier.
for l=1:length(r_rate)
   optimal_k(l)=kstar(n,r_rate(l),rho,phi,sigma_g,c_2,lamda);
end
figure
plot(r_rate,optimal_k)
xlabel('Real rate of interest')
ylabel('Trading barrier')
grid on
title('The effect of real interest rate level on trading barrier')
% The effect of an increase in real interest rate level on the bubble size.
for l=1:length(r_rate)
   optimal_k(l)=kstar(n,r_rate(l),rho,phi,sigma_g,c_2,lamda);
   bubble(l)=(1/(r(l)+lamda))*(h(n,r_rate(l),rho,-optimal_k(l),phi,sigma_g))/...
(hbarplus(n,r_rate(l),rho,optimal_k(l),phi,sigma_g)+hbarminus(n,r_rate(l),rho,optimal_k(l),…
phi,sigma_g));
end
figure
plot(r_rate,bubble)
xlabel('Real rate of interest')
ylabel('The size of bubble')
grid on
title('The effect of interest rate level on the size of bubble')
% The effect of an increase in real interest rate level on the expected duration between
trades.
for l=1:length(r_rate)
   optimal_k(l)=kstar(n,r_rate(l),rho,phi,sigma_g,c_2,lamda);
   duration_between_trades(l)=duration(n,r_rate(l),rho,optimal_k(l),phi,sigma_g);
end
figure
plot(r_rate,duration_between_trades)
xlabel('Real rate of interest')
ylabel('The expected duration between trades')
grid on
title('The effect of interest rate on the expected duration between trades')
```

```
% The effect of an increase in real interest rate level on the extra volatility component.
for l=1:length(r_rate)
    optimal_k(l)=kstar(n,r_rate(l),rho,phi,sigma_g,c_2,lamda);
  extravolatility(l)=(sigma_g/(r_rate(l)+lamda))*…
(hbarplus(n,r_rate(l),rho,optimal_k(l),phi,sigma_g)/(hbarplus(n,r_rate(l),rho,optimal_k(l),…
phi,sigma_g)+ hbarminus(n,r_rate(l),rho,optimal_k(l),phi,sigma_g)));
end
figure
plot(r_rate,extravolatility)
xlabel('Real rate of interest rate')
ylabel('Extra volatility component')
grid on
title('The effect of interest rate level on the extra volatility component')
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Step2: Write the codes for the effect of an increase in resale cost on (a) Trading barrier, (b) Bubble, (c) Duration between trades, and (d) Extra volatility component.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Parameters setting
sigma_f=  ;% The volatility of fundamentals.
sigma_d= ;% The volatility of dividend (in this study, it is the volatility of rents).
sigma_s= ;% The volatility of signals.
f_bar= ;% The long-run fundamental.
r_rate= ;% Real interest rate
phi= ; % Overconfidence level.
lamda= ; % Mean reverting parameter.
n=; % Number of term/s in pochhammer function.
c_1= ; % The building cost.
c_2= [ ] ; % Vector of resale costs.
%Setting rho
rho1=(lamda+(phi*(sigma_f/sigma_s)))^2;
rho2=(1-(phi^2))
rho3=(sigma_f^2)*((2/(sigma_s^2))+(1/(sigma_d^2)));
rho=sqrt(rho1+(rho2*rho3))
%Setting gamma
gamma1=(lamda+(phi*(sigma_f/sigma_s)))^2;
gamma2=(1-(phi^2));
gamma3=(2*((sigma_f^2)/(sigma_s^2)))+((sigma_f^2)/(sigma_d^2));
```

```
gamma4=sqrt(gamma1+(gamma2*gamma3));
gamma5=lamda+(phi*(sigma_f/sigma_s));
gamma6=(1/(sigma_d^2))+(2/(sigma_s^2));
gamma=(gamma4-gamma5)/gamma6;
%Setting sigma_g
sigma_g=(sqrt(2))*phi*sigma_f;
%The effect of an increase in resale cost on trading barrier.
for l=1:length(c_2)
   k_star(l)=kstar(n,r_rate,rho,phi,sigma_g,c_2(l),lamda);
end
figure
plot(c_2,k_star)
xlabel('Resale cost')
ylabel('Trading barrier')
grid on
title('The effect of resale cost on the trading barrier')
% The effect of an increase in resale cost on the bubble size.
for l=1:length(c_2)
   k_star(l)=kstar(n,r_rate,rho,phi,sigma_g,c_2(l),lamda)
   bubble(l)=(1/(r_rate+lamda))*(h(n,r_rate,rho,-k_star(l),phi,sigma_g))/...
(hbarplus(n,r_rate,rho,k_star(l),phi,sigma_g)+hbarminus(n,r_rate,rho,k_star(l),phi,sigma_g))
end
figure
plot(c_2,bubble)
xlabel('Resale cost')
ylabel('The size of bubble')
grid on
title('The effect of resale cost on the size of bubble')
% The effect of an increase in resale cost on the expected duration between trades.
for l=1:length(c_2)
   k_star(l)=kstar(n,r_rate,rho,phi,sigma_g,c_2(l),lamda);
   duration_between_trades(l)=duration(n,r_rate,rho,k_star(l),phi,sigma_g);
end
figure
plot(c_2,duration_between_trades)
xlabel('Resale cost')
ylabel('The expected duration between trades')
grid on
```

```
title('The effect of resale cost on the expected duration between trades')
% The effect of an increase in resale cost on the extra volatility component.
for l=1:length(c_2)
    k_star(l)=kstar(n,r_rate,rho,phi,sigma_g,c_2(l),lamda);
    extravolatility(l)=(sigma_g/(r_rate+lamda))*( hbarplus(n,r_rate,rho,k_star(l),...
    phi,sigma_g)/(hbarplus(n,r_rate,rho,k_star(l),phi,sigma_g)+ ...
    hbarminus(n,r_rate,rho,k_star(l),phi,sigma_g)));
end
figure
plot(c_2,extravolatility)
xlabel('Resale cost')
ylabel('Extra volatility component')
grid on
title('The effect of resale cost on the extra volatility component')
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Step3: Write the codes for the effect of an increase in overconfidence level on (a) Trading barrier, (b) Bubble, (c) Duration between trades, and (d) Extra volatility component.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Parameters setting
sigma_f=  ;% The volatility of fundamentals.
sigma_d= ;% The volatility of dividend (in this study, it is the volatility of rents).
sigma_s= ;% The volatility of signals.
f_bar= ;% The long-run fundamental.
r_rate= ;% Real interest rate
phi=[ ] ; % Vector of overconfidence levels.
lamda= ; % Mean reverting parameter.
n=; % Number of term/s in pochhammer function.
c_1= ; % The building cost.
c_2= ; % The resale cost.
%The effect of an increase in overconfidence level on trading barrier.
for l=1:length(phi)
    rho1(l)=(lamda+(phi(l)*(sigma_f/sigma_s)))^2;
    rho2(l)=(1-(phi(l)^2));
    rho3(l)=(sigma_f^2)*((2/(sigma_s^2))+(1/(sigma_d^2)));
    rho(l)=sqrt(rho1(l)+(rho2(l)*rho3(l)));
    sigma_g(l)=(sqrt(2))*phi(l)*sigma_f;
    optimal_k(l)=kstar(n,r_rate,rho(l),phi(l),sigma_g(l),c_2,lamda);
```

```
end
figure
plot(phi,optimal_k)
xlabel('Overconfidence level')
ylabel('Trading barrier')
grid on
title('The effect of overconfidence level on trading barrier')
```

%The effect of an increase in overconfidence level on the size of bubble.

```
for l=1:length(phi)
    rho1(l)=(lamda+(phi(l)*(sigma_f/sigma_s)))^2;
    rho2(l)=(1-(phi(l)^2));
    rho3(l)=(sigma_f^2)*((2/(sigma_s^2))+(1/(sigma_d^2)));
    rho(l)=sqrt(rho1(l)+(rho2(l)*rho3(l)));
    sigma_g(l)=(sqrt(2))*phi(l)*sigma_f;
    optimal_k(l)=kstar(n,r_rate,rho(l),phi(l),sigma_g(l),c_2,lamda);
    bubble(l)=(1/(r_rate+lamda))*(h(n,r_rate,rho(l),-optimal_k(l),phi(l),sigma_g(l)))/...
(hbarplus(n,r_rate,rho(l),optimal_k(l),phi(l),sigma_g(l))+hbarminus(n,r_rate,rho(l),…
optimal_k(l),phi(l),sigma_g(l)));
end
figure
plot(phi,bubble)
xlabel('Overconfidence level')
ylabel('The size of bubble')
grid on
title('The effect of overconfidence level on the size of bubble')
```

%The effect of an increase in overconfidence level on the expected duration between trades.

```
for l=1:length(phi)
    rho1(l)=(lamda+(phi(l)*(sigma_f/sigma_s)))^2;
    rho2(l)=(1-(phi(1,l)^2));
    rho3(l)=(sigma_f^2)*((2/(sigma_s^2))+(1/(sigma_d^2)));
    rho(l)=sqrt(rho1(l)+(rho2(l)*rho3(l)));
    sigma_g(l)=(sqrt(2))*phi(l)*sigma_f;
    optimal_k(l)=kstar(n,r_rate,rho(l),phi(l),sigma_g(l),c_2,lamda);
    duration_between_trades(l)=duration(n,r_rate,rho(l),optimal_k(l),phi(l),sigma_g(l));
end
plot(phi,duration_between_trades)
xlabel('Overconfidence level')
```

```
ylabel('The expected duration between trades')
grid on
title('The effect of overconfidence level on the expected duration between trades')
%The effect of an increase in overconfidence level on the extra volatility component.
for l=1:length(phi)
    rho1(l)=(lamda+(phi(l)*(sigma_f/sigma_s)))^2;
    rho2(l)=(1-(phi(l)^2));
    rho3(l)=(sigma_f^2)*((2/(sigma_s^2))+(1/(sigma_d^2)));
    rho(l)=sqrt(rho1(l)+(rho2(l)*rho3(l)));
    sigma_g(l)=(sqrt(2))*phi(1,l)*sigma_f;
    optimal_k(l)=kstar(n,r_rate,rho(l),phi(l),sigma_g(l),c_2,lamda);
    extravolatility(l)=(sigma_g(l)/(r_rate+lamda))*( hbarplus(n,r_rate,rho(l),...
    optimal_k(l),phi(l),sigma_g(l))/(hbarplus(n,r_rate,rho(l),optimal_k(l),...
    phi(l),sigma_g(l))+ hbarminus(n,r_rate,rho(l),optimal_k(l),phi(l),sigma_g(l))));
end
figure
plot(phi,extravolatility)
xlabel('Overconfidence level')
ylabel('Extra volatility component')
grid on
title('The effect of overconfidence level on the extra volatility component')
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Step4: Write the codes for the effect of an increase in information causing by a decrease in volatility of signals on (a) Trading barrier, (b) Bubble, (c) Duration between trades, and (d) Extra volatility component.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Parameters setting
sigma_f=  ;% The volatility of fundamentals.
sigma_d= ;% The volatility of dividend (in this study, it is the volatility of rents).
sigma_s= [ ] ;% Vector of  the volatility of signals.
f_bar= ;% The long-run fundamental.
r_rate= ;% Real interest rate.
phi= ; % Overconfidence level.
lamda= ; % Mean reverting parameter.
n=; % Number of term/s in pochhammer function.
c_1= ; % The building cost.
c_2= ; % The resale cost.
```

```
%Setting gamma
gamma1=(lamda+(phi*(sigma_f/sigma_s)))^2;
gamma2=(1-(phi^2));
gamma3=(2*((sigma_f^2)/(sigma_s^2)))+((sigma_f^2)/(sigma_d^2));
gamma4=sqrt(gamma1+(gamma2*gamma3));
gamma5=lamda+(phi*(sigma_f/sigma_s));
gamma6=(1/(sigma_d^2))+(2/(sigma_s^2));
gamma=(gamma4-gamma5)/gamma6;
%The effect of a decrease in volatility of signals on the trading barrier.
for l=1:length(sigma_s)
   rho1(l)=(lamda+(phi*(sigma_f/sigma_s(1,l))))^2;
   rho2(l)=(1-(phi^2));
   rho3(l)=(sigma_f^2)*((2/(sigma_s(1,l)^2))+(1/(sigma_d^2)));
   rho(l)=sqrt(rho1(l)+(rho2(l)*rho3(l)));
   sigma_g(l)=(sqrt(2))*phi*sigma_f;
   optimal_k(l)=kstar(n,r_rate,rho(l),phi,sigma_g(l),c_2,lamda);
end
figure
is=sigma_f./sigma_s
plot(is,optimal_k)
xlabel('Information in signals')
ylabel('Trading barrier')
grid on
title('The effect of information in signals on trading barrier')
% The effect of a decrease in volatility of signals on the size of bubble.
for l=1:length(sigma_s)
   rho1(l)=(lamda+(phi*(sigma_f/sigma_s(1,l))))^2;
   rho2(l)=(1-(phi^2));
   rho3(l)=(sigma_f^2)*((2/(sigma_s(1,l)^2))+(1/(sigma_d^2)));
   rho(l)=sqrt(rho1(l)+(rho2(l)*rho3(l)));
   sigma_g(l)=(sqrt(2))*phi*sigma_f;
   optimal_k(l)=kstar(n,r_rate,rho(l),phi,sigma_g(l),c_2,lamda);
   bubble(l)=(1/(r_rate+lamda))*(h(n,r_rate,rho(l),-optimal_k(l),phi,sigma_g(l)))/...
(hbarplus(n,r_rate,rho(l),optimal_k(l),phi,sigma_g(l))+hbarminus(n,r_rate,rho(l),…
optimal_k(l),phi,sigma_g(l)));
end
figure
is=sigma_f./sigma_s
```

```
plot(is,bubble)
xlabel('Information in signals')
ylabel('The size of bubble')
grid on
title('The effect of information in signals on the size of bubble')
```

% The effect of a decrease in volatility of signals on the expected duration between trades

```
for l=1:length(sigma_s)
   rho1(l)=(lamda+(phi*(sigma_f/sigma_s(1,l))))^2;
   rho2(l)=(1-(phi^2));
   rho3(l)=(sigma_f^2)*((2/(sigma_s(1,l)^2))+(1/(sigma_d^2)));
   rho(l)=sqrt(rho1(l)+(rho2(l)*rho3(l)));
   sigma_g(l)=(sqrt(2))*phi*sigma_f;
   optimal_k(l)=kstar(n,r_rate,rho(l),phi,sigma_g(l),c_2,lamda);
   duration_between_trades(l)=duration(n,r_rate,rho(l),optimal_k(l),phi,sigma_g(l));
end
is=sigma_f./sigma_s
plot(is,duration_between_trades)
xlabel('Information in signals')
ylabel('The expected duration between trades')
grid on
title('The effect of information in signals on the expected duration between trades')
```

% The effect of a decrease in volatility of signals on the extra volatility component

```
for l=1:length(sigma_s)
   rho1(l)=(lamda+(phi*(sigma_f/sigma_s(1,l))))^2;
   rho2(l)=(1-(phi^2));
   rho3(l)=(sigma_f^2)*((2/(sigma_s(1,l)^2))+(1/(sigma_d^2)));
   rho(l)=sqrt(rho1(l)+(rho2(l)*rho3(l)));
   sigma_g(l)=(sqrt(2))*phi*sigma_f;
   optimal_k(l)=kstar(n,r_rate,rho(l),phi,sigma_g(l),c_2,lamda);
   extravolatility(l)=(sigma_g(l)/(r_rate+lamda))*(hbarplus(n,r_rate,rho(l),...
   optimal_k(l),phi,sigma_g(l))/(hbarplus(n,r_rate,rho(l),optimal_k(l),...
   phi,sigma_g(l))+hbarminus(n,r_rate,rho(l),optimal_k(l),phi,sigma_g(l))));
end
figure
is=sigma_f./sigma_s
plot(is,extravolatility)
xlabel('Information in signals')
```

```
ylabel('Extra volatility component')
grid on
title('The effect of information in signals on the extra volatility component')
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Step5: Write the codes for the effect of an increase in information causing by an increase in volatility of fundamentals on (a) Trading barrier, (b) Bubble, (c) Duration between trades, and (d) Extra volatility component.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Parameters setting
sigma_f= [ ]  ;% Vector of volatility of fundamentals.
sigma_d= ;% The volatility of dividend (in this study, it is the volatility of rents).
sigma_s= ;% The volatility of signals.
f_bar= ;% The long-run fundamental.
r_rate= ;% Real interest rate.
phi= ; % Overconfidence level.
lamda= ; % Mean reverting parameter.
n=; % Number of term/s in pochhammer function.
c_1= ; % The building cost.
c_2= ; % The resale cost.
%Setting gamma
gamma1=(lamda+(phi*(sigma_f/sigma_s)))^2;
gamma2=(1-(phi^2));
gamma3=(2*((sigma_f^2)/(sigma_s^2)))+((sigma_f^2)/(sigma_d^2));
gamma4=sqrt(gamma1+(gamma2*gamma3));
gamma5=lamda+(phi*(sigma_f/sigma_s));
gamma6=(1/(sigma_d^2))+(2/(sigma_s^2));
gamma=(gamma4-gamma5)/gamma6;
%The effect of an increase in volatility of fundamentals on the trading barrier.
for l=1:length(sigma_f)
   rho1(l)=(lamda+(phi*(sigma_f(1,l)/sigma_s)))^2;
   rho2(l)=(1-(phi^2));
   rho3(l)=(sigma_f(1,l)^2)*((2/(sigma_s^2))+(1/(sigma_d^2)));
   rho(l)=sqrt(rho1(l)+(rho2(l)*rho3(l)));
   sigma_g(l)=(sqrt(2))*phi*sigma_f(1,l);
   optimal_k(l)=kstar(n,r_rate,rho(l),phi,sigma_g(l),c_2,lamda);
end
figure
is=sigma_f./sigma_s
```

```
plot(is,optimal_k)
xlabel('Information in signals')
ylabel('Trading barrier')
grid on
title('The effect of information in signals on trading barrier')
```

% The effect of an increase in volatility of fundamentals on the size of bubble.

```
for l=1:length(sigma_f)
    rho1(l)=(lamda+(phi*(sigma_f(1,l)/sigma_s)))^2;
    rho2(l)=(1-(phi^2));
    rho3(l)=(sigma_f(1,l)^2)*((2/(sigma_s^2))+(1/(sigma_d^2)));
    rho(l)=sqrt(rho1(l)+(rho2(l)*rho3(l)));
    sigma_g(l)=(sqrt(2))*phi*sigma_f(1,l);
    optimal_k(l)=kstar(n,r_rate,rho(l),phi,sigma_g(l),c_2,lamda);
    bubble(l)=(1/(r_rate+lamda))*(h(n,r_rate,rho(l),-optimal_k(l),phi,sigma_g(l)))/...
(hbarplus(n,r_rate,rho(l),optimal_k(l),phi,sigma_g(l))+hbarminus(n,r_rate,rho(l),optimal_k(l),…
phi,sigma_g(l)));
end
figure
is=sigma_f./sigma_s
plot(is,bubble)
xlabel('Information in signals')
ylabel('The size of bubble')
grid on
title('The effect of information in signals on the size of bubble')
```

% The effect of an increase in volatility of fundamentals on the expected duration between trades.

```
for l=1:length(sigma_f)
    rho1(l)=(lamda+(phi*(sigma_f(1,l)/sigma_s)))^2;
    rho2(l)=(1-(phi^2));
    rho3(l)=(sigma_f(1,l)^2)*((2/(sigma_s^2))+(1/(sigma_d^2)));
    rho(l)=sqrt(rho1(l)+(rho2(l)*rho3(l)));
    sigma_g(l)=(sqrt(2))*phi*sigma_f(1,l);
    optimal_k(l)=kstar(n,r_rate,rho(l),phi,sigma_g(l),c_2,lamda);
    duration_between_trades(l)=duration(n,r_rate,rho(l),optimal_k(l),phi,sigma_g(l));
end
is=sigma_f./sigma_s
plot(is,duration_between_trades)
xlabel('Information in signals')
```

```
ylabel('The expected duration between trades')
grid on
title('The effect of information in signals on the expected duration between trades')
```

% The effect of an increase in volatility of fundamentals on the extra volatility component.

```
for l=1:length(sigma_f)
    rho1(l)=(lamda+(phi*(sigma_f(1,l)/sigma_s)))^2;
    rho2(l)=(1-(phi^2));
    rho3(l)=(sigma_f(1,l)^2)*((2/(sigma_s^2))+(1/(sigma_d^2)));
    rho(l)=sqrt(rho1(l)+(rho2(l)*rho3(l)));
    sigma_g(l)=(sqrt(2))*phi*sigma_f(1,l);
    optimal_k(l)=kstar(n,r_rate,rho(l),phi,sigma_g(l),c_2,lamda);
    extravolatility(l)=(sigma_g(l)/(r_rate+lamda))*(hbarplus(n,r_rate,rho(l),...
    optimal_k(l),phi,sigma_g(l))/(hbarplus(n,r_rate,rho(l),optimal_k(l),...
    phi,sigma_g(l))+hbarminus(n,r_rate,rho(l),optimal_k(l),phi,sigma_g(l))));
end
figure
is=sigma_f./sigma_s
plot(is,extravolatility)
xlabel('Information in signals')
ylabel('Extra volatility component')
grid on
title('The effect of information in signals on the extra volatility component')
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Note that the codes for the expected duration between trades are:

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function expected_duration=duration1(n,r_rate,rho,x,phi,sigma_g)
%---------function expected duration------------
expected_duration=h(n,r_rate,rho,-x,phi,sigma_g)/h(n,r_rate,rho,x,phi,sigma_g);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function durationstar=duration(n,r_rate,rho,x,phi,sigma_g)
%------- function value of duration---------------------------
durationstar=log(duration1(n,r_rate,rho,x,phi,sigma_g))/(-r_rate);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

After we have already written codes for analyzing the policy simulations on resale option, in the next step, we turn to write codes for studying the policy simulations on building option and the optimal stopping time to develop land to be building.

Based on the characteristics of the building option which has no closed form solution, we therefore start with codes to solve for the value of building option by using Finite Difference Method (FDM). These codes are provided below:

The Codes for Solving the Value of Building Option
by Using Finite Difference Method (FDM)

%%%

Step1: Write an M-file for valuating the immediate gain from developing land to be building. However, this gain is composed of two components. One of the two components is the value of resale option. We therefore firstly write an M-file for the resale option value.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function  resale_option=resale(n,r_rate,rho,x,phi,sigma_g,lamda,k_star,bubble_value,c_2)
% the function value of the resale option.
if x<k_star
    resale_option=(bubble_value*h(n,r_rate,rho,x,phi,sigma_g))/h(n,r_rate,rho,…
-k_star,phi,sigma_g);
else
    resale_option=(x/(r_rate+lamda))+((bubble_value*h(n,r_rate,rho,…
-x,phi,sigma_g))/h(n,r_rate,rho,-k_star,phi,sigma_g))-c_2;
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Step2: Next, we write an M-file for valuating the immediate gain from developing land to be building.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
Function total_gain=gain(T,fmax,gmax,dt,df,dg,sigma_f,sigma_d,sigma_s,delta_f,delta_g,
f_bar,r_rate,phi,lamda,n,Ra,c_1,c_2,rho_s_x,g_adjust)
%rho=; % The drift term of the difference in beliefs.
rho1=(lamda+(phi*(sigma_f/sigma_s)))^2;
rho2=(1-(phi^2));
rho3=(sigma_f^2)*((2/(sigma_s^2))+(1/(sigma_d^2)));
rho=sqrt(rho1+(rho2*rho3));
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%gamma=;  % The stationary variance.
gamma1=(lamda+(phi*(sigma_f/sigma_s)))^2;
gamma2=(1-(phi^2));
gamma3=(2*((sigma_f^2)/(sigma_s^2)))+((sigma_f^2)/(sigma_d^2));
gamma4=sqrt(gamma1+(gamma2*gamma3));
gamma5=lamda+(phi*(sigma_f/sigma_s));
gamma6=(1/(sigma_d^2))+(2/(sigma_s^2));
gamma=(gamma4-gamma5)/gamma6;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%sigma_g=;% The volatility of the difference in beliefs.
sigma_g=(sqrt(2))*phi*sigma_f;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%sigma_conditional f=;  %  The volatility of the conditional mean of the beliefs of agents.
w1=((phi*sigma_s*sigma_f)+gamma)/sigma_s;
w2=gamma/sigma_s;
w3=gamma/sigma_d;
w=(w1^2)+(w2^2)+(w3^2);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Setting up grid and adjust increments if necessary.
N_f=round(fmax/df);
df=(fmax/N_f);
N_g=round(gmax/dg);
dg=(gmax/N_g);
N_t=round(T/dt);
dt=(T/N_t);
matval=zeros(N_f*(2*N_g),N_t);
vetf=linspace(df,fmax,N_f);
veti=vetf/df;
vetg=linspace(dg,(2*gmax),(2*N_g));% Using 2*gmax in order to cover [-gmax,gmax]
vetj=vetg/dg;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
for i=1:length(veti)
   for j=1:length(vetj)
      veti_adjust(i,j)=i;  % The matrix which each element in row i is equal to i for all i
=1,...,N_x.
   end
end
```

```
veti_adjust=veti_adjust'; % The transpose matrix veti_adjust.
veti_final=veti_adjust(:); % The column vector which comes from the matrix veti_adjust.
for i=1:length(veti)
   for j=1:length(vetj)
      vetj_adjust(i,j)=j;  %The matrix which each element in row i is equal to j for all j =
1,2,...,N_y.
   end
end
vetj_adjust=vetj_adjust'; % The transpose matrix vetj_adjust.
vetj_final=vetj_adjust(:);   % The column vector which comes from the matrix vetj_adjust.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Setting the value of the conditional mean of the beliefs of agents
%in group A and the difference in beliefs
gain_1=zeros(N_f*(2*N_g),1);
d1=vetj_final*dg;            % The value of the difference in beliefs.
gain_2=zeros(N_f*(2*N_g),1);
d2=veti_final*df;            % The value of the conditional mean of the beliefs of agents.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
k_star=kstar(n,r_rate,rho,phi,sigma_g,c_2,lamda) % The resale option trading barrier.
bubble_value=(1/(r_rate+lamda))*h(n,r_rate,rho,k_star,phi,sigma_g)/((hbarplus(n,r_rate,…
rho,k_star,phi,sigma_g)+hbarminus(n,r_rate,rho,k_star,phi,sigma_g)));
for j=1:length(gain_1)
gain_1(j,1)=resale(n,r_rate,rho,((g_adjust)+d1(j,1)),phi,sigma_g,lamda,k_star,bubble_value…
,c_2); % The value of resale option
end
for i=1:length(gain_2)
    gain_2(i,1) =((f_bar-Ra)/r_rate)+(((d2(i,1))-f_bar)/(r_rate+lamda))-c_1;  % The value of
option from the fundamentals.
end
total_gain=gain_1+gain_2;       % The value of immediate gain.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Finally, we solve the partial differential equation to find the value of building option by using Finite Difference Method.

%%

Step3: Write an M-file for solving the value of building option

%%

```
function
value_FDM=FDM(T,fmax,gmax,dt,df,dg,sigma_f,sigma_d,sigma_s,delta_f,delta_g,f_bar,r_rat
e,phi,lamda,n,Ra,c_1,c_2,rho_s_x,g_adjust)
%Solving the value of building option.
%The definition of parameters.
%T=;                %The terminal time.
%fmax=;             %The maximum value of the conditional mean of the beliefs of agents in
group A.
%gmax=;             %The maximum value of the difference in beliefs.
%dt=;               %The change in time.
%df=;               % The change in conditional mean of the beliefs of agents in group A.
%dg=;               % The change in the difference in beliefs.
%sigma_f=;          %The volatility of fundamental variable.
%sigma_d=;          %The volatility of rental variable.
%sigma_s=;          %The volatility of signals.
%f_bar=;            %The long-run mean of fundamental value.
%r_rate=;           %The real interest rate.
%phi=;              %Overconfidence parameter.
%lamda=;            %Mean reversion parameter.
%n=;                % Term/s in kummer function.
%Ra=;               %The real return on vacant land.
%c_1=;              %The building cost.
%c_2=;              %The resale cost.
%delta_f=;          % The risk-adjust discount rate for the conditional mean of the beliefs of
agents in group A
%delta_g=;          % The risk-adjust discount rate for the difference in beliefs.
%rho_s_x=;          %  The correlation between the conditional mean of the beliefs of agents in
group A and the difference in beliefs.
%g_adjust=;         %Using g_adjust to adjust the value of the difference in beliefs.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%rho=;              %The drift term of the difference in beliefs.
rho1=(lamda+(phi*(sigma_f/sigma_s)))^2;
rho2=(1-(phi^2));
rho3=(sigma_f^2)*((2/(sigma_s^2))+(1/(sigma_d^2)));
rho=sqrt(rho1+(rho2*rho3));
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%gamma=;            % The stationary variance.
gamma1=(lamda+(phi*(sigma_f/sigma_s)))^2;
```

```
gamma2=(1-(phi^2));
gamma3=(2*((sigma_f^2)/(sigma_s^2)))+((sigma_f^2)/(sigma_d^2));
gamma4=sqrt(gamma1+(gamma2*gamma3));
gamma5=lamda+(phi*(sigma_f/sigma_s));
gamma6=(1/(sigma_d^2))+(2/(sigma_s^2));
gamma=(gamma4-gamma5)/gamma6;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%sigma_g=;          %The volatility of the difference in beliefs.
sigma_g=(sqrt(2))*phi*sigma_f;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%sigma_conditional f=;  % The volatility of the conditional mean of the beliefs of agents in
group A.
w1=((phi*sigma_s*sigma_f)+gamma)/sigma_s;
w2=gamma/sigma_s;
w3=gamma/sigma_d;
w=(w1^2)+(w2^2)+(w3^2);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Setting up grid and adjust increments if necessary.
N_f=round(fmax/df);
df=(fmax/N_f);
N_g=round(gmax/dg);
dg=(gmax/N_g);
N_t=round(T/dt);
dt=(T/N_t);
matval=zeros(N_f*(2*N_g),N_t);
vetf=linspace(df,fmax,N_f);
veti=vetf/df;
vetg=linspace(dg,(2*gmax),(2*N_g));% Using 2*gmax in order to cover[-gmax,gmax].
vetj=vetg/dg;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
for i=1:length(veti)
   for j=1:length(vetj)
      veti_adjust(i,j)=i;
   end
end
veti_adjust=veti_adjust';
veti_final=veti_adjust(:);
for i=1:length(veti)
```

```
   for j=1:length(vetj)
       vetj_adjust(i,j)=j;
   end
end
vetj_adjust=vetj_adjust';
vetj_final=vetj_adjust(:);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Setting the value of the conditional mean of the beliefs of agents
%in group A and the differences in beliefs.
gain_1=zeros(N_f*(2*N_g),1);
d1=vetj_final*dg;           % The value of the difference in beliefs.
gain_2=zeros(N_f*(2*N_g),1);
d2=veti_final*df;           % The value of the conditional mean of the beliefs of agents.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
k_star=kstar(n,r_rate,rho,phi,sigma_g,c_2,lamda);
bubble_value=(1/(r_rate+lamda))*h(n,r_rate,rho, -k_star,phi,sigma_g)/…
((hbarplus(n,r_rate,rho,k_star,phi,sigma_g)+hbarminus...
(n,r_rate,rho,k_star,phi,sigma_g)));
for j=1:length(gain_1)
   gain_1(j,1)=resale(n,r_rate,rho,((-g_adjust)+d1(j,1)),phi,sigma_g,lamda,k_star,…
bubble_value,c_2);
end
for i=1:length(gain_2)
    gain_2(i,1) =((f_bar-Ra)/r_rate)+(((d2(i,1))-f_bar)/(r_rate+lamda))-c_1;
end
total_gain=gain_1+gain_2;       % The value of immediate gain.
matval(:,N_t)=max(total_gain,0); %  The value of building option at time T.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Setting the coefficient in matrix M1
A1=(rho_s_x)*(sqrt(w))*sigma_g;
A2=2*df*dg;
A3=A1/A2;
A=ones(N_f*(2*N_g),1)*A3;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
B1=1/(dg^2);
B2=0.5*(sigma_g^2);
B3=(rho_s_x)*(sqrt(w)*sigma_g*dg)/(2*df);
B4=((r_rate-(delta_g-rho))*(-g_adjust+(vetj_final*dg))/(2*dg));
```

```
B=(B1*(B2-B3))-B4;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
C1=1/(df^2);
C2=w/2;
C3=(rho_s_x)*(sqrt(w)*sigma_g*df)/(2*dg);
C4=f_bar-(veti_final*df);
C5=C4./(veti_final*df);
C6=(r_rate-(delta_f-lamda*(C5))).*(veti_final*df);
C7=C6/(2*df);
C=(C1*(C2-C3))-C7;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
D1=-2/(df^2);
D2=w/2;
D3=(rho_s_x)*(sqrt(w)*sigma_g*df)/(2*dg);
D4=2/(dg^2);
D5=(sigma_g^2)/2;
D6=(rho_s_x)*(sqrt(w)*sigma_g*dg)/(2*df);
D7=(rho_s_x)*(sqrt(w)*sigma_g)/(df*dg);
D8=(D1*(D2-D3))-(D4*(D5-D6))-D7-r_rate;
D=ones(N_f*(2*N_g),1)*D8;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
E1=1/(df^2);
E2=w/2;
E3=(rho_s_x)*(sqrt(w)*sigma_g*df)/(2*dg);
E4=C7;
E=(E1*(E2-E3))+E4;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%F
F1=1/(dg^2);
F2=(sigma_g^2)/2;
F3=(rho_s_x)*(sqrt(w)*sigma_g*dg)/(2*df);
F4=((r_rate-(delta_g-rho))*(-g_adjust+(vetj_final*dg))/(2*dg));
F=(F1*(F2-F3))+F4;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
G1=(rho_s_x)*sqrt(w)*sigma_g;
G2=G1/(2*df*dg);
G=ones(N_f*(2*N_g),1)*G2;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Constructing the matrix M1.
```

```
M1=-diag(A(1:(N_f*2*N_g)-141),-141)-diag(C(1:(N_f*2*N_g)-140),-140)-…
diag(B(1:(N_f*2*N_g)-1),-1)...
   -diag(D(1:(N_f*2*N_g)))-diag(F(2:(N_f*2*N_g)),1)-diag(E(141:(N_f*2*N_g)),140)-…
diag(G(142:(N_f*2*N_g)),141);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Adjusting the matrix M.
%VectorA
for i=2:29
   M1((i*140)+1,(i-1)*140)=0;
end
M1(4200,4060-1)=A(4060-1,1)+G(4060-1,1);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%VectorC
for i=1:140
   M1(4060+i,3920+i)=C(3920+i,1)+E(3920+i,1);
end
for i=1:139
   M1(4060+i,3920+i+1)=G(3920+i+1,1);
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%VectorB
for i=1:29
   M1((i*140)+1,i*140)=0;
end
for i=1:30
   M1(i*140,(i*140)-1)=B((i*140)-1,1)+F((i*140)-1,1);
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%VectorF
for i=1:29
   M1(i*140,(i*140)+1)=0;
end
%VectorE
for i=1:29
   M1(i*140,((i+1)*140)-1)=G(((i+1)*140)-1,1);
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%VectorG
```

```
for i=1:28
    M1(i*140,((i+1)*140)+1)=0;
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Adding vectorM2
M2=zeros(N_f*2*N_g,1)
M2(1,1)= A(1,1)*max((resale(n,r_rate,rho,(-g_adjust),phi,sigma_g,lamda,k_star,…
bubble_value,c_2)+((f_bar-Ra)/r_rate)+(((0)-f_bar)/(r_rate+lamda))-c_1),0)+…
B(1,1)*max((resale(n,r_rate,rho,(-g_adjust),phi,sigma_g,lamda,k_star,…
bubble_value,c_2)+((f_bar-Ra)/r_rate)+(((df)-f_bar)/(r_rate+lamda))-c_1),0)…
+C(1,1)*max((resale(n,r_rate,rho,(-g_adjust+dg),phi,sigma_g,lamda,k_star,…
bubble_value,c_2)+((f_bar-Ra)/r_rate)+(((0)-f_bar)/(r_rate+lamda))-c_1),0);
for i=2:140
    M2(i,1)=A(i,1)*max((resale(n,r_rate,rho,(-g_adjust+(i*dg)),phi,sigma_g,lamda,k_star,…
bubble_value,c_2)+((f_bar-Ra)/r_rate)+(((0)-f_bar)/(r_rate+lamda))-c_1),0)+C(2,1)*…
max((resale(n,r_rate,rho,(-g_adjust+(i*2*dg)),phi,sigma_g,lamda,k_star,bubble_value,c_2)…
+((f_bar-Ra)/r_rate)+(((0)-f_bar)/(r_rate+lamda))-c_1),0);
end
for i=1:29
    M2((i*140)+1,1)=A((i*140)+1,1)*max((resale(n,r_rate,rho,(-g_adjust),phi,sigma_g,…
lamda,k_star,bubble_value,c_2)+((f_bar-Ra)/r_rate)+(((1*df)-f_bar)/(r_rate+lamda))-…
c_1),0)+B((i*140)+1,1)*max((resale(n,r_rate,rho,(-g_adjust),phi,sigma_g,lamda,k_star,…
bubble_value,c_2)+((f_bar-Ra)/r_rate)+(((2*df)-f_bar)/(r_rate+lamda))-c_1),0);
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
L_H=(1/(dt))*eye(N_f*(2*N_g))+M1;
R_H=(1/(dt))*eye(N_f*(2*N_g));
[L,U]=lu(L_H);
%Solving the sequence of linear systems.
for l=N_t-1:-1:1
    matval1(1:(N_f*2*N_g),l)=(U\(L\(R_H*matval(1:(N_f*2*N_g),l+1)+M2)));
    matval2(1:(N_f*2*N_g),l)=max(total_gain(1:(N_f*2*N_g),1),0);
    matval(1:(N_f*2*N_g),l)=max(matval1(1:(N_f*2*N_g),l),matval2(1:(N_f*N_g*2),l));
end
value_FDM=matval;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

When we have already solved the value of building option, we then apply the Monte Carlo technique to generate the conditional mean of the beliefs of agents in group A and group B. The codes for generating these paths are presented below:

The Codes for Generating the Conditional Mean of the Beliefs of Agents in Group A and B by Using Monte Carlo Technique

%%

Step1: Write an M-file for generating the conditional mean of the beliefs of agents in group A.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function
spaths_conditionalf_A=brownianA(T,sigma_d,N_sim,N,sigma_f,sigma_s,lamda,f_bar,phi,con
ditionalf0_A)
%Simulating the value of the conditional mean of the beliefs of agents in group A
%The definition of parameters
%T=;                    %The terminal time T.
%sigma_d=;              %The volatility of rental variable.
%N_sim=;                %Number of simulation.
%N=;                    %Number of time steps.
delt=T/N;               %Time steps.
randn('seed',0);        % Generate the same normally distributed random numbers.
%sigma_f=;              %The volatility of fundamental variable.
%lamda=;                %Mean reversion parameter.
%f_bar=;                %The long-run mean of fundamental value.
%phi=;                  %Overconfidence parameter.
%sigma_s=;              %The volatility of signals.
%conditionalf0_A=;   %Initial value of the conditional mean of the beliefs of agents in group A.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%gamma=; The stationary variance.
gamma1=(lamda+(phi*(sigma_f/sigma_s)))^2;
gamma2=(1-(phi^2));
gamma3=(2*((sigma_f^2)/(sigma_s^2)))+((sigma_f^2)/(sigma_d^2));
gamma4=sqrt(gamma1+(gamma2*gamma3));
gamma5=lamda+(phi*(sigma_f/sigma_s));
```

```
gamma6=(1/(sigma_d^2))+(2/(sigma_s^2));
gamma=(gamma4-gamma5)/gamma6;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%sigma_g=; The volatility of the difference in beliefs
sigma_g=(sqrt(2))*phi*sigma_f;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
spaths_conditionalf_A=zeros(N_sim,N);          % Generate the matrix size N_sim,1+N where each row represents  path i for all i = 1,…,N_sim.
spaths_conditionalf_A(:,1)=conditionalf0_A;     % The first column represents the initial value of the conditional mean of the beliefs of agents in group A.
for i=1:N_sim
   for j=1:N-1
        dz_A_A(i,j+1)=sqrt(delt)*randn;            %The Weiner term.
        dz_B_A(i,j+1)=sqrt(delt)*randn;           % The Weiner term.
        dz_d_A(i,j+1)=sqrt(delt)*randn;           % The Weiner term.
        drift_conditionalf_A(i,j)=-lamda*(spaths_conditionalf_A(i,j)-f_bar)*delt;  % The drift term.
spaths_conditionalf_A(i,j+1)=spaths_conditionalf_A(i,j)+(drift_conditionalf_A(i,j))+(((phi*…
sigma._s*sigma_f)+gamma)/(sigma_s))*(dz_A_A(i,j+1))+(gamma/(sigma_s))*…
( dz_B_A(i,j+1))+(gamma/(sigma_d))*(dz_d_A(i,j+1));
      end
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Step2: Write an M-file for generating the conditional mean of the beliefs of agents in group B.

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function
spaths_conditionalf_B=brownianB(T,sigma_d,N_sim,N,sigma_f,sigma_s,lamda,f_bar,phi,con
ditionalf0_B)
%Simulating the value of the conditional mean of the beliefs of agents in group A
%The definition of parameters
%T=;                      %The terminal time T.
%sigma_d=;                %The volatility of rental variable.
%N_sim=;                  %Number of simulation.
%N=;                      %Number of time steps
delt=T/N;                 %Time steps.
randn('seed',0);          % Generate the same normally distributed random numbers.
%sigma_f=;                %The volatility of fundamental variable.
%lamda=;                  %Mean reversion parameter.
```

```
%f_bar=;                    %The long-run mean of fundamental value.
%phi=;                      %Overconfidence parameter.
%sigma_s=;                  %The volatility of signals.
%conditionalf0_B=;     %Initial value of conditional mean of the beliefs of agents in group B.
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%gamma=; The stationary variance.
gamma1=(lamda+(phi*(sigma_f/sigma_s)))^2;
gamma2=(1-(phi^2));
gamma3=(2*((sigma_f^2)/(sigma_s^2)))+((sigma_f^2)/(sigma_d^2));
gamma4=sqrt(gamma1+(gamma2*gamma3));
gamma5=lamda+(phi*(sigma_f/sigma_s));
gamma6=(1/(sigma_d^2))+(2/(sigma_s^2));
gamma=(gamma4-gamma5)/gamma6;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%sigma_g=; The volatility of the difference in beliefs.
sigma_g=(sqrt(2))*phi*sigma_f;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
spaths_conditionalf_B=zeros(N_sim,N);              % Generate the matrix size N_sim,1+N
where each row represents  path i for all i = 1,…,N_sim.
spaths_conditionalf_B(:,1)=conditionalf0_B;         % The first column represents the initial
value of the conditional mean of the beliefs of agents in group B.
for i=1:N_sim
   for j=1:N-1
       dz_A_B(i,j+1)=sqrt(delt)*randn;    % The Weiner term.
       dz_B_B(i,j+1)=sqrt(delt)*randn;     % The Weiner term.
       dz_d_B(i,j+1)=sqrt(delt)*randn;       % The Weiner term.
       drift_conditionalf_B(i,j)=-lamda*(spaths_conditionalf_B(i,j)-f_bar)*delt;   % The drift term.
    spaths_conditionalf_B(i,j+1)=spaths_conditionalf_B(i,j)+(drift_conditionalf_B(i,j))+…
(((phi*sigma_s*sigma_f)+gamma)/(sigma_s))*(dz_B_B(i,j+1))+(gamma/(sigma_s))*…
( dz_A_B(i,j+1))+(gamma/(sigma_d))*( dz_d_B(i,j+1));
      end
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Based on one of our objectives, we would like to analyze the policy simulations on optimal stopping time to develop land to be building. Since we have had the value of building option and paths of the conditional mean of the beliefs of agents in group A and B, we then finally write the codes to identify the optimal stopping time for each path of them.

Here are the codes for finding the optimal stopping time to develop land to be building.

The Codes for Finding the Optimal Stopping Time to Develop Land to be Building

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function
time=optimal(T,sigma_d,N_sim,N,sigma_f,sigma_s,lamda,f_bar,phi,conditionalf0_A,condition
alf0_B,df,dg,value_FDM,g_adjust,total_gain)
spaths_conditionalf_A=brownianA(T,sigma_d,N_sim,N,sigma_f,sigma_s,lamda,f_bar,phi,con
ditionalf0_A);
spaths_conditionalf_A1=roundn(spaths_conditionalf_A,-2); % Round number to the nearest
one-hundredth.
spaths_conditionalf_B=brownianB(T,sigma_d,N_sim,N,sigma_f,sigma_s,lamda,f_bar,phi,con
ditionalf0_B);
spaths_conditionalf_B1=roundn(spaths_conditionalf_B,-2); % Round number to the nearest
one-hundredth.
difference=spaths_conditionalf_B-spaths_conditionalf_A;
difference1=roundn(difference,-3); % Round number to the nearest one-thousandth.
for i=1:N_sim
  pathf=spaths_conditionalf_A1(i,:);
  pathg=difference1(i,:);
  for t=1:T
    f(t)=pathf(1,t);
    g(t)=pathg(1,t);
    if f(t)<=0
      indexf(t)=1
    else if f(t)>0.3
        indexf(t)=30
        else
          indexf(t)=round(f(t)/df)
      end
      if g(t)<=-0.7
          indexg(t)=1
```

```
            else if g(t)>0.7
                indexg(t)=140
              else
                indexg(t)=round((g(t)+g_adjust)/dg);
              end
        indexFDM(t)=max(indexg(t)+((indexf(t)-1)*140),1);
        valueoption(t)=value_FDM(indexFDM(t),t);
        immediatelygain(t)=total_gain(indexFDM(t),1);
        if valueoption(t)==immediatelygain(t)
          time(i)=t;,break,
        else
          time(i)=0;
        end
              end
            end
          end
        end
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Finally, to generate the density function of the optimal stopping time, we apply Distribution Fitting Tool (DFITTOOL). DFITTOOL displays a window for fitting distributions to data. We can create a data set by importing data from our workspace, and we can fit distributions and display them over plots of the empirical distribution of the data. Therefore, we apply this tool to fit the density function of the optimal stopping time.

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